

## A2 SKILLS CHECKS

Half Term 2A (ANSWERS)	Week 1
<p>1</p> <p><math>1^{\text{st}} \text{ term} = 5</math></p> <p><math>2^{\text{nd}} \text{ term} = 8</math></p> <p><math>S_{10} = \frac{10}{2}(2 \times 5 + 3(10 - 1))</math></p> <p><math>= 185</math></p>	
<p>2</p> <p><math>u = 4x^2 - x^3 \quad y = u^4</math></p> <p><math>\frac{du}{dx} = 8x - 3x^2 \quad \frac{dy}{du} = 4u^3</math></p> <p><math>\frac{dy}{dx} = 4(8x - 3x^2)(4x^2 - x^3)^3</math></p>	
<p>3</p> <p><math>(1 + 2x)^{-2}</math></p> <p><math>= 1 - 2(2x) + \frac{(-2)(-3)}{2!}(2x)^2 + \frac{(-2)(-3)(-4)}{3!}(2x)^3</math></p> <p><math>= 1 - 4x + 12x^2 - 32x^3</math></p>	
<p>4</p> <p><math>\sin^2 \theta = \frac{3}{4}</math></p> <p><math>\sin \theta = \pm \frac{\sqrt{3}}{2}</math></p> <p><math>\theta = -\frac{2\pi}{3}, -\frac{\pi}{3}, \frac{\pi}{3}, \frac{2\pi}{3}</math></p>	
<p>5</p> <p><math>\int_{-1}^3 3x^2 - 6x + 7 dx = [x^3 - 3x^2 + 7x]</math></p> <p><math>(3^3 - 3 \times 3^2 + 7 \times 3) - ((-1)^3 - 3 \times (-1)^2 + 7 \times (-1))</math></p> <p><math>= 32</math></p>	

1

$$a = 12$$

$$r = 0.2 \quad S_n = \frac{12(1-0.2^n)}{1-0.2}$$

$$S_n = 15(1 - 0.2^n)$$

2

$$u = 2x - 2x^3 \quad y = \ln u$$

$$\frac{du}{dx} = 2 - 6x^2 \quad \frac{dy}{du} = \frac{1}{u}$$

$$\frac{dy}{dx} = \frac{2-6x^2}{2x-2x^3}$$

3

$$(1 - 4x)^{\frac{1}{2}}$$

$$= 1 + \frac{1}{2}(-4x) + \frac{\left(\frac{1}{2}\right)\left(-\frac{1}{2}\right)}{2!}(-4x)^2 + \frac{\left(\frac{1}{2}\right)\left(-\frac{1}{2}\right)\left(-\frac{3}{2}\right)}{3!}(-4x)^3$$

$$= 1 - 2x - 2x^2 - 4x^3$$

4

$$\text{Arc length} = 5 \times \frac{2\pi}{3}$$

$$= \frac{10\pi}{3}$$

$$\text{Perimeter} = 10 + \frac{10\pi}{3}$$

5

$$\int_{-4}^{-1} 5 - 3x^2 dx = [5x - x^3]$$

$$(5 \times -1 - (-1)^3) - (5 \times -4 - (-4)^3)$$

$$= -48$$

$$\begin{aligned}
 1 \quad & a + 9d = 104 \\
 & a + 13d = 152 \\
 & 4d = 48 \\
 & d = 12 \\
 & a = -4 \qquad -4 + 12(n - 1) = 368 \\
 & \qquad \qquad \qquad n = 32
 \end{aligned}$$

$$\begin{aligned}
 2 \quad & u = 2x^3 - 2x \quad y = u^{\frac{1}{2}} \\
 & \frac{du}{dx} = 6x^2 - 2 \quad \frac{dy}{du} = \frac{1}{2} u^{-\frac{1}{2}} \\
 & \frac{dy}{dx} = \frac{6x^2 - 2}{2\sqrt{2x^3 - 2x}}
 \end{aligned}$$

$$\begin{aligned}
 3 \quad & (1 - 2x)^{-1} = 1 + 2x + 4x^2 + 8x^3 \\
 & 2(1 - 2x)^{-1} = 2 + 4x + 8x^2 + 16x^3
 \end{aligned}$$

$$\begin{aligned}
 4 \quad & 4\sin\theta = \frac{3\cos\theta}{\sin\theta} \\
 & 2\sin^2\theta - 3\cos\theta = 0 \\
 & 2 - 2\cos^2\theta - 3\cos\theta = 0 \\
 & 2\cos^2\theta + 3\cos\theta - 2 = 0 \\
 & \cos\theta \neq -2 \quad \cos\theta = \frac{1}{2} \quad \theta = \frac{\pi}{3}, \frac{5\pi}{3}
 \end{aligned}$$

$$\begin{aligned}
 5 \quad & \int_1^3 4x^2 - 4x + 1 dx \\
 & \left[ \frac{4x^3}{3} - 2x^2 + x \right] \\
 & = \left( \frac{4 \times 27}{3} - 2 \times 9 + 3 \right) - \left( \frac{4 \times 1}{3} - 2 \times 1 + 1 \right) \\
 & = 20 \frac{2}{3}
 \end{aligned}$$

$$1 \quad 5 = 5k + 3$$

$$5k = 2$$

$$k = 0.4$$

$$2 \quad u = 2e^{2x} - 1 \quad y = u^3$$

$$\frac{du}{dx} = 4e^{2x} \quad \frac{dy}{dx} = 3u^2$$

$$\frac{dy}{dx} = 12e^{2x}(2e^{2x} - 1)^2$$

$$3 \quad (2 - x)^{-1} = 1 + \frac{1}{4}x + \frac{1}{8}x^2 + \frac{1}{16}x^3$$

$$(x + 1)\left(\frac{1}{2} + \frac{1}{4}x + \frac{1}{8}x^2 + \frac{1}{16}x^3\right)$$

$$= \frac{1}{2} + \frac{3}{4}x^2 + \frac{3}{8}x^2 + \frac{3}{16}x^3$$

$$4 \quad 8\sin^2 2\theta \cos 2\theta = \frac{\sin^2 2\theta}{\cos^2 2\theta}$$

$$8\cos^3 2\theta = 1$$

$$\cos 2\theta = \frac{1}{2}$$

$$2\theta = \frac{\pi}{3}, \frac{5\pi}{3}$$

$$\theta = \frac{\pi}{6}, \frac{5\pi}{6}$$

$$5 \quad \int_1^3 6x^2 + 4 \, dx$$

$$= [2x^3 + 4x]$$

$$(2 \times 27 + 4 \times 3) - (2 \times 1 + 4 \times 1)$$

$$= 60$$

$$\begin{aligned}
 1 \quad & ar = 10 \\
 & ar^3 = 62.5 \\
 & r^2 = 6.25 \\
 & r = 2.5 \qquad S_5 = \frac{4(2.5^5 - 1)}{2.5 - 1} \\
 & a = 4 \\
 & \qquad \qquad \qquad = 257.75
 \end{aligned}$$

$$\begin{aligned}
 2 \quad & u = \cos x \quad y = u^3 \\
 & \frac{du}{dx} = -\sin x \qquad \frac{dy}{du} = 3u^2 \\
 & \frac{dy}{dx} = -3 \sin x \cos^2 x
 \end{aligned}$$

$$\begin{aligned}
 3 \quad & (1 + 3x)^{-1} = 1 - 3x + 9x^2 - 27x^3 \\
 & 3(3 + x)^{-1} = 1 - \frac{1}{3}x + \frac{1}{9}x^2 - \frac{1}{27}x^3 \\
 & 1 - 3x + 9x^2 - 27x^3 \dots + 1 - \frac{1}{3}x + \frac{1}{9}x^2 - \frac{1}{27}x^3 \\
 & = 2 - \frac{10}{3}x + \frac{82}{9}x^2 - \frac{730}{27}x^3
 \end{aligned}$$

$$\begin{aligned}
 4 \quad & r \times \frac{\pi}{6} = 8 \\
 & r = \frac{48}{\pi} \\
 & \text{Area of triangle} = \frac{1}{2} \times \frac{48}{\pi} \times \frac{48}{\pi} \times \sin \frac{\pi}{6} \\
 & \qquad \qquad \qquad = 58.4 \text{ cm}^2
 \end{aligned}$$

$$\begin{aligned}
 5 \quad & \int_1^4 2x^{-\frac{1}{2}} dx \\
 & = [4\sqrt{x}] \\
 & = 4 \times 2 - 4 \times 1 = 4
 \end{aligned}$$

$$1 \quad S_1 = 25 \quad S_2 = 54 \quad S_3 = 87$$

$$1^{\text{st}} \text{ term} = 25$$

$$2^{\text{nd}} \text{ term} = 29$$

$$3^{\text{rd}} \text{ term} = 33 \quad u_n = 25 + 4(n - 1)$$

$$2 \quad u = \ln x + 4 \quad y = u^{-3}$$

$$\frac{du}{dx} = \frac{1}{x} \quad \frac{dy}{du} = -3u^{-4}$$

$$\frac{dy}{dx} = \frac{-3}{x(\ln x + 4)^4}$$

$$3 \quad (1 - x)^{\frac{1}{2}} = 1 - \frac{1}{2}x - \frac{1}{8}x^2$$

$$(1 + x)^{-\frac{1}{2}} = 1 - \frac{1}{2}x + \frac{3}{8}x^2$$

$$(1 - \frac{1}{2}x - \frac{1}{8}x^2)(1 - \frac{1}{2}x + \frac{3}{8}x^2)$$

$$= 1 - \frac{1}{2}x - \frac{1}{2}x^2$$

$$4 \quad A_1 = \frac{1}{2} \times 4 \times \theta \quad A_2 = 10\theta$$

$$10\theta = \frac{1}{2} \times 81\theta - \frac{1}{2} \times (OX)^2\theta$$

$$(OX)^2 = 61$$

$$OX = \sqrt{61}$$

$$5 \quad x^2 - 4x + 4 = 9$$

$$x^2 - 4x - 5 = 0$$

$$(x - 5)(x + 1) = 0 \quad x = -1 \text{ and } x = 5$$

$$\int_{-1}^5 9 - x^2 + 4x - 4 \, dx$$

$$= \left[ 5x - \frac{x^3}{3} + 2x^2 \right]$$

$$= \left( 25 - \frac{125}{3} + 50 \right) - \left( -5 + \frac{1}{3} + 2 \right) = 36$$

$$1 \quad a = 2 \quad r = 3$$

$$\frac{2(3^n - 1)}{3 - 1} > 200000$$

$$3^n > 200001$$

$$n > \frac{\ln(200001)}{\ln 3} \quad n > 11.11 \quad 12 \text{ terms needed}$$

$$2 \quad u = 2 - \sin 2x \quad y = \ln u$$

$$\frac{du}{dx} = -2\cos 2x \quad \frac{dy}{du} = \frac{1}{u}$$

$$\frac{dy}{dx} = -2\cos 2x \times \frac{1}{2 - \sin 2x}$$

$$\frac{dy}{dx} = \frac{-2\cos 2x}{2 - \sin 2x}$$

$$3 \quad (4 + x)^{\frac{1}{2}} = 2 + \frac{1}{4}x - \frac{1}{64}x^2$$

$$(1 - x)^{-\frac{1}{2}} = 1 + \frac{1}{2}x + \frac{3}{8}x^2$$

$$(2 + \frac{1}{4}x - \frac{1}{64}x^2)(1 + \frac{1}{2}x + \frac{3}{8}x^2)$$

$$= 2 + \frac{5}{4}x + \frac{55}{64}x^2$$

$$4 \quad 12^2 = 10^2 + 10^2 - 2 \times 10 \times 10 \times \cos \theta$$

$$\theta = 1.287 \text{ radians}$$

$$\text{Perimeter} = 10 \times 1.287 + 12$$

$$= 24.9 \text{ cm (3 s.f)}$$

$$5 \quad \frac{dy}{dx} = 4 - 10x^{-2}$$

$$y = 4x + 10x^{-1} + c$$

$$\text{When } x = 5 \quad y = 16$$

$$16 = 20 + 2 + c$$

$$c = -6$$

$$y = 4x + \frac{10}{x} - 6$$

