Half Term 3A (ANSWERS)	Week 1
1	$u = x^2 \quad \frac{du}{dx} = 2x$ $v = (x-1)^{\frac{1}{2}} \quad \frac{dv}{dx} = \frac{1}{2\sqrt{x-1}}$ $\frac{dy}{dx} = 2x\sqrt{x-1} + \frac{x^2}{2\sqrt{x-1}} \qquad \frac{dy}{dx} = \frac{5x^2-4x}{2\sqrt{x-1}}$
2	$u = x^2 \quad \frac{du}{dx} = 2x$ $v = x^2 - 4 \quad \frac{dv}{dx} = 2x$ $\frac{dy}{dx} = \frac{2x(x^2-4)-2x^3}{(x^2-4)^2} \qquad \frac{dy}{dx} = \frac{-8x}{(x^2-4)^2}$
3	$xy - 2y = x + 2$ $xy - x = 2y + 2$ $x(y - 1) = 2y + 2$ $f^{-1}(x) = \frac{2x+2}{x-1} \qquad f^{-1}(x) \neq -2$
4	*Midpoint at (-2, 0)* *Gradient of line* $= -\frac{4}{3}$ *Equation of the perpendicular bisector* $y = \frac{3}{4}(x + 2) \qquad 4y = 3x + 6$
5	$\int \frac{1}{3}x^2 - 4x + 1 \; dx = \left[\frac{1}{9}x^3 - 2x^2 + x + c\right]$ $x = 3 \quad y = 18$ $18 = 3 - 18 + 3 + c$ $c = 30$ $y = \frac{1}{9}x^3 - 2x^2 + x + 30$

Half Term 3A (ANSWERS)	Week 2

1	$u = e^x \quad \frac{du}{dx} = e^x$ $v = x^2 \quad \frac{dv}{dx} = 2x$ $\frac{dy}{dx} = x^2e^x + 2xe^x$
2	$u = x^3 \quad \frac{du}{dx} = 3x^2$ $v = x^2 + 3x \quad \frac{dv}{dx} = 2x + 3$ $\frac{dy}{dx} = \frac{3x^2(x^2+3) - x^3(2x+3)}{(x^2+3x)^2} \qquad \frac{dy}{dx} = \frac{x(x+6)}{(x+3)^2}$
3	$4(x^2 + 2) - 3$ $4((x + 1)^2 - 1) - 3$ $4(x + 1)^2 - 7$ Vertex at $(-1, -7)$ Range $f(x) \geq -7$
4	$y = k - x$ $x^2 + (k - x)^2 = 2x$ $x^2 + x^2 - 2kx + k^2 - 2x = 0$ $2x^2 - (2k + 2)x + k^2 = 0 \qquad (2k + 2)^2 - 4 \times 2 \times k^2 = 0$ $b^2 - 4ac = 0 \qquad 4k^2 + 8k + 4 - 8k^2 = 0$ $8k + 4 - 4k^2 = 0$ $k = 1 \pm \sqrt{2}$
5	$y\ 16 - 4x^2 = 0 \quad x = 2\ x = -2$ $\int_{-2}^{2} 16 - 4x^2 dx$ $\left[16x - \frac{4}{3}x^3\right] \qquad \left(32 - \frac{4}{3} \times 8\right) - \left(-32 + \frac{4}{3} \times 8\right) = 42\frac{2}{3}$

Half Term 3A (ANSWERS)	Week 3

1	$u = x^2 \quad \frac{du}{dx} = 2x$ $v = sinx \quad \frac{dv}{dx} = cosx$ $\frac{dy}{dx} = 2xsinx + x^2cosx$
2	$u = x^2 \quad \frac{du}{dx} = 2x$ $v = (x+1)^{\frac{1}{2}} \quad \frac{dv}{dx} = \frac{1}{2\sqrt{x+1}}$ $\frac{dy}{dx} = \frac{2x\sqrt{x+1} - \frac{x^2}{2\sqrt{x+1}}}{x+1} \qquad \frac{dy}{dx} = \frac{3x^2+4x}{2(x+1)^{\frac{3}{2}}}$
3	$fg(x) = 9x^2 + 1 \quad gf(x) = 3x^2 + 3$ $9x^2 + 1 = 3x^2 + 3$ $6x^2 = 2$ $x = \pm\frac{\sqrt{3}}{3}$
4	$x^2 + (y - 2)^2 - 4 - 14 = 0$ $Centre\ (0, 2) \quad radius = \sqrt{18}$ $Distance = \sqrt{18 - 4^2}$ $= \sqrt{2}$
5	$\int_2^4 1 + \frac{4}{x^3}\,dx = \left[x - \frac{2}{x^2}\right]$ $\left(4 - \frac{2}{16}\right) - \left(2 - \frac{2}{4}\right)$ $= 2\frac{3}{8}$

Half Term 3A (ANSWERS)	Week 4

1	$u = \sqrt{x}$ $\quad \frac{du}{dx} = \frac{1}{2\sqrt{x}}$ $v = e^{2x}$ $\quad \frac{dv}{dx} = 2e^{2x}$ $\frac{dy}{dx} = \frac{e^{2x}}{2\sqrt{x}} + 2e^{2x}\sqrt{x}$ $\qquad \frac{dy}{dx} = \frac{e^{2x}(4x+1)}{2\sqrt{x}}$
2	$u = e^x$ $\quad \frac{du}{dx} = e^x$ $v = x^2 + 1$ $\quad \frac{dv}{dx} = 2x$ $\frac{dy}{dx} = \frac{e^x(x^2+1) - 2xe^x}{(x^2+1)^2}$ $\qquad \frac{dy}{dx} = \frac{e^x(x^2-2x+1)}{(x^2+1)^2}$
3	$f(x) = 16 - (6x + x^2)$ $= 16 - ((x+3)^2 - 9)$ $= 25 - (x+3)^2$ *Vertex and (-3, 25)* *Range f(x)* $\leq$ *25*
4	*Circle Centre (-1,3)* *Gradient of radius =* $\frac{2}{3}$ *Gradient of tangent =* $-\frac{3}{2}$ *Equation of tangent :* $y - 5 = -\frac{3}{2}(x - 2)$ $2y + 3x = 16$
5	$\int_0^4 c\sqrt{x}\, dx = \left[\frac{2c}{3}x^{\frac{3}{2}}\right]$ $\left(\frac{2\times 8c}{3}\right) - 0 = 64$ *c = 12*

Half Term 3A (ANSWERS)	Week 5
1	$u = (x+3)^3 \quad \frac{du}{dx} = 3(x+3)^2$ $v = x^2 \quad \frac{dv}{dx} = 2x$ $\frac{dy}{dx} = 3x^2(x+3)^2 + 2x(x+3)^3$
2	$u = e^{2x} + x \qquad \frac{du}{dx} = 2e^{2x} + 1$ $v = x + 1 \qquad \frac{dv}{dx} = 1$ $\frac{dy}{dx} = \frac{(2e^{2x}+1)(x+1) - (e^{2x}+1)}{(x+1)^2} \qquad \frac{dy}{dx} = \frac{e^{2x}(2x+1)+1}{(x+1)^2}$
3	$y = \frac{ax+b}{x-b}$ $xy - by = ax + b$ $xy - ax = b + by$ $x(y - a) = b + by$ $f^{-1}(x) = \frac{b+bx}{x-a}$
4	$(x-4)^2 + (y+3)^2 = 8$ Gradient of normal = 1 Gradient of radius = -1 Centre (4, -3) Radius $= 2\sqrt{2}$ Points on circumference (2, -1) (6 , -5) $y = x - 3 \qquad y = x - 11$
5	$\int_1^2 \frac{8}{x^3} + x^3 dx = \left[-\frac{4}{x^2} + \frac{1}{4}x^4\right]$ $(-1+4) - (-4 + \frac{1}{4})$ $= 6\,\frac{3}{4}$

Half Term 3A (ANSWERS)	Week 6
1	$u = \sqrt[3]{x}$ $\quad \frac{du}{dx} = \frac{1}{3}x^{-\frac{2}{3}}$ $v = e^{3x}$ $\quad \frac{dv}{dx} = 3e^{3x}$ $\frac{dy}{dx} = \frac{e^{3x}}{3\sqrt[3]{x^2}} + 3e^{3x}\sqrt[3]{x}$ $\qquad \frac{dy}{dx} = \frac{e^{3x}(9x+1)}{3\sqrt[3]{x^2}}$
2	$u = \sqrt{x}$ $\quad \frac{du}{dx} = \frac{1}{2\sqrt{x}}$ $v = x^2 + 2x$ $\quad \frac{dv}{dx} = 2x + 2$ $\frac{dy}{dx} = \frac{\frac{x^2+2x}{2\sqrt{x}} - \sqrt{x}(2x+2)}{(x^2+2x)^2}$ $\qquad \frac{dy}{dx} = -\frac{3x+2}{2x^{\frac{3}{2}}(x+2)^2}$
3	f(x) : domain x ∈ ℝ Range : f(x) > 1 $f^{-1}(x)$: Domain x > 1 Range $f^{-1}(x)$ ∈ ℝ
4	A : x = -1 y = 0 (-1 , -2) B : x = 2 y = 4 (2, 4) Length of AB = $\sqrt{3^2 + 6^2}$ $= 3\sqrt{5}$
5	$x^4 + 4 = 20$ $x^4 = 16$ $x = \pm 2$ $\int_{-2}^{2} 20 - x^4 - 4\, dx = \left[16x - \frac{1}{5}x^5\right]$ $= 51\frac{1}{5}$

Half Term 3A (ANSWERS)	Week 7

1	$u = x^2 \quad \frac{du}{dx} = 2x$ $v = cosx \quad \frac{dv}{dx} = -sinx$ $\frac{dy}{dx} = 2xcosx - x^2sinx$
2	$u = e^{2x} \quad \frac{du}{dx} = 2e^{2x}$ $v = x^2 - x \quad \frac{dv}{dx} = 2x - 1$ $\frac{dy}{dx} = \frac{2e^{2x}(x^2-x)-e^{2x}(2x-1)}{(x^2-x)^2}$ $\qquad \frac{dy}{dx} = \frac{e^{2x}(2x^2-4x+1)}{(x^2-x)^2}$
3	$gf(x) = e^{2(\ln(3x-1))} - 1$ $= (3x-1)^2 - 1$ $= 9x^2 - 6x$ $9x^2 - 6x = 0$ $3x(3x-2) = 0$ $x \neq 0$ or $x = \frac{2}{3}$
4	$l_1 : (y-5) = \frac{1}{3}(x-6) \quad 3y - x = 9 \quad y = 2 \quad x = -3 \quad q = -3$ $l_2 : 4 \times -3 + 2p - 6 = 0$ $-18 + 2p = 0$ $p = 9$
5	$\int 4x^2 - \frac{4}{x^3}\, dx = \frac{4}{3}x^3 + \frac{2}{x^2} + c$ $x = -1 \quad y = 0$ $-\frac{4}{3} + 2 + c = 0 \quad c = -\frac{2}{3} \quad y = \frac{4}{3}x^3 + \frac{2}{x^2} - \frac{2}{3}$