Half Term 3C (ANSWERS)	Week 1
1	$u = 4x^2 - x^3 \quad y = u^4$ $\frac{du}{dx} = 8x - 3x^2 \quad \frac{dy}{du} = 4u^3$ $\frac{dy}{dx} = 4(8x - 3x^2)(4x^2 - x^3)^3$
2	$2cos^2x - 1 - 3cosx - 4 = 0$ $2cos^2x - 2cosx - 5 = 0$ $(2cosx - 5)(cosx + 1) = 0$ $cosx = \frac{5}{2}$ $(No\ solutions)$ $cosx = -1 \quad x = -\pi, \pi$
3	$\frac{dx}{dt} = 2 \quad \frac{dy}{dt} = \frac{1}{2\sqrt{t}} \quad \frac{dy}{dt} = \frac{1}{4\sqrt{t}}$ $t = 1 \quad x = 2 \quad y = 4$ *Gradient of tangent* = ¼ $(y - 4) = \frac{1}{4}(x - 2)$ $4y - 16 = x - 2 \quad 4y = x + 14$
4	$xy - 2y = x + 2$ $xy - x = 2y + 2$ $x(y - 1) = 2y + 2$ $f^{-1}(x) = \frac{2x+2}{x-1} \quad f^{-1}(x) \neq -2$
5	*Midpoint at (-2, 0)* *Gradient of line* = $-\frac{4}{3}$ *Equation of the perpendicular bisector* $y = \frac{3}{4}(x + 2) \quad 4y = 3x + 6$

Half Term 3C (ANSWERS)	Week 2
1	$u = 2x - 2x^3 \quad y = \ln u$ $\frac{du}{dx} = 2 - 6x^2 \qquad \frac{dy}{du} = \frac{1}{u}$ $\frac{dy}{dx} = \frac{2-6x^2}{2x-2x^3}$
2	$2sinx - 5cosx = R(sinxcos\theta - cosxsin\theta)$ $cos\theta = 2 \qquad sin\theta = 5$ $tan\theta = \frac{5}{2} \qquad \theta = 1.19\ (3\ sf)$ $R = \sqrt{5^2 + 2^2} \qquad R = \sqrt{29}$ $2sinx - 5cosx = \sqrt{29}sin(x - 1.19)$
3	$x = 1 - \frac{1}{t} \qquad y = 1 + \frac{1}{t}$ $\frac{1}{t} = 1 - x \qquad \frac{1}{t} = y - 1$ $1 - x = y - 1$ $x + y = 2$
4	$4(x^2 + 2) - 3$ $4((x + 1)^2 - 1) - 3$ $4(x + 1)^2 - 7$ Vertex at (-1, -7) Range $f(x) \geq -7$
5	$y = k - x$ $x^2 + (k - x)^2 = 2x$ $x^2 + x^2 - 2kx + k^2 - 2x = 0$ $2x^2 - (2k + 2)x + k^2 = 0 \qquad (2k + 2)^2 - 4 \times 2 \times k^2 = 0$ $b^2 - 4ac = 0 \qquad 4k^2 + 8k + 4 - 8k^2 = 0$ $8k + 4 - 4k^2 = 0$ $k = 1 \pm \sqrt{2}$

Half Term 3C (ANSWERS)	Week 3

1	$u = 2x^3 - 2x \quad y = u^{\frac{1}{2}}$ $\frac{du}{dx} = 6x^2 - 2 \quad \frac{dy}{du} = \frac{1}{2}u^{-\frac{1}{2}}$ $\frac{dy}{dx} = \frac{6x^2-2}{2\sqrt{2x^3-2x}}$
2	$3\cos x - 7\sin x$ $\cos\theta = 3 \quad \sin\theta = 7$ $\tan\theta = \frac{7}{3} \quad \theta = 1.17$ $R = \sqrt{3^2 + 7^2}$ $R = \sqrt{58}$ $3\cos x - 7\sin x = \sqrt{58}\cos(x + 1.17)$
3	$\frac{dx}{dt} = 2t \quad \frac{dy}{dt} = 1$ $\quad t = 4 \quad x = 18 \quad y = 2$ $\frac{dy}{dx} = \frac{1}{2t}$ Gradient of tangent $= \frac{1}{8}$ Gradient of normal $= -8$ $\quad y - 2 = -8(x - 18)$ $y + 8x = 146$
4	$fg(x) = 9x^2 + 1 \quad gf(x) = 3x^2 + 3$ $9x^2 + 1 = 3x^2 + 3$ $6x^2 = 2$ $x = \pm\frac{\sqrt{3}}{3}$
5	$x^2 + (y - 2)^2 - 4 - 14 = 0$ Centre $(0, 2)$ radius $= \sqrt{18}$ Distance $= \sqrt{18 - 4^2}$ $= \sqrt{2}$

1

$u = 2e^{2x} - 1 \quad y = u^3$

$\frac{du}{dx} = 4e^{2x} \quad \frac{dy}{dx} = 3u^2$

$$\frac{dy}{dx} = 12e^{2x}(2e^{2x} - 1)^2$$

2

$\frac{cosec\theta}{cosec\theta - sin\theta} = \frac{1}{sin\theta} \div \left(\frac{1}{sin\theta} - sin\theta\right)$

$= \frac{1}{sin\theta} \div \left(\frac{1 - sin^2\theta}{sin\theta}\right)$

$= \frac{1}{sin\theta} \times \left(\frac{sin\theta}{cos^2\theta}\right)$

$= sec^2\theta$

3

$t = 2 \quad x = 12 \quad y = 12$

$\frac{dx}{dt} = 6t \quad \frac{dy}{dt} = 6$

$\frac{dy}{dx} = \frac{1}{t}$

Gradient of tangent = ½ Gradient of normal = -2

$y - 12 = -2(x - 12)$

$y + 2x = 36$

$t = \frac{y}{6} \quad x = \frac{3y^2}{36}$

$= \frac{y^2}{12}$

$y + \frac{2y^2}{12} = 36$

$2y^2 + 12y - 432 = 0$

$y = 12 \quad y = -18 \ (t = -3)$

$(27, -18)$

4

$f(x) = 16 - (6x + x^2)$

$= 16 - ((x + 3)^2 - 9)$

$= 25 - (x + 3)^2$

Vertex and (-3, 25)

Range $f(x) \leq 25$

5

Circle Centre (-1,3)

Gradient of radius $= \frac{2}{3}$

Gradient of tangent $= -\frac{3}{2}$

Equation of tangent : $y - 5 = -\frac{3}{2}(x - 2)$

$2y + 3x = 16$

Half Term 3C (ANSWERS)	Week 5
1	$u = cos\,x \quad y = u^3$ $\frac{du}{dx} = -\,sin\,x \qquad \frac{dy}{du} = 3u^2$ $\frac{dy}{dx} = -3\,sin\,x\,cos^2x$
2	$5 \times 2sin2\theta cos2\theta = 3sin2\theta$ $sin2\theta(10cos2\theta\ - 3) = 0$ $sin2\theta = 0 \quad 2\theta = 0\,, \pi$ $cos2\theta = \frac{3}{10} \quad 2\theta = 1.27,\ 5.02$ $\theta = 0.635,\ \frac{\pi}{2},\ 2.51$
3	$sint = \frac{x}{\sqrt{2}} \quad cost = \frac{y}{2}$ $\left(\frac{x}{\sqrt{2}}\right)^2 + \left(\frac{y}{2}\right)^2 = 1$ $\frac{x^2}{2} + \frac{y^2}{4} = 1 \qquad 2x^2 + y^2 = 4$
4	$y = \frac{ax+b}{x-b}$ $xy\ - by = ax + b$ $xy\ - ax = b + by$ $x(y\ - a) = b + by$ $f^{-1}(x) = \frac{b+bx}{x-a}$
5	$(x - 4)^2 + (y + 3)^2 = 8$ Gradient of normal = 1 Gradient of radius = -1 Centre (4, -3) Radius = $2\sqrt{2}$ Points on circumference (2, -1) (6 , -5) $y = x - 3 \qquad y = x - 11$

Half Term 3C (ANSWERS)	Week 6

1	$u = ln\, x + 4 \qquad y = u^{-3}$ $\frac{du}{dx} = \frac{1}{x} \qquad \frac{dy}{du} = -3u^{-4}$ $\frac{dy}{dx} = \frac{-3}{x(lnx+4)^4}$
2	$1 + tan^2\theta = \frac{cos^2\theta + sin^2\theta}{cos^2\theta} \qquad 1 - tan^2\theta = \frac{cos^2\theta - sin^2\theta}{cos^2\theta}$ $\frac{1-tan^2\theta}{1+tan^2\theta} = \frac{cos^2\theta - sin^2\theta}{cos^2\theta} \times \frac{cos^2\theta}{cos^2\theta + sin^2\theta}$ $= cos^2\theta - sin^2\theta$ $= (1 - sin^2\theta) - sin^2\theta$ $= 1 - 2sin^2\theta$
3	$\frac{dx}{dt} = -\frac{1}{2t^2} \qquad \frac{dy}{dt} = -\frac{2}{(2t+2)^2}$ $\frac{dy}{dx} = -\frac{4t^2}{(2t+2)^2}$
4	f(x) : domain $x \in \mathbb{R}$ Range : $f(x) > 1$ $f^{-1}(x)$: Domain $x > 1$ Range $f^{-1}(x) \in \mathbb{R}$
5	$A : x = -1 \; y = 0 \quad (-1, -2)$ $B : x = 2 \; y = 4 \quad (2, 4)$ $Length\ of\ AB = \sqrt{3^2 + 6^2}$ $= 3\sqrt{5}$

Half Term 3C (ANSWERS)	Week 7

1	$u = 2 - \sin 2x \quad y = \ln u$ $\frac{du}{dx} = -2\cos 2x \quad \frac{dy}{du} = \frac{1}{u}$ $\frac{dy}{dx} = -2\cos 2x \times \frac{1}{2-\sin 2x}$ $\frac{dy}{dx} = \frac{-2\cos 2x}{2-\sin 2x}$
2	$2cosec^2\theta - 2 + cosec\theta + 1 = 0$ $2cosec^2\theta + cosec\theta - 1 = 0$ $(2cosec\,\theta - 1)(cosec\theta + 1) = 0$ $cosec\,\theta = \frac{1}{2}$ *(no solutions)* $cosec\,\theta = -1 \quad \theta = -\frac{\pi}{2}, \frac{3\pi}{2}$
3	$\frac{dx}{dt} = -e^t \quad \frac{dy}{dt} = 2e^{2t} \quad \frac{dy}{dx} = \frac{2e^{2t}}{-e^t}$ $t = 0$ gradient $= -2$ $x = 1 \quad y = 6$ Equation of tangent $(y - 6) = -2(x - 1)$ $y + 2x = 8$
4	$gf(x) = e^{2(\ln(3x-1))} - 1$ $= (3x - 1)^2 - 1$ $= 9x^2 - 6x$ $9x^2 - 6x = 0$ $3x(3x - 2) = 0$ $x \neq 0$ or $x = \frac{2}{3}$
5	$l_1 : (y - 5) = \frac{1}{3}(x - 6) \quad 3y - x = 9 \quad y = 2 \quad x = -3 \quad q = -3$ $l_2 : 4 \times -3 + 2p - 6 = 0$ $-18 + 2p = 0$ $p = 9$