Half Term 1 (ANSWERS)	Week 1

1

$$\frac{3\sqrt{3}-2}{2-\sqrt{3}} \times \frac{2+\sqrt{3}}{2+\sqrt{3}} = 5 + 4\sqrt{3}$$

2

$6C4 \times (2x)^4(-1)^6$

240

3

$\int x^{-\frac{1}{2}}\, dx = 2\sqrt{x}$

$= 2\sqrt{2} - 2\sqrt{1}$

$= 2(\sqrt{2} - 1)$

4

Gradient of tangent = 2

$\frac{dy}{dx} = 4 - 2x \qquad 4 - 2x = 2$

$x = 1$

$x = 1 \quad y = 4 - 1 - 1$

$= 2$ *Point of contact at (1,2)*

5

x = 0 y = 8

y = 0 $12e^x - 4e^{2x} = 0$

$4e^x(3 - e^x) = 0$

$4e^x = 0$ (no solutions)

$e^x = 3$

x = ln3 (0, 8) and (ln 3, 0)

Half Term 1 (ANSWERS)	Week 2

1

$(x - a)^2 + (y - b)^2 = 25$

$a^2 + b^2 = 25$ $\quad$ $(8 - a)^2 + b^2 = 25$

$a^2 = 25 - b^2$ $\quad$ $64 - 16a + a^2 + b^2 = 25$

$64 - 16a + 25 - b^2 + b^2 = 25$

$a = 4$

$16 = 25 - b^2$

$b = \pm 3$ $\quad$ (4 , 3) and (4, -3)

2

$sin(2\theta) + 2 = 2\left(1 - sin^2(2\theta)\right)$

$2sin^2(2\theta) + sin(2\theta) = 0$

$sin(2\theta)\,(2\,sin(2\theta) + 1) = 0$

$sin(2\theta) = 0$ $\quad$ $sin(2\theta) = -\frac{1}{2}$

$2\theta = 0°, 180°, 210°, 330°, 360°$

$\theta = 0°, 90°, 105°, 165°, 180°$

3

$\frac{dy}{dx} = 3ax^2 - 3$ $\quad$ x = 2 $\quad$ 12a - 3 = 45

$a = 4$

(2, 15) $\quad$ 15 = 8a - 6 + c

15 = 32 – 6 + c

c = - 11

4

$(x - 1)(ax^2 + bx + c) = 2x^3 - 5x^2 - 6x + 9$

$a = 2$

$c = -9$

Equating the coefficients of x^2 $\quad$ $-a + b = -5$

$b = -3$

$(x - 1)(2x + 3)(x - 3)$

5

$y = (x + 1)^3 + 3$

$y = x^3 + 3x^2 + 3x + 1 + 3$

$= x^3 + 3x^2 + 3x + 4$

Half Term 1 (ANSWERS)	Week 3

1

$\int x^{-2} + 2x^{-3} + x^{-4}\, dx = -x^{-1} - x^{-2} - \frac{1}{3}x^{-3}$

$\left[-\frac{1}{2} - \frac{1}{4} - \frac{1}{24}\right] - \left[-1 - 1 - \frac{1}{3}\right] = 1\frac{13}{24}$

2

$8\left(x^2 + \frac{1}{2}x\right) + 3$

$8\left(x + \frac{1}{4}\right)^2 - \frac{1}{2} + 3$

$8\left(x + \frac{1}{4}\right)^2 + \frac{5}{2}$

3

$f(x) = 8x^3$

$f(x+h) = 8(x+h)^3$

$= 8(x^3 + 3hx^2 + 3h^2x + h^3)$

$= 8x^3 + 24hx^2 + 24h^2x + 8h^3$

$\frac{dy}{dx} = \lim_{h\to 0} \frac{8x^3+24hx^2+24h^2x+8h^3-8x^3}{h}$

$= \lim_{h\to 0}(24x^2 + 24hx + 8h^2)$

$= 24x^2$

4

$\int x^{-2} - x^{-3} dx = -\frac{1}{x} + \frac{1}{x^2} + c$

$x = -1,\ y = 6$

$6 = \frac{1}{1} + \frac{1}{1} + c$

$c = 4 \qquad y = 4 - \frac{1}{x} + \frac{1}{x^2}$

5

$4^{(2x+1)} = \frac{1}{2^x}$

$2^{(4x+2)} = 2^{-x}$

$4x + 2 = -x$

$5x = -2$

$x = -\frac{2}{5}$

1

$b^2 - 4ac > 0$
$(k + 4)^2 - 4k > 0$
$K^2 + 8k + 16 - 4k > 0$
$k^2 + 4k + 16 > 0$
$(k + 2)^2 + 12 > 0$ *for all values of k*

2

$y^2 = 4^2 + 8^2 - 2 \times 4 \times 8 \times cos(60°)$
$y = 4\sqrt{3}$
$\frac{sin\, x}{4} = \frac{sin60}{4\sqrt{3}}$
$sin\, x = ½$

3

$\int 3\sqrt{x} + 2x\, dx = 2x^{\frac{3}{2}} + x^2$
$2 \times 4^{\frac{3}{2}} + 4^2 - 2 \times 1^{\frac{3}{2}} - 1^2$
$= 29$

4

$y = x^2 - x^3$
$\frac{dy}{dx} = 2x - 3x^2$
$x = \frac{1}{2} \quad \frac{dy}{dx} = \frac{1}{4} \quad y = \frac{1}{8}$
$y - \frac{1}{8} = \frac{1}{4}\left(x - \frac{1}{2}\right) \qquad y = \frac{1}{4}x$

5

Translation $\begin{bmatrix} 2 \\ 1 \end{bmatrix}$

Half Term 1 (ANSWERS)	Week 5

1

$\int 5 - 3x\, dx = 5x - \frac{3x^2}{2} (+c)$

$10a - 6a^2 - 5a + \frac{3a^2}{2} = \frac{1}{2}$

$10a - 9a^2 = 1$

$9a^2 - 10a + 1 = 0$

$(a - 1)(9a - 1) = 0$

$a=1 \;\; a = \frac{1}{9}$

2

$\frac{dy}{dx} = 3x^2 - 8x - 35$

$3x^2 - 8x - 35 < 0$

$(3x + 7)(x - 5) < 0$

$-\frac{7}{3} < x < 5$

3

$\frac{\sin(2\theta - 20°)}{\cos(2\theta - 20°)} = \frac{3}{\sqrt{3}}$

$\tan(2\theta - 20°) = \frac{3}{\sqrt{3}}$

$2\theta - 20° = 60°, 240°$

$\theta = 40º, 130º$

4

Centre of circle (5, 4)

Gradient of radius at B $= \frac{4}{3}$

Gradient of tangent at B $= -\frac{3}{4}$

$y - 8 = -\frac{3}{4}(x - 8)$

$3x + 4y = 56$

5

$2^8 + 8C1(2^7)\left(-\frac{1}{4}x\right) + 8\,C2(2^6)(-\frac{1}{4}x)^2$

$256 - 256x + 112x^2$

1

$b^2 - 4ac > 0$

$16k^2 + 12k > 0$

$4k(4k + 3) > 0$

$k < -\frac{3}{4}$ *or* $k > 0$

2

$\overrightarrow{AB} = 4i + 8j$

$|AB| = \sqrt{4^2 + 8^2}$

$= 4\sqrt{5}$

3

$5 - 2cos3\theta = 8(1 - cos^2 3\theta)$
$8cos^2 3\theta - 2cos3\theta - 3 = 0$
$cos3\theta = \frac{3}{4}$ $cos3\theta = -\frac{1}{2}$
$3\theta = 41.4°, 120°, 240°, 318.6°, 401.4°, 480°$
$\theta = 13.8°, 40°, 80°, 106.2°, 133.8°, 160°$

4

$2e^{2x} + 3e^{x} - 2 = 0$

$(2e^x - 1)(e^x + 2) = 0$

$e^x = ½$ $e^{2x} = -2$ *(no solutions)*

$x = ln ½$

5

$f(1) = a - 1 - b + 6$ $a - b = -5$

$f(-2) = -8a - 4 + 2b + 6$ $8a - 2b = 2$ $\underline{4a - b = 1}$

$3a + 0 = 6$

$a = 2$ $b = 7$

Half Term 1 (ANSWERS)	Week 7

1

$12x^2 - 32x + 5 > 0$

$(2x - 5)(6x - 1) > 0$

$x < \frac{1}{6} \quad or \quad x > \frac{5}{2}$

2

$\frac{3\sqrt{2}-1}{\sqrt{2}-1} \times \frac{\sqrt{2}+1}{\sqrt{2}+1} = 5 + 2\sqrt{2}$

3

$1 + 10C1(-2x) + 10C2(-2x)^2 + 10C3(-2x)^3$

$= 1 - 20x + 180x^2 - 960x^3$

4

$y = 3x^{\frac{3}{2}} + 16x^{-1}$

$\frac{dy}{dx} = \frac{9}{2}x^{\frac{1}{2}} - \frac{16}{x^2}$

$x = 4 \quad \frac{dy}{dx} = 8$

$x = 4 \quad y = 28$

$y - 28 = 8(x - 4)$

$y = 8x - 4$

5

$\sqrt{20^2 + 21^2} = \sqrt{29}$

$\frac{20}{\sqrt{29}}i - \frac{21}{\sqrt{29}}j$