

Half Term 2B (ANSWERS)	Week 1
<p>1</p> <p>$1^{\text{st}} \text{ term} = 5$</p> <p>$2^{\text{nd}} \text{ term} = 8$</p> <p>$S_{10} = \frac{10}{2}(2 \times 5 + 3(10 - 1))$</p> <p>$= 185$</p>	
<p>2</p> <p>$(1 + 2x)^{-2}$</p> <p>$= 1 - 2(2x) + \frac{(-2)(-3)}{2!}(2x)^2 + \frac{(-2)(-3)(-4)}{3!}(2x)^3$</p> <p>$= 1 - 4x + 12x^2 - 32x^3$</p>	
<p>3</p> <p>$\sin^2 \theta = \frac{3}{4}$</p> <p>$\sin \theta = \pm \frac{\sqrt{3}}{2}$</p> <p>$\theta = -\frac{2\pi}{3}, -\frac{\pi}{3}, \frac{\pi}{3}, \frac{2\pi}{3}$</p>	
<p>4</p> <p>$A(x + 1) + B(x + 3) = 2$</p> <p>$x = -1 \quad 2B = 2 \quad B = 1$</p> <p>$x = -3 \quad -2A = 2 \quad A = -1$</p> <p>$\frac{2}{(x+3)(x+1)} = -\frac{1}{x+3} + \frac{1}{x+1}$</p>	
<p>5</p> <p>$\int_{-1}^3 3x^2 - 6x + 7 \, dx = [x^3 - 3x^2 + 7x]$</p> <p>$(3^3 - 3 \times 3^2 + 7 \times 3) - ((-1)^3 - 3 \times (-1)^2 + 7 \times (-1))$</p> <p>$= 32$</p>	

1

$$a = 12$$

$$r = 0.2 \quad S_n = \frac{12(1-0.2^n)}{1-0.2}$$

$$S_n = 15(1 - 0.2^n)$$

2

$$(1 - 4x)^{\frac{1}{2}}$$

$$= 1 + \frac{1}{2}(-4x) + \frac{\left(\frac{1}{2}\right)\left(-\frac{1}{2}\right)}{2!}(-4x)^2 + \frac{\left(\frac{1}{2}\right)\left(-\frac{1}{2}\right)\left(-\frac{3}{2}\right)}{3!}(-4x)^3$$

$$= 1 - 2x - 2x^2 - 4x^3$$

3

$$\text{Arc length} = 5 \times \frac{2\pi}{3}$$

$$= \frac{10\pi}{3}$$

$$\text{Perimeter} = 10 + \frac{10\pi}{3}$$

4

$$2\sin x - 5\cos x = R(\sin x \cos \theta - \cos x \sin \theta)$$

$$\cos \theta = 2 \quad \sin \theta = 5$$

$$\tan \theta = \frac{5}{2} \quad \theta = 1.19 \text{ (3 sf)}$$

$$R = \sqrt{5^2 + 2^2} \quad R = \sqrt{29}$$

$$2\sin x - 5\cos x = \sqrt{29}\sin(x - 1.19)$$

5

$$y = 2x^{\frac{5}{2}} + 3x^2$$

$$\frac{dy}{dx} = 5x^{\frac{3}{2}} + 6x$$

$$\frac{d^2y}{dx^2} = \frac{15}{2}\sqrt{x} + 6$$

$$15\sqrt{x} + 12 + a\sqrt{x} = 12$$

$$a = -15$$

$$\begin{aligned}
 1 \quad & a + 9d = 104 \\
 & a + 13d = 152 \\
 & 4d = 48 \\
 & d = 12 \\
 & a = -4 \qquad -4 + 12(n - 1) = 368 \\
 & \qquad \qquad \qquad n = 32
 \end{aligned}$$

$$\begin{aligned}
 2 \quad & (1 - 2x)^{-1} = 1 + 2x + 4x^2 + 8x^3 \\
 & 2(1 - 2x)^{-1} = 2 + 4x + 8x^2 + 16x^3
 \end{aligned}$$

$$\begin{aligned}
 3 \quad & 4\sin\theta = \frac{3\cos\theta}{\sin\theta} \\
 & 2\sin^2\theta - 3\cos\theta = 0 \\
 & 2 - 2\cos^2\theta - 3\cos\theta = 0 \\
 & 2\cos^2\theta + 3\cos\theta - 2 = 0 \\
 & \cos\theta \neq -2 \quad \cos\theta = \frac{1}{2} \qquad \theta = \frac{\pi}{3}, \frac{5\pi}{3}
 \end{aligned}$$

$$\begin{aligned}
 4 \quad & 3\cos x - 7\sin x = R(\cos x \cos\theta - \sin x \sin\theta) \\
 & \cos\theta = \frac{3}{R} \qquad \sin\theta = \frac{7}{R} \\
 & \tan\theta = \frac{7}{3} \qquad \theta = 1.17 \text{ (3 sf)} \\
 & R = \sqrt{7^2 + 3^2} \qquad R = \sqrt{58} \\
 & 3\cos x - 7\sin x = \sqrt{58}\sin(x - 1.17)
 \end{aligned}$$

$$\begin{aligned}
 5 \quad & \int_1^3 4x^2 - 4x + 1 dx \\
 & \left[\frac{4x^3}{3} - 2x^2 + x \right] \\
 & = \left(\frac{4 \times 27}{3} - 2 \times 9 + 3 \right) - \left(\frac{4 \times 1}{3} - 2 \times 1 + 1 \right) \\
 & = 20 \frac{2}{3}
 \end{aligned}$$

$$1 \quad 5 = 5k + 3$$

$$5k = 2$$

$$k = 0.4$$

$$2 \quad (2 - x)^{-1} = 1 + \frac{1}{4}x + \frac{1}{8}x^2 + \frac{1}{16}x^3$$

$$(x + 1)\left(\frac{1}{2} + \frac{1}{4}x + \frac{1}{8}x^2 + \frac{1}{16}x^3\right)$$

$$= \frac{1}{2} + \frac{3}{4}x^2 + \frac{3}{8}x^2 + \frac{3}{16}x^3$$

$$3 \quad 8\sin^2 2\theta \cos 2\theta = \frac{\sin^2 2\theta}{\cos^2 2\theta}$$

$$8\cos^3 2\theta = 1$$

$$\cos 2\theta = \frac{1}{2}$$

$$2\theta = \frac{\pi}{3}, \frac{5\pi}{3}$$

$$\theta = \frac{\pi}{6}, \frac{5\pi}{6}$$

$$4 \quad \frac{\operatorname{cosec} \theta}{\operatorname{cosec} \theta - \sin \theta} = \sec^2 \theta$$

$$\operatorname{cosec} \theta - \sin \theta$$

$$= \frac{1 - \sin^2 \theta}{\sin \theta}$$

$$= \frac{\cos^2 \theta}{\sin \theta} \quad \frac{1}{\sin \theta} \times \frac{\sin \theta}{\cos^2 \theta} = \frac{1}{\cos^2 \theta} = \sec^2 \theta$$

$$5 \quad \frac{dy}{dx} = 8x + \frac{5}{x^2}$$

$$x = -1 \quad \text{gradient of the tangent} = -3$$

$$\text{Gradient of the normal} = \frac{1}{3}$$

$$x = -1 \quad y = 10$$

$$(y - 10) = \frac{1}{3}(x + 1) \quad 3y - x = 31$$

$$\begin{aligned}
 1 \quad & ar = 10 \\
 & ar^3 = 62.5 \\
 & r^2 = 6.25 \\
 & r = 2.5 \qquad S_5 = \frac{4(2.5^5 - 1)}{2.5 - 1} \\
 & a = 4 \\
 & \qquad \qquad \qquad = 257.75
 \end{aligned}$$

$$\begin{aligned}
 2 \quad & (1 + 3x)^{-1} = 1 - 3x + 9x^2 - 27x^3 \\
 & 3(3 + x)^{-1} = 1 - \frac{1}{3}x + \frac{1}{9}x^2 - \frac{1}{27}x^3 \\
 & 1 - 3x + 9x^2 - 27x^3 \dots + 1 - \frac{1}{3}x + \frac{1}{9}x^2 - \frac{1}{27}x^3 \\
 & = 2 - \frac{10}{3}x + \frac{82}{9}x^2 - \frac{730}{27}x^3
 \end{aligned}$$

$$\begin{aligned}
 3 \quad & r \times \frac{\pi}{6} = 8 \\
 & r = \frac{48}{\pi} \\
 & \text{Area of triangle} = \frac{1}{2} \times \frac{48}{\pi} \times \frac{48}{\pi} \times \sin \frac{\pi}{6} \\
 & \qquad \qquad \qquad = 58.4 \text{ cm}^2
 \end{aligned}$$

$$\begin{aligned}
 4 \quad & 5 \times 2 \sin 2\theta \cos 2\theta = 3 \sin 2\theta \\
 & \sin 2\theta (10 \cos 2\theta - 3) = 0 \\
 & \sin 2\theta = 0 \quad 2\theta = 0, \pi \\
 & \cos 2\theta = \frac{3}{10} \quad 2\theta = 1.27, 5.02 \\
 & \theta = 0.635, \frac{\pi}{2}, 2.51
 \end{aligned}$$

$$\begin{aligned}
 5 \quad & y = 48x^{-1} + x^3 \\
 & \frac{dy}{dx} = -\frac{48}{x^2} + 3x^2 \qquad \frac{d^2y}{dx^2} = \frac{144}{x^3} + 6x \quad x = 2 \quad \frac{d^2y}{dx^2} > 0 \quad (\text{min}) \\
 & -\frac{48}{x^2} + 3x^2 = 0 \qquad \qquad \qquad x = -2 \quad \frac{d^2y}{dx^2} < 0 \quad (\text{max}) \\
 & 3x^4 = 48 \quad x = \pm 2 \qquad \qquad \qquad \text{Max point at } (-2, -32)
 \end{aligned}$$

$$1 \quad S_1 = 25 \quad S_2 = 54 \quad S_3 = 87$$

$$1^{\text{st}} \text{ term} = 25$$

$$2^{\text{nd}} \text{ term} = 29$$

$$3^{\text{rd}} \text{ term} = 33 \quad u_n = 25 + 4(n - 1)$$

$$2 \quad (1 - x)^{\frac{1}{2}} = 1 - \frac{1}{2}x - \frac{1}{8}x^2$$

$$(1 + x)^{-\frac{1}{2}} = 1 - \frac{1}{2}x + \frac{3}{8}x^2$$

$$(1 - \frac{1}{2}x - \frac{1}{8}x^2)(1 - \frac{1}{2}x + \frac{3}{8}x^2)$$

$$= 1 - \frac{1}{2}x - \frac{1}{2}x^2$$

$$3 \quad A_1 = \frac{1}{2} \times 4 \times \theta \quad A_2 = 10\theta$$

$$10\theta = \frac{1}{2} \times 81\theta - \frac{1}{2} \times (OX)^2\theta$$

$$(OX)^2 = 61$$

$$OX = \sqrt{61}$$

$$4 \quad 1 + \tan^2\theta = \frac{\cos^2\theta + \sin^2\theta}{\cos^2\theta} \quad 1 - \tan^2\theta = \frac{\cos^2\theta - \sin^2\theta}{\cos^2\theta}$$

$$\frac{1 - \tan^2\theta}{1 + \tan^2\theta} = \frac{\cos^2\theta - \sin^2\theta}{\cos^2\theta} \times \frac{\cos^2\theta}{\cos^2\theta + \sin^2\theta}$$

$$= \frac{\cos^2\theta - \sin^2\theta}{\cos^2\theta + \sin^2\theta}$$

$$= (1 - \sin^2\theta) - \sin^2\theta$$

$$= 1 - 2\sin^2\theta$$

$$5 \quad x^2 - 4x + 4 = 9$$

$$x^2 - 4x - 5 = 0$$

$$(x - 5)(x + 1) = 0 \quad x = -1 \text{ and } x = 5$$

$$\int_{-1}^5 9 - x^2 + 4x - 4 \, dx$$

$$= \left[5x - \frac{x^3}{3} + 2x^2 \right]$$

$$= \left(25 - \frac{125}{3} + 50 \right) - \left(-5 + \frac{1}{3} + 2 \right) = 36$$

$$1 \quad a = 2 \quad r = 3$$

$$\frac{2(3^n - 1)}{3 - 1} > 200000$$

$$3^n > 200001$$

$$n > \frac{\ln(200001)}{\ln 3} \quad n > 11.11 \quad 12 \text{ terms needed}$$

$$2 \quad (4 + x)^{\frac{1}{2}} = 2 + \frac{1}{4}x - \frac{1}{64}x^2$$

$$(1 - x)^{-\frac{1}{2}} = 1 + \frac{1}{2}x + \frac{3}{8}x^2$$

$$(2 + \frac{1}{4}x - \frac{1}{64}x^2)(1 + \frac{1}{2}x + \frac{3}{8}x^2)$$

$$= 2 + \frac{5}{4}x + \frac{55}{64}x^2$$

$$3 \quad 12^2 = 10^2 + 10^2 - 2 \times 10 \times 10 \times \cos \theta$$

$$\theta = 1.287 \text{ radians}$$

$$\text{Perimeter} = 10 \times 1.287 + 12$$

$$= 24.9 \text{ cm (3 s.f)}$$

$$4 \quad 2\operatorname{cosec}^2\theta - 2 + \operatorname{cosec}\theta + 1 = 0$$

$$2\operatorname{cosec}^2\theta + \operatorname{cosec}\theta - 1 = 0$$

$$(2\operatorname{cosec}\theta - 1)(\operatorname{cosec}\theta + 1) = 0$$

$$\operatorname{cosec}\theta = \frac{1}{2} \text{ (no solutions)}$$

$$\operatorname{cosec}\theta = -1 \quad \theta = -\frac{\pi}{2}, \frac{3\pi}{2}$$

$$5 \quad \frac{dy}{dx} = 4 - 10x^{-2}$$

$$y = 4x + 10x^{-1} + c$$

$$\text{When } x = 5 \quad y = 16$$

$$16 = 20 + 2 + c$$

$$c = -6$$

$$y = 4x + \frac{10}{x} - 6$$

