FORMULAE GIVEN AND TO LEARN - PURE

Topic	GIVEN	LEARN
TOPIC 1 PROOF		

<table>
<tr><th>Topic</th><th>GIVEN</th><th>LEARN</th></tr>
<tr>
<td>TOPIC 2:
ALGEBRA & FUNCTIONS</td>
<td></td>
<td>

Indices rules

$a^0 = 1$

$a^{-m} = \frac{1}{a^m}$

$a^m \times a^n = a^{m+n}$

$a^m \div a^n = a^{m-n}$

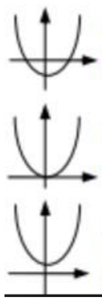
$(a^m)^n = a^{mn}$

$a^{m/n} = \sqrt[n]{a^m} = \left(\sqrt[n]{a}\right)^m$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

<table>
<tr><th>S.No.</th><th>Form of the rational function</th><th>Form of the partial fraction</th></tr>
<tr><td>1.</td><td>$\frac{px+q}{(x-a)(x-b)}, a \neq b$</td><td>$\frac{A}{x-a} + \frac{B}{x-b}$</td></tr>
<tr><td>2.</td><td>$\frac{px+q}{(x-a)^2}$</td><td>$\frac{A}{x-a} + \frac{B}{(x-a)^2}$</td></tr>
<tr><td>3.</td><td>$\frac{px^2+qx+r}{(x-a)(x-b)(x-c)}$</td><td>$\frac{A}{x-a} + \frac{B}{x-b} + \frac{C}{x-c}$</td></tr>
<tr><td>4.</td><td>$\frac{px^2+qx+r}{(x-a)^2(x-b)}$</td><td>$\frac{A}{x-a} + \frac{B}{(x-a)^2} + \frac{C}{x-b}$</td></tr>
<tr><td>5.</td><td>$\frac{px^2+qx+r}{(x-a)(x^2+bx+c)}$
where $x^2 + bx + c$ cannot be factorised further</td><td>$\frac{A}{x-a} + \frac{Bx+C}{x^2+bx+c}$,</td></tr>
</table>

Discriminant



$b^2 - 4ac > 0$
two roots, real and unequal

$b^2 - 4ac = 0$
one root, real and equal

$b^2 - 4ac < 0$
no roots, not real

If (x-p) is a factor of f(x) then f(p) =0

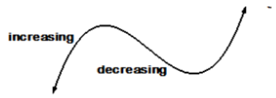
</td>
</tr>
</table>

<table>
<tr><th>Topic</th><th>GIVEN</th><th>LEARN</th></tr>
<tr><td>**TOPIC 3: COORDINATE GEOMETRY**</td><td></td><td>
<table>
<tr><td>**Slope formula**</td><td>$slope, m = \frac{change\ in\ y}{change\ in\ x} = \frac{rise}{run} = \frac{y_2 - y_1}{x_2 - x_1}$
Parallel lines have equal slopes.
The slopes of perpendicular lines are opposite reciprocals of each other</td></tr>
<tr><td>**General Form**</td><td>$Ax + By = C$</td></tr>
<tr><td>**Slope Intercept Form**</td><td>$y = mx + b$
where *m* is the slope and *b* is the y-intercept</td></tr>
<tr><td>**Point Slope Form**</td><td>$(y - y_1) = m(x - x_1)$
where *m* is the slope</td></tr>
<tr><td>**Midpoint Formula**</td><td>$\left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}\right)$</td></tr>
<tr><td>**Distance Formula**</td><td>$\sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$</td></tr>
</table>
Equation of a circle
$(x - a)^2 + (y - b)^2 = r^2$ Centre (a, b) radius = r
Know circle theorems from GCSE
</td></tr>
</table>

<table>
<tr><th>Topic</th><th>GIVEN</th><th>LEARN</th></tr>
<tr><td>**TOPIC 4: SEQUENCES AND SERIES**</td><td>
Arithmetic series
$S_n = \frac{1}{2}n(a + l) = \frac{1}{2}n[2a + (n - 1)d]$
Geometric series
$S_n = \frac{a(1 - r^n)}{1 - r}$
$S_\infty = \frac{a}{1 - r}$ for $|r| < 1$
Binomial series
$(a + b)^n = a^n + \binom{n}{1}a^{n-1}b + \binom{n}{2}a^{n-2}b^2 + \ldots + \binom{n}{r}a^{n-r}b^r + \ldots + b^n \quad (n \in \mathbb{N})$
where $\binom{n}{r} = {}^nC_r = \frac{n!}{r!(n - r)!}$
$(1 + x)^n = 1 + nx + \frac{n(n - 1)}{1 \times 2}x^2 + \ldots + \frac{n(n - 1)\ldots(n - r + 1)}{1 \times 2 \times \ldots \times r}x^r + \ldots \quad (|x| < 1, n \in \mathbb{R})$
</td><td>
nth Term of an Arithmetic Sequence Formula
$a_n = a_1 + (n - 1)d$
where *d* is the common difference
Geometric Sequence
A geometric sequence has a common ratio.
The formula for the n^{th} term is
$a_n = ar^{n-1}$
where a_n = n^{th} term of the sequence
a = first term of the sequence
r = common ratio
</td></tr>
</table>

Topic	GIVEN	LEARN
TOPIC 5: TRIGONOMETRY	**Trigonometric identities** $\sin(A \pm B) = \sin A \cos B \pm \cos A \sin B$ $\cos(A \pm B) = \cos A \cos B \mp \sin A \sin B$ $\tan(A \pm B) = \dfrac{\tan A \pm \tan B}{1 \mp \tan A \tan B} \quad (A \pm B \neq (k + \tfrac{1}{2})\pi)$ Small angle approximations $\sin\theta \approx \theta$ $\cos\theta \approx 1 - \dfrac{\theta^2}{2}$ $\tan\theta \approx \theta$ where θ is measured in radians	Sine rule $\dfrac{a}{\sin A} = \dfrac{b}{\sin B} = \dfrac{c}{\sin C}$ $\dfrac{\sin A}{a} = \dfrac{\sin B}{b} = \dfrac{\sin C}{c}$ Cosine rule $a^2 = b^2 + c^2 - 2bc\cos A$ $\quad \cos A = \dfrac{b^2 + c^2 - a^2}{2bc}$ Area of a triangle = $\frac{1}{2} a\, b \sin C$ Reciprocal Identities : $\sin\theta = \dfrac{1}{\csc\theta}$ $\quad \csc\theta = \dfrac{1}{\sin\theta}$ $\cos\theta = \dfrac{1}{\sec\theta}$ $\quad \sec\theta = \dfrac{1}{\cos\theta}$ $\tan\theta = \dfrac{1}{\cot\theta}$ $\quad \cot\theta = \dfrac{1}{\tan\theta}$ Pythagorean Identities : $\sin^2\theta + \cos^2\theta = 1$ $1 + \tan^2\theta = \sec^2\theta$ $1 + \cot^2\theta = \csc^2\theta$ Cofuntion Identities : $\sin\theta = \cos\left(\dfrac{\pi}{2} - \theta\right), \quad \cos\theta = \sin\left(\dfrac{\pi}{2} - \theta\right)$ $\sec\theta = \csc\left(\dfrac{\pi}{2} - \theta\right), \quad \csc\theta = \sec\left(\dfrac{\pi}{2} - \theta\right)$ $\tan\theta = \cot\left(\dfrac{\pi}{2} - \theta\right), \quad \cot\theta = \tan\left(\dfrac{\pi}{2} - \theta\right)$ Even Odd Identities : $\sin(-\theta) = -\sin\theta, \quad \csc(-\theta) = -\csc\theta$ $\tan(-\theta) = -\tan\theta, \quad \cot(-\theta) = -\cot\theta$ $\cos(-\theta) = \cos\theta, \quad \sec(-\theta) = \sec t\theta$ Quotient Identities : $\tan\theta = \dfrac{\sin\theta}{\cos\theta} \quad \cot\theta = \dfrac{\cos\theta}{\sin\theta}$ **Radians 180° = π radians** Area of sector $\boldsymbol{A = \frac{1}{2} r^2 \theta}$ Arc length $\boldsymbol{l = r\theta}$ **Double angle formulae** $\sin 2A = 2\sin A\cos A$ $\cos 2A = \cos^2 A - \sin^2 A$ $= 1 - 2\sin^2 A$ $= 2\cos^2 A - 1$

Topic	GIVEN	LEARN
TOPIC 6: EXPONENTIALS & LOGARITHMS	**Logarithms and exponentials** $\log_a x = \frac{\log_b x}{\log_b a}$ $e^{x \ln a} = a^x$	$\log_a(a) = 1$ $\log_a(1) = 0$ $\log_a(mn) = \log_a(m) + \log_a(n)$ $\log_a(m^p) = p\log_a(m)$ $\log_a\left(\frac{m}{n}\right) = \log_a(m) - \log_a(n)$ $\log_a\left(\frac{1}{n}\right) = -\log_a(n)$ $\log_a(m) = n$ then $a^n = m$ $a^{\log_a(m)} = m$ $\log_a(0)$ cannot exist $\log_a(\text{negative})$ cannot exist

Topic	GIVEN	LEARN
TOPIC 7: DIFFERENTIATION	First Principles $$f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$$ **f(x)** — **f′(x)** $\tan kx$ — $k\sec^2 kx$ $\sec kx$ — $k\sec kx \tan kx$ $\cot kx$ — $-k\operatorname{cosec}^2 kx$ $\operatorname{cosec} kx$ — $-k\operatorname{cosec} kx \cot kx$ $\frac{f(x)}{g(x)}$ — $\frac{f'(x)g(x) - f(x)g'(x)}{(g(x))^2}$	Chain rule - $\frac{dy}{dx} = \frac{du}{dx} \times \frac{dy}{du}$ Product rule - $$\frac{d}{dx}uv = u\frac{dv}{dx} + v\frac{du}{dx}$$ $$\frac{dx}{dy} = \frac{1}{\frac{dy}{dx}}$$ **Trigonometric Functions** $\frac{d}{dx}(\sin x) = \cos x$ $\quad$ $\frac{d}{dx}(\cos x) = -\sin x$ **Exponential and Logarithmic Functions** $\frac{d}{dx}e^x = e^x$ $\quad$ $\frac{d}{dx}\ln x = \frac{1}{x}$ $\frac{d}{dx}a^x = a^x \ln a$ $\quad$ $\frac{d}{dx}(\log_a x) = \frac{1}{x \ln a}$ **Parametric Equations** If $x = f(t)$ and $y = g(t)$ are differentiable, then $$y' = \frac{dy}{dx} = \frac{dy/dt}{dx/dt}$$ **An increasing function has a positive gradient $y' > 0$** $\quad \frac{dy}{dx} > 0$ **A decreasing function has a negative gradient $y' < 0$** $\quad \frac{dy}{dx} < 0$  $\frac{d^2y}{dx^2} > 0 \Rightarrow$ minimum $\frac{d^2y}{dx^2} < 0 \Rightarrow$ maximum **Stationary points have gradient = zero** $\frac{dy}{dx} = 0$ $f'(x) = 0$ $y' = 0$

Topic	GIVEN	LEARN
TOPIC 8: INTEGRATION	**f(x)** — $\int f(x)\,dx$ $\sec^2 kx$ — $\frac{1}{k}\tan kx$ $\tan kx$ — $\frac{1}{k}\ln\lvert\sec kx\rvert$ $\cot kx$ — $\frac{1}{k}\ln\lvert\sin kx\rvert$ $\operatorname{cosec} kx$ — $-\frac{1}{k}\ln\lvert\operatorname{cosec} kx+\cot kx\rvert$, $\frac{1}{k}\ln\lvert\tan(\frac{1}{2}kx)\rvert$ $\sec kx$ — $\frac{1}{k}\ln\lvert\sec kx+\tan kx\rvert$, $\frac{1}{k}\ln\lvert\tan(\frac{1}{2}kx+\frac{1}{4}\pi)\rvert$ $\int u\frac{dv}{dx}dx = uv - \int v\frac{du}{dx}dx$	(1) (i) $\int x^n dx = \frac{x^{n+1}}{n+1}+c, n\neq -1$ (ii) $\int (ax+b)^n dx = \frac{1}{a}.\frac{(ax+b)^{n+1}}{n+1}+c\ ,\ n\neq -1$ (2) (i) $\int \frac{1}{x}dx = \log\lvert x\rvert + c$ (ii) $\int \frac{1}{ax+b}dx = \frac{1}{a}(\log\lvert ax+b\rvert) + c$ (3) $\int e^x dx = e^x + c$ (4) $\int a^x dx = \frac{a^x}{\log_e a}+c$ (5) $\int \sin x\,dx = -\cos x + c$ (6) $\int \cos x\,dx = \sin x + c$ $\int f'(x)(f(x))^n dx = (f(x))^n + c$ $\int f'(x)e^{f(x)}dx = e^{f(x)}+c$ $\int \frac{f'(x)}{f(x)}dx = \ln\lvert f(x)\rvert + c$ Helpful Trig. Things. $cos^2x = \frac{1}{2}(\cos 2x\ + 1)$ $sin^2x = \frac{1}{2}(1 - \cos 2x)$ $\int sin^n x\, cosx\, dx = \frac{sin^{n+1}x}{n+1}+c$ $\int cos^n x\, sinx\, dx = \frac{-cos^{n+1}x}{n+1}+c$

Topic	GIVEN	LEARN
TOPIC 9: NUMERICAL METHODS	The trapezium rule: $\int_a^b y\,dx \approx \frac{1}{2}h\{(y_0 + y_n) + 2(y_1 + y_2 + \ldots + y_{n-1})\}$, where $h = \frac{b-a}{n}$ The Newton-Raphson iteration for solving $f(x) = 0 : x_{n+1} = x_n - \frac{f(x_n)}{f'(x_n)}$	

Topic	GIVEN	LEARN
TOPIC 10: VECTORS		▪ **If the vector $\mathbf{a} = x\mathbf{i} + y\mathbf{j} + z\mathbf{k}$ makes an angle θ_x with the positive x-axis then $\cos\theta_x = \frac{x}{\lvert\mathbf{a}\rvert}$ and similarly for the angles θ_y and θ_z** ▪ **The distance from the origin to the point (x, y, z) is $\sqrt{x^2 + y^2 + z^2}$.** ▪ **The distance between the points (x_1, y_1, z_1) and (x_2, y_2, z_2) is $\sqrt{(x_1 - x_2)^2 + (y_1 - y_2)^2 + (z_1 - z_2)^2}$** ▪ **To multiply a column vector by a scalar, multiply each component by the scalar: $\lambda\begin{pmatrix}p\\q\end{pmatrix} = \begin{pmatrix}\lambda p\\\lambda q\end{pmatrix}$** ▪ **To add two column vectors, add the x-components and the y-components: $\begin{pmatrix}p\\q\end{pmatrix} + \begin{pmatrix}r\\s\end{pmatrix} = \begin{pmatrix}p+r\\q+s\end{pmatrix}$** ▪ **A unit vector in the direction of a is $\frac{\mathbf{a}}{\lvert\mathbf{a}\rvert}$** ▪ **$\overrightarrow{AB} = \overrightarrow{OB} - \overrightarrow{OA}$, where $\overrightarrow{OA}$ and $\overrightarrow{OB}$ are the position vectors of A and B respectively.**

FORMULAE GIVEN AND TO LEARN - APPLIED

Topic	GIVEN	LEARN
TOPIC 1: STATISTICAL SAMPLING		Stratified sample size = $\frac{group\ frequency}{total\ frequncy} \times sample\ size$

Topic	GIVEN	LEARN
TOPIC 2: DATA PRESENTAION AND INTERPRETATION	Standard deviation = √(Variance) Interquartile range = $\text{IQR} = \text{Q}_3 - \text{Q}_1$ $\text{S}_{xx} = \Sigma(x_i - \bar{x})^2 = \Sigma x_i^2 - \frac{(\Sigma x_i)^2}{n}$ Standard deviation = $\sqrt{\frac{\text{S}_{xx}}{n}}$ or $\sqrt{\frac{\sum x^2}{n} - \bar{x}^2}$	**Standard deviation** (with frequencies) = $\sqrt{\frac{\sum fx^2}{\sum f} - \bar{x}^2}$ If **code** is $y = \frac{x-a}{b}$, then $\bar{x} = b\bar{y} + a$ and $\sigma_x = b\sigma_y$ Linear interpolation – Use the boxes and the proportions within the boxes **Histograms** - $frequency\ density \propto \frac{frequency}{class\ width}$ $Area \propto frequency$ **Outliers** - outlier$> Q_3 + k(IQR)$ outlier $< Q_1 - k(IQR)$ outlier $> \mu + 2\sigma$ outlier $< \mu - 2\sigma$ (you should be told this in the question, and which one to use)

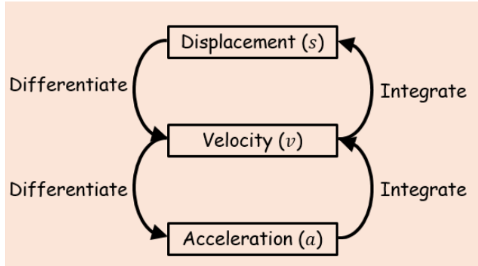
Topic	GIVEN	LEARN
TOPIC 3: PROBABILITY	**Probability** $\text{P}(A') = 1 - \text{P}(A)$ $\text{P}(A \cup B) = \text{P}(A) + \text{P}(B) - \text{P}(A \cap B)$ $\text{P}(A \cap B) = \text{P}(A)\text{P}(B \mid A)$ For independent events A and B, $\text{P}(B \mid A) = \text{P}(B)$ $\text{P}(A \mid B) = \text{P}(A)$ $\text{P}(A \cap B) = \text{P}(A)\ \text{P}(B)$	

<table>
<tr><th>Topic</th><th>GIVEN</th><th>LEARN</th></tr>
<tr>
<td>TOPIC 4: STATISTICAL DISTRIBUTIONS</td>
<td>
<table>
<tr><th>Distribution of X</th><th>$P(X = x)$</th><th>Mean</th><th>Variance</th></tr>
<tr><td>Binomial $B(n, p)$</td><td>$\binom{n}{x} p^x (1-p)^{n-x}$</td><td>$np$</td><td>$np(1-p)$</td></tr>
</table>
The values z in the table are those which a random variable $Z \sim N(0, 1)$ exceeds with probability p; that is, $P(Z > z) = 1 - \Phi(z) = p$.
<table>
<tr><th>p</th><th>z</th><th>p</th><th>z</th></tr>
<tr><td>0.5000</td><td>0.0000</td><td>0.0500</td><td>1.6449</td></tr>
<tr><td>0.4000</td><td>0.2533</td><td>0.0250</td><td>1.9600</td></tr>
<tr><td>0.3000</td><td>0.5244</td><td>0.0100</td><td>2.3263</td></tr>
<tr><td>0.2000</td><td>0.8416</td><td>0.0050</td><td>2.5758</td></tr>
<tr><td>0.1500</td><td>1.0364</td><td>0.0010</td><td>3.0902</td></tr>
<tr><td>0.1000</td><td>1.2816</td><td>0.0005</td><td>3.2905</td></tr>
</table>
</td>
<td>Normal distribution - $Z = \frac{X-\mu}{\sigma}$</td>
</tr>
</table>

<table>
<tr><th>Topic</th><th>GIVEN</th><th>LEARN</th></tr>
<tr>
<td>TOPIC 5: STATISTICAL HYPOTHESIS TESTING</td>
<td>For a random sample of n observations from $N(\mu, \sigma^2)$

$\frac{\bar{X} - \mu}{\sigma / \sqrt{n}} \sim N(0, 1)$</td>
<td>Normal distribution of the sample mean –

$Z = \frac{\bar{X} - \mu}{\frac{\sigma}{\sqrt{n}}}$</td>
</tr>
</table>

Topic	GIVEN	LEARN
TOPIC 7: KINEMATICS	For motion in a straight line with constant acceleration: $v = u + at$ $s = ut + \frac{1}{2}at^2$ $s = vt - \frac{1}{2}at^2$ $v^2 = u^2 + 2as$ $s = \frac{1}{2}(u + v)t$	For motion in a straight line with non constant acceleration  Kinematics with vectors $\boldsymbol{r} = \boldsymbol{r_0} + \boldsymbol{v}t$ (with constant velocity, so a=0) $\boldsymbol{v} = \boldsymbol{u} + \boldsymbol{a}t$ (constant acceleration) $\boldsymbol{r} = \boldsymbol{u}t + \frac{1}{2}\boldsymbol{a}t^2$ (constant acceleration)

Topic	GIVEN	LEARN
TOPIC 8: FORCES & NEWTONS LAWS		F=ma **F**=ma (with **F** and **a** as vectors) $F \leq \mu R$ $F_{max} = \mu R$ (limiting equilibrium) Projectiles • **Time of flight** $= \frac{2U\sin\alpha}{g}$ • **Time to reach greatest height** $= \frac{U\sin\alpha}{g}$ • **Range on horizontal plane** $= \frac{U^2\sin 2\alpha}{g}$ • **Equation of trajectory:** $y = x\tan\alpha - gx^2\frac{(1+\tan^2\alpha)}{2U^2}$

Topic	GIVEN	LEARN
TOPIC 9: MOMENTS		Moment of Force about a point P = $\lvert Force \rvert \times perpendicular\ distance$