Half Term 3B (ANSWERS)	Week 1
1	$u = 4x^2 - x^3 \qquad y = u^4$ $\frac{du}{dx} = 8x - 3x^2 \qquad \frac{dy}{du} = 4u^3$ $\frac{dy}{dx} = 4(8x - 3x^2)(4x^2 - x^3)^3$
2	*Midpoint at (-2, 0)* *Gradient of line* $= -\frac{4}{3}$ *Equation of the perpendicular bisector* $y = \frac{3}{4}(x + 2) \qquad 4y = 3x + 6$
3	$xy - 2y = x + 2$ $xy - x = 2y + 2$ $x(y - 1) = 2y + 2$ $f^{-1}(x) = \frac{2x+2}{x-1} \qquad f^{-1}(x) \neq -2$
4	$A(x + 1) + B(x + 3) = 2$ $x = -1 \quad 2B = 2 \quad B = 1$ $x = -3 \quad -2A = 2 \quad A = -1$ $\frac{2}{(x+3)(x+1)} = -\frac{1}{x+3} + \frac{1}{x+1}$
5	$\int \frac{1}{3}x^2 - 4x + 1 \; dx = \left[\frac{1}{9}x^3 - 2x^2 + x + c\right]$ $x = 3 \quad y = 18$ $18 = 3 - 18 + 3 + c$ $c = 30$ $y = \frac{1}{9}x^3 - 2x^2 + x + 30$

Half Term 3B (ANSWERS)	Week 2

1

$u = 2x - 2x^3 \quad y = \ln u$

$\frac{du}{dx} = 2 - 6x^2 \qquad \frac{dy}{du} = \frac{1}{u}$

$\frac{dy}{dx} = \frac{2-6x^2}{2x-2x^3}$

2

$y = k - x$

$x^2 + (k - x)^2 = 2x$

$x^2 + x^2 - 2kx + k^2 - 2x = 0$

$2x^2 - (2k + 2)x + k^2 = 0$ $\qquad (2k + 2)^2 - 4 \times 2 \times k^2 = 0$

$b^2 - 4ac = 0$ $\qquad 4k^2 + 8k + 4 - 8k^2 = 0$

$8k + 4 - 4k^2 = 0$

$k = 1 \pm \sqrt{2}$

3

$4(x^2 + 2) - 3$

$4((x + 1)^2 - 1) - 3$

$4(x + 1)^2 - 7$

Vertex at $(-1, -7)$ $\qquad$ Range $f(x) \geq -7$

4

$A(x - 4) + B(x - 1) = x + 2$

$x = 4 \quad 3B = 6 \quad B = 2$

$x = 1 \quad -3A = 3 \quad A = -1$

$\frac{x+2}{(x-1)(x-4)} = -\frac{1}{x-1} + \frac{2}{x-4}$

5

$16 - 4x^2 = 0 \quad x = 2 \ x = -2$

$$\int_{-2}^{2} 16 - 4x^2 dx$$

$$\left[16x - \frac{4}{3}x^3\right] \qquad \left(32 - \frac{4}{3} \times 8\right) - \left(-32 + \frac{4}{3} \times 8\right) = 42\frac{2}{3}$$

Half Term 3B (ANSWERS)	Week 3

1	$u = 2x^3 - 2x \quad y = u^{\frac{1}{2}}$ $\frac{du}{dx} = 6x^2 - 2 \quad \frac{dy}{du} = \frac{1}{2}u^{-\frac{1}{2}}$ $\frac{dy}{dx} = \frac{6x^2-2}{2\sqrt{2x^3-2x}}$
2	$x^2 + (y - 2)^2 - 4 - 14 = 0$ $Centre\ (0, 2) \quad radius = \sqrt{18}$ $Distance = \sqrt{18 - 4^2}$ $= \sqrt{2}$
3	$fg(x) = 9x^2 + 1 \quad gf(x) = 3x^2 + 3$ $9x^2 + 1 = 3x^2 + 3$ $6x^2 = 2$ $x = \pm\frac{\sqrt{3}}{3}$
4	$A(x - 3)^2 - B(x + 1)(x - 3) + C(x + 1) = 16$ $x = -1 \quad 16A = 16 \quad A = 1$ $x = 3 \quad 4C = 16 \quad C = 4$ $x = 0 \quad 9A + 3B + C = 16 \quad B = 1$ $\frac{16}{(x+1)(x-3)^2} = \frac{1}{x+1} - \frac{1}{x-3} + \frac{4}{(x-3)^2}$
5	$\int_2^4 1 + \frac{4}{x^3}\ dx = \left[x - \frac{2}{x^2}\right]$ $\left(4 - \frac{2}{16}\right) - \left(2 - \frac{2}{4}\right)$ $= 2\frac{3}{8}$

Half Term 3B (ANSWERS)	Week 4

1	$u = 2e^{2x} - 1 \quad y = u^3$ $\frac{du}{dx} = 4e^{2x} \quad \frac{dy}{dx} = 3u^2$ $\frac{dy}{dx} = 12e^{2x}(2e^{2x} - 1)^2$
2	*Circle Centre (-1,3)* *Gradient of radius* $= \frac{2}{3}$ *Gradient of tangent* $= -\frac{3}{2}$ *Equation of tangent :* $y - 5 = -\frac{3}{2}(x - 2)$ $2y + 3x = 16$
3	$f(x) = 16 - (6x + x^2)$ $= 16 - ((x + 3)^2 - 9)$ $= 25 - (x + 3)^2$ *Vertex and (-3, 25)* *Range* $f(x) \leq 25$
4	$A(1 - 2x) + C = 3x - 1$ $x = ½ \quad C = ½$ $x = 0 \quad A + ½ = -1 \quad A = -\frac{3}{2}$ $\frac{3x-1}{(1-2x)^2} = \frac{-3}{2(1-2x)} + \frac{1}{2(1-2x)^2}$
5	$\int_0^4 c\sqrt{x}\, dx = \left[\frac{2c}{3}x^{\frac{3}{2}}\right]$ $\left(\frac{2\times 8c}{3}\right) - 0 = 64$ $c = 12$

Half Term 3B (ANSWERS)	Week 5

1	$u = \cos x \quad y = u^3$ $\frac{du}{dx} = -\sin x \qquad \frac{dy}{du} = 3u^2$ $\frac{dy}{dx} = -3\sin x \cos^2 x$
2	$(x - 4)^2 + (y + 3)^2 = 8$ Gradient of normal = 1 Gradient of radius = -1 Centre (4, -3) Radius $= 2\sqrt{2}$ Points on circumference (2, -1) (6 , -5) $y = x - 3 \quad y = x - 11$
3	$y = \frac{ax+b}{x-b}$ $xy - by = ax + b$ $xy - ax = b + by$ $x(y - a) = b + by$ $f^{-1}(x) = \frac{b+bx}{x-a}$
4	$Ax(x + 2) + B(x - 1) = 3x^2 + 10x - 4$ $x = -2 \quad -3B = -12 \quad B = 4$ $x = 1 \quad 3A = 9 \quad A = 3$ $\frac{3x^2+10x-4}{(x-1)(x+2)} = \frac{3x}{x-1} + \frac{4}{x+2}$
5	$\int_1^2 \frac{8}{x^3} + x^3 dx = \left[-\frac{4}{x^2} + \frac{1}{4}x^4\right]$ $(-1 + 4) - (-4 + \frac{1}{4})$ $= 6\,\frac{3}{4}$

Half Term 3B (ANSWERS)	Week 6
1	$u = \ln x + 4 \quad y = u^{-3}$ $\frac{du}{dx} = \frac{1}{x} \quad \frac{dy}{du} = -3u^{-4}$ $\frac{dy}{dx} = \frac{-3}{x(\ln x + 4)^4}$
2	$A: x = -1 \;\; y = 0 \quad (-1, -2)$ $B: x = 2 \;\; y = 4 \quad (2, 4)$ Length of AB $= \sqrt{3^2 + 6^2}$ $= 3\sqrt{5}$
3	f(x) : domain $x \in \mathbb{R}$ Range : $f(x) > 1$ $f^{-1}(x)$: Domain $x > 1$ Range $f^{-1}(x) \in \mathbb{R}$
4	$Ax(2x^2 - 1) + Bx + C = 6x^3 - x + 6$ Comparing coefficients $x^3 : A = 3$ $x : -A + B = -1 \quad B = 2$ $C = 6$ $\frac{6x^3 - x + 6}{2x^2 - 1} = 3x + \frac{2x + 6}{2x^2 - 1}$
5	$x^4 + 4 = 20$ $x^4 = 16$ $x = \pm 2$ $\int_{-2}^{2} 20 - x^4 - 4 \, dx = \left[16x - \frac{1}{5}x^5\right]$ $= 51\frac{1}{5}$

Half Term 3B (ANSWERS)	Week 7
1	$u = 2 - \sin 2x \quad y = \ln u$ $\frac{du}{dx} = -2\cos 2x \quad \frac{dy}{du} = \frac{1}{u}$ $\frac{dy}{dx} = -2\cos 2x \times \frac{1}{2-\sin 2x}$ $\frac{dy}{dx} = \frac{-2\cos 2x}{2-\sin 2x}$
2	$l_1 : (y - 5) = \frac{1}{3}(x - 6) \quad 3y - x = 9 \quad y = 2 \quad x = -3 \quad q = -3$ $l_2 : 4 \times -3 + 2p - 6 = 0$ $-18 + 2p = 0$ $p = 9$
3	$gf(x) = e^{2(\ln(3x-1))} - 1$ $= (3x - 1)^2 - 1$ $= 9x^2 - 6x$ $9x^2 - 6x = 0$ $3x(3x - 2) = 0$ $x \neq 0 \text{ or } x = \frac{2}{3}$
4	$A(1 - 2x) + B(1 + x) = x$ $x = \frac{1}{2} \quad \frac{3}{2}B = \frac{1}{2} \qquad B = \frac{1}{3}$ $x = -1 \quad 3A = -1 \qquad A = -\frac{1}{3}$ $\frac{x}{(1+x)(1-2x)} = -\frac{1}{3(1+x)} + \frac{1}{3(1-2x)}$
5	$\int 4x^2 - \frac{4}{x^3}\,dx = \frac{4}{3}x^3 + \frac{2}{x^2} + c$ $x = -1 \quad y = 0$ $-\frac{4}{3} + 2 + c = 0 \qquad c = -\frac{2}{3} \qquad y = \frac{4}{3}x^3 + \frac{2}{x^2} - \frac{2}{3}$