

## AS SKILLS CHECKS

Half Term 6 (ANSWERS)	Week 1
<p>1</p> $(x - 2)^2 + (y - 3)^2 = 25$ <p>Centre (2,3)</p> $y = 0$ $(x - 2)^2 + 9 = 25$ $x^2 - 4x - 12 = 0$ $x = 6 \text{ or } x = -2$	$x = 6 \quad y = 0$ $\text{Gradient of normal} = -\frac{3}{4}$ $\text{Gradient of tangent} = \frac{4}{3}$ $3y = 4x - 24$
<p>2</p> $y = 4x^{\frac{3}{2}} + \frac{1}{2}x^{-\frac{1}{2}}$ $\frac{dy}{dx} = 6\sqrt{x} + \frac{1}{4\sqrt{x^3}}$	
<p>3</p> $(x + 3)^2 - 9 + 16$ $(x + 3)^2 + 7$ $\text{Greatest value} = \frac{1}{7}$	
<p>4</p> <p>(-4,4) to (2,6) Gradient = <math>\frac{1}{3}</math></p> <p>(2,6) to (6,-6) Gradient = -3</p> <p>Perpendicular</p> $\text{Area} = \frac{1}{2}(\sqrt{6^2 + 2^2} \times \sqrt{4^2 + 12^2})$ $= 40 \text{ units}^2$	
<p>5</p> $2^4 \times (-3x^3) + 2 \times {}_4C_3 \times 2 \times (3x)^3$ $= 384 x^3$	

$$1 \quad (-k)^2 - 4 \times 4 \times (k - 3) \geq 0$$

$$k^2 - 16k + 48 \geq 0$$

$$k \leq 4 \quad \text{or} \quad k \geq 12$$

$$2 \quad \cos^2 \theta = (\sqrt{2} - 1)^2$$

$$= 3 - 2\sqrt{2}$$

$$\sin^2 \theta = 1 - \cos^2 \theta$$

$$= 1 - (3 - 2\sqrt{2})$$

$$= 2\sqrt{2} - 2$$

$$3 \quad x = 1 \quad y = 5 \quad A(1, 5)$$

$$\frac{dy}{dx} = 1 - \frac{4}{x^2} \quad x = 1 \quad \frac{dy}{dx} = -3$$

*Gradient of normal at A is  $\frac{1}{3}$*

$$\text{Equation of the normal } (y - 5) = \frac{1}{3}(x - 1)$$

$$3y = x + 14$$

$$y = 0 \quad x = -14$$

$$4 \quad \log_a \frac{n^2}{3-n} = \log_a 4$$

$$\frac{n^2}{3-n} = 4$$

$$n^2 + 4n - 12 = 0$$

$$n = 2 \quad (n = -6 \text{ not possible})$$

$$5 \quad \left(\frac{1}{2}\right)^x = 2^{-x}$$

*Reflection in the y - axis*

$$1 \quad 4y = 6x + 2 \quad 4y^2 = 9x^2 + 6x + 1$$

$$9x^2 - 9x^2 - 6x - 1 + 9x - 6x - 2 = 1$$

$$-3x = 4$$

$$x = -\frac{4}{3} \quad y = -\frac{3}{2}$$

$$2 \quad y = \frac{(x-3)^2}{3\sqrt{x}}$$

$$= \frac{x^2 - 6x + 9}{3\sqrt{x}}$$

$$Y = \frac{1}{3}x^{\frac{3}{2}} - 2x^{\frac{1}{2}} + 3x^{-\frac{1}{2}}$$

$$\frac{dy}{dx} = \frac{1}{2}\sqrt{x} - \frac{1}{\sqrt{x}} - \frac{3}{2\sqrt{x^3}}$$

$$3 \quad 9^x - 3(3^{x+1}) = 0$$

$$3^{2x} - 3(3^x \times 3) = 0$$

$$3^{2x} - 9(3^x) = 0$$

$$\text{Let } y = 3^x \quad y^2 - 9y = 0$$

$$y(y - 9) = 0 \quad y = 0 \text{ and } y = 9$$

$$3^x = 0 \quad \text{or} \quad 3^x = 9$$

$$x = 2$$

$$4 \quad \sin x = 1 - 2(1 - \sin^2 x)$$

$$2\sin^2 x - \sin x - 1 = 0$$

$$\sin x = 1 \quad \text{or} \quad \sin x = -\frac{1}{2}$$

$$x = 90^\circ, 210^\circ, 330^\circ$$

$$5 \quad 4x - x^2 = 0$$

$$x = 0 \text{ or } x = 4$$

$$\int_0^4 4x - x^2 dx = \left[ 2x^2 - \frac{1}{3}x^3 \right]$$

$$= 10\frac{2}{3}$$

1  $(x - 4)^2 + (y - 3)^2 = 5$   
Centre (4,3)  
Gradient of normal =  $-\frac{1}{2}$   
Gradient of the tangent = 2  
 $y - 4 = 2(x - 2)$   
 $y = 2x$

2  $2^8 + 8 \times 2^7 \times (-3x) + 28 \times 2^6 \times (-3x)^2$   
  
 $256 - 3072x + 16128x^2$

3  $f(x) = (x + 3)(x + 4)(x - 1)$

4  $\vec{AB} = 6i + 3j$   
 $|\vec{AB}| = \sqrt{6^2 + 3^2}$   
 $= 3\sqrt{5}$

5  $t = 0 \quad 20 = Ae^0$   
 $A = 20$   
 $t = 5 \quad M = 10$   
 $10 = 20e^{-5k}$   
 $e^{-5k} = 0.5$   
 $-5k = \ln(0.5) \quad k = 0.139$

$$1 \quad (2^{-3})^{2x} = (2^4)^{3x-1}$$

$$-6x = 4(3x-1)$$

$$x = \frac{2}{9}$$

$$2 \quad x^2 + 1 = 10$$

$$x = \pm 3$$

$$\int_{-3}^3 10 - x^2 - 1 \, dx = \left[ 9x - \frac{1}{3}x^3 \right]$$

$$= 36$$

$$3 \quad 2\theta = (-45^\circ), 135^\circ, 315^\circ, 495^\circ, 675^\circ$$

$$\theta = 67.5^\circ, 157.5^\circ, 247.5^\circ, 337.5^\circ$$

$$4 \quad y = x + 4 + \frac{4}{x} \quad x = 1 \quad y = 9$$

$$\frac{dy}{dx} = 1 - \frac{4}{x^2}$$

$$\text{Gradient of tangent} = -3$$

$$\text{Gradient of the normal} = \frac{1}{3}$$

$$y - 9 = \frac{1}{3}(x - 1) \quad 3y = x + 26$$

$$5 \quad x \times {}_7C_3 \times (2x)^3 - 4 \times {}_7C_4 \times (2x)^4$$

$$-1960 x^4$$

$$1 \quad \sqrt{13^2 - 5^2} = 12 \quad \cos\theta = \frac{12}{13}$$

$$2 \quad \log_4(n(n+6)) = \log_4 16$$

$$n^2 + 6n = 16$$

$$n^2 + 6n - 16 = 0$$

$$(n+8)(n-2) = 0$$

$$n = -8 \text{ (no solution)}$$

$$n = 2$$

$$3 \quad \frac{(3+\sqrt{2})(\sqrt{2}-1)}{(\sqrt{2}+1)(\sqrt{2}-1)} - \frac{(1+\sqrt{2})}{(1-\sqrt{2})(1+\sqrt{2})}$$

$$-1 + 2\sqrt{2} - (-1 - \sqrt{2})$$

$$= 3\sqrt{2}$$

$$4 \quad \frac{dy}{dx} = 3x^2 + 6x \quad \frac{d^2y}{dx^2} = 6x + 6$$

$$3x^2 + 6x = 0$$

$$3x(x+2) = 0$$

$$x = 0 \text{ or } x = -2$$

$$x = 0 \quad \frac{d^2y}{dx^2} > 0 \text{ minimum}$$

$$(0, 72)$$

$$5 \quad y = \int 3 + \frac{12}{x^4} dx = 3x - \frac{4}{x^3} + c$$

$$x = 1, y = 13 - \frac{4}{1} + c = 1$$

$$c = 2$$

$$f(x) = 3x - \frac{4}{x^3} + 2$$

$$\begin{aligned}
 1 \quad & 4(1 - \cos^2\theta) - 2 = 7\cos\theta \\
 & 2 - 4\cos^2\theta = 7\cos\theta \\
 & 4\cos^2\theta + 7\cos\theta - 2 = 0 \\
 & \cos\theta = \frac{1}{4} \quad \cos\theta = -2 \text{ (no solutions)} \\
 & \theta = -75.5^\circ, 75.5^\circ
 \end{aligned}$$

$$\begin{aligned}
 2 \quad & x^2 + 2 = 2x + 5 \\
 & x^2 - 2x - 3 = 0 \\
 & (x - 3)(x + 1) = 0 \\
 & x = -1 \text{ and } x = 3 \\
 & \int_{-1}^3 (2x + 5 - x^2 - 2) dx = \left[ x^2 + 3x - \frac{1}{3}x^3 \right] = 9 - \left(-1\frac{2}{3}\right) = 10\frac{2}{3}
 \end{aligned}$$

$$\begin{aligned}
 3 \quad & |2i - 4j| = \sqrt{2^2 + 4^2} \\
 & = \sqrt{20} \\
 & 2\sqrt{5}i - 4\sqrt{5}
 \end{aligned}$$

$$\begin{aligned}
 4 \quad & e^{2x-2} = 20 \\
 & 2x - 2 = \ln 20 \\
 & x = \frac{\ln 20 + 2}{2} \\
 & x = 2.498
 \end{aligned}$$

$$\begin{aligned}
 5 \quad & (x + 2)^2 + (y - 2)^3 - 8 = 24 \\
 & \text{Centre of the circle } (-2, 2) \\
 & \text{Gradient of the normal} = -1 \\
 & \text{Gradient of the tangent} = 1 \quad y + 2 = x - 2 \\
 & y = x - 4
 \end{aligned}$$

