Chapter 2 Wave–particle duality

2.1 Early theories of light

Learning objectives

→ Describe Newton's corpuscular theory of light.

→ State what corpuscular theory predicted should be observed in Young's double slits experiment.

→ Explain what Young's double slits experiment tells you about the nature of light.

Newton's corpuscular theory of light

Newton imagined a light ray as a stream of tiny particles which he referred to as 'corpuscles'. He developed his ideas to explain reflection and refraction.

- When a light ray is **reflected** by a plane mirror, Newton said the corpuscles bounce off the mirror without loss of speed. To explain the law of reflection, he said that the normal component of velocity, v_N, of each corpuscle is reversed and the component of velocity parallel to the mirror, $v_{\uparrow\uparrow}$ is unchanged. Figure 1 shows the idea. Since the magnitude of normal and parallel components of velocity are unchanged on reflection, it can be shown that the angle of reflection, r, is equal to the angle of incidence, i.

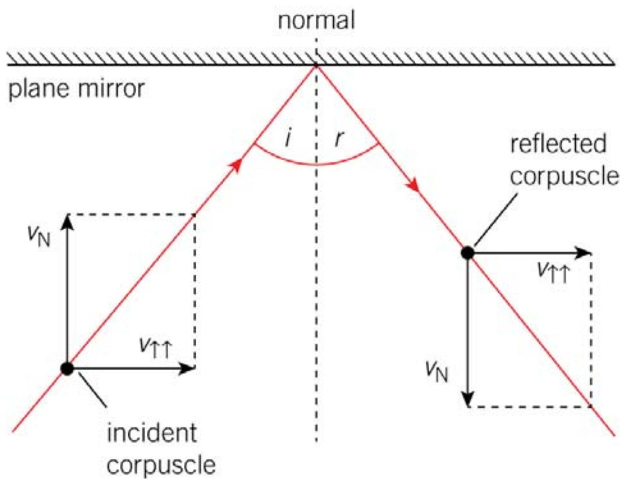

Figure 1 *Reflection of light according to Newton*

- When a light ray is **refracted** on passing from air into a transparent substance, Newton said the corpuscles are attracted into the substance so they travel faster in the substance than in air. To explain the law of refraction, he said that the component of velocity perpendicular to the boundary of each corpuscle is increased as the corpuscle crosses the boundary into the substance and the component of velocity parallel to the boundary is unchanged. Figure 2 shows the idea.

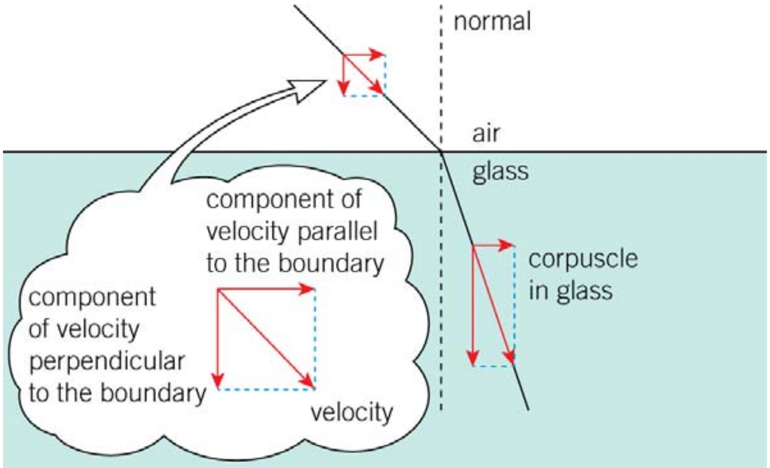

Figure 2 *Refraction of light according to Newton*

Link

Refraction was looked at in Topic 5.1, Refraction of light, in Year 1 of the *AQA Physics* student book.

The wave theory of light put forward by Huygens at roughly the same time also explained reflection and refraction of light. Huygens' explanation of reflection and refraction assumed light waves travel *slower* in a transparent substance than in air. The wave theory of light was rejected by most scientists in favour of Newton's corpuscular theory because:

- It was not possible to measure the speed of light in air or in a transparent substance at that time so there was no experimental evidence showing whether light travels faster or slower in a transparent substance than in air.
- Newton had a much stronger scientific reputation than Huygens so most scientists thought Newton's theory was correct.
- The wave theory of light was considered in terms of longitudinal waves so could not explain polarisation of light.

The significance of Young's double slits experiment

Newton's corpuscular theory was the accepted theory of light for over 150 years until Thomas Young in 1803 showed that light passed through double slits produces an interference pattern. Since interference is a wave property, Young's experiment challenged the accepted theory that light consists of corpuscles. Figure 3 represents light waves from a single slit S passing through double slits where the waves spread out because they undergo diffraction as they pass through the slits. The double slits act as coherent sources of waves so the wavefronts from each of the slits produce an interference pattern where they overlap consisting of parallel equally spaced bright and dark fringes.

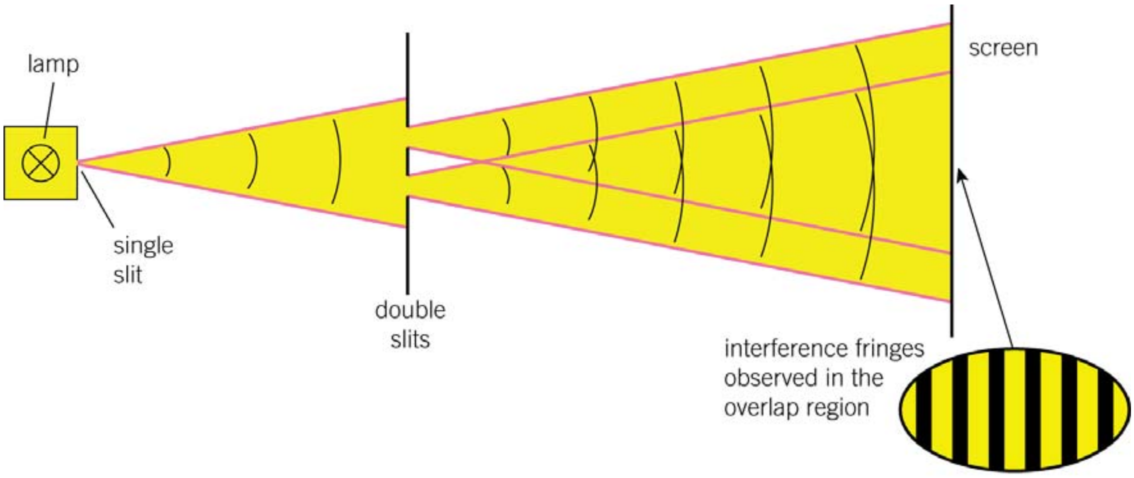

Figure 3 *Young's double slits*

- Each bright fringe is formed where light waves from each slit arrive in phase so they reinforce one another.
- Each dark fringe is formed where light waves from each slit arrive 180° out of phase and therefore cancel each other out.

The number of fringes observed depends on the slit spacing and the width of each slit. The further the slits are from each other (i.e., the larger the slit spacing), the smaller the fringe spacing is. The narrower the slits, the greater the amount of diffraction so the overlap region is greater. Therefore, more fringes are observed using widely spaced slits compared with closely spaced slits of the same width.

Newton's corpuscular theory of light predicted that corpuscles would pass through each slit so two bright fringes would be seen corresponding to light passing through each slit. Figure 4 shows the idea. Clearly, the corpuscular theory could not explain the observed interference patterns whereas wave theory could.

Link

Interference was looked at in Topic 5.4, Double slit interference, in Year 1 of the *AQA Physics* student book.

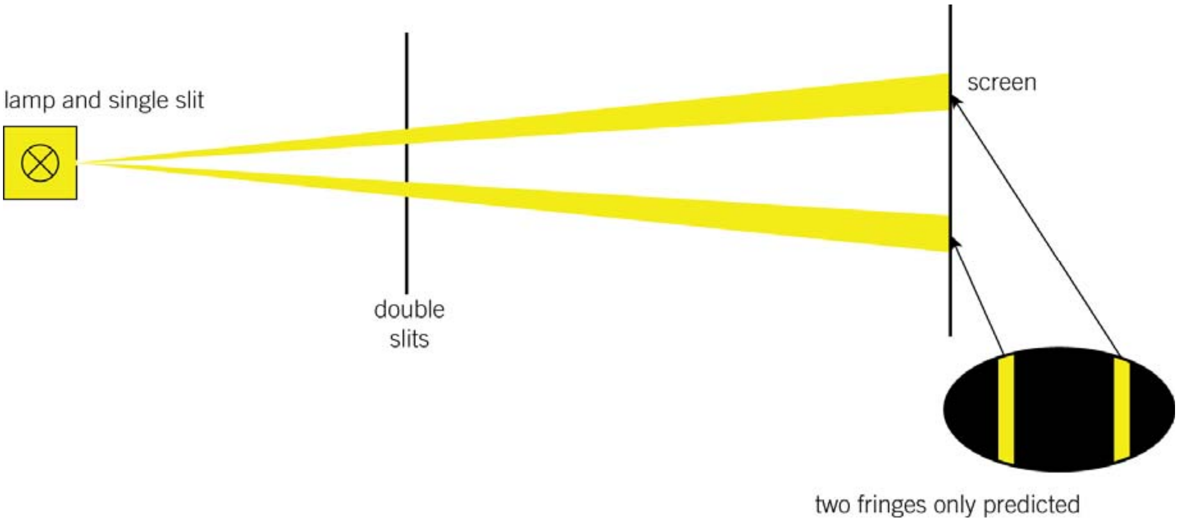

Figure 4 *The corpuscular theory prediction*

Young's double slits experiment showed light undergoes interference which is a property of waves. However, the wave theory of light was not accepted even after Young demonstrated interference of light for the reasons stated earlier. This was until it was shown by Armand Fizeau and Leon Foucault in separate experiments in 1850 that light in water travels slower than light in air. With this experimental evidence, scientists realised that light is a waveform and it must be transverse rather than longitudinal in order to explain polarisation.

Summary questions

1 State two differences between Newton's theory of light and that of Huygens.

2 Explain with the aid of a diagram how Newton explained the refraction of a light ray when the light ray passes from air into glass.

3 **a** Describe the fringe pattern observed in Young's double slits experiment and explain why it could not be explained using Newton's theory of light.

b Use the wave theory of light to explain the formation of the interference fringes.

4 **a** State one reason why Newton's theory of light was accepted in favour of Huygens' theory.

b Give one reason why the wave theory of light was not accepted immediately after Young first demonstrated that interference is a property of light.

2.2 The discovery of electromagnetic waves

Learning objectives

→ Define electromagnetic waves.
→ Explain what Maxwell proved about the speed of electromagnetic waves.
→ Describe how radio waves were first produced and detected.
→ Describe how the speed of electromagnetic waves was first measured accurately.

Maxwell's theory of electromagnetic waves

The wave theory of light developed into the theory of electromagnetic waves. This was as a result of theoretical work by James Clark Maxwell who showed mathematically in 1865 that a changing current in a wire creates waves of changing electric and magnetic fields that radiate from the wire. Maxwell showed that the waves are transverse in nature and that the electric waves are in phase with and perpendicular to the magnetic waves as shown in Figure 1.

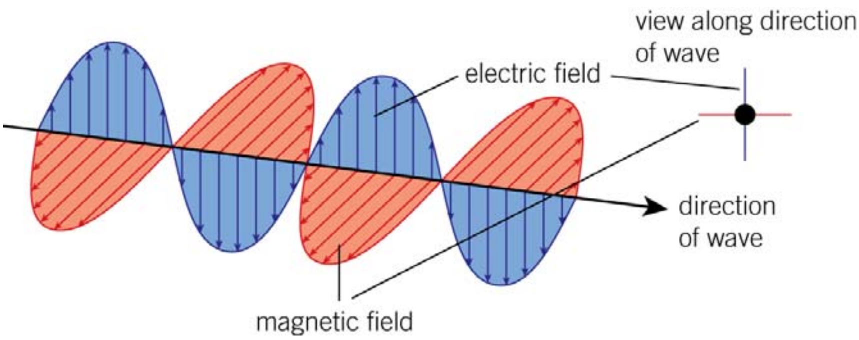

Figure 1 *Electromagnetic waves*

In effect, an alternating current in a wire creates an alternating magnetic field, which generates an alternating electric field, which generates an alternating magnetic field further from the wire, which generates an alternating electric field, which generates an alternating magnetic field yet further from the wire, and so on. Maxwell knew that the strength of the electric field depends on the permittivity of free space, ε_0, and he knew that the magnetic field strength depends on the equivalent magnetic constant, the permeability of free space, μ_0. He showed mathematically that the speed of electromagnetic waves in free space, c, is given by

$$c = \frac{1}{\sqrt{\mu_0 \varepsilon_0}}$$

The speed of the waves in free space can therefore be calculated using the values of ε_0 and μ_0 determined from separate electric field and magnetic field measurements. Given $\varepsilon_0 = 8.85 \times 10^{-12}$ F m^{-1} and $\mu_0 = 4\pi \times 10^{-7}$ T m A^{-1}, prove for yourself using the given values of ε_0 and μ_0 that the above formula gives 3.0×10^8 m s^{-1} which is the speed of light in free space.

Maxwell's mathematical theory showed that light consists of electromagnetic waves and it also predicted the existence of electromagnetic waves outside the boundaries of the then known spectrum which was from infrared to ultraviolet radiation. Many years later, the separate discoveries of X-rays and of radio waves confirmed the correctness of Maxwell's predictions.

Study tip

If you are asked to describe an electromagnetic wave, a diagram is helpful as well as a written description. Remember that the electric and magnetic waves are in phase, perpendicular to each other and perpendicular to the direction in which the wave is travelling.

More about μ_0

The information here is *not* part of the specification for this option. For information about ε_0, see Topic 22.2, Electric field strength, in Year 2 of the *AQA Physics* student book.

1 The magnetic flux density B inside a long current-carrying solenoid is uniform and is proportional to the current I and to the number of turns per unit length n of the solenoid. The permeability of free space, μ_0, may be defined as the magnetic flux density per unit current ÷ the number of turns per unit length of the solenoid. In other words, μ_0 is the constant of proportionality in the equation $B = \mu_0 nI$.

2 From the above definition of μ_0, it can then be shown that the magnetic flux density B at distance d from a long straight wire carrying current I is given by

$$B = \frac{\mu_0 I}{2\pi d}$$

For two such parallel wires X and Y at distance d apart carrying currents I_X and I_Y, using $F = BIL$ gives the force per unit length on each wire due to their magnetic interaction $\frac{F}{L}\left(= BI\right) = \frac{\mu_0 I_X I_Y}{2\pi d}$

The wires attract if the currents are in the same direction and repel if the currents are in opposite directions, as shown in Figure 2.

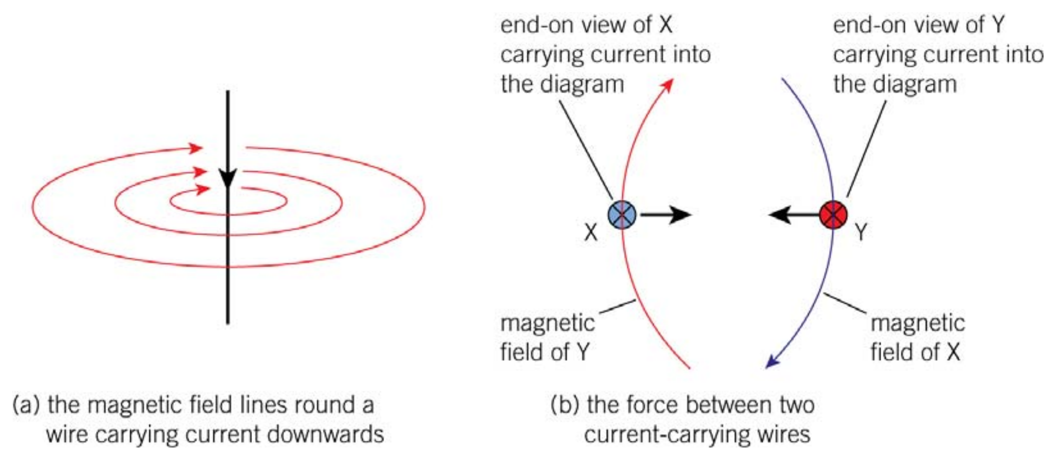

Figure 2 *The magnetic force between two current carrying wires*

The ampere is defined as that current in two parallel wires, one metre apart, which gives a force per unit length on each wire of 2×10^{-7} N m^{-1}. Substituting this value of $\frac{F}{L}$ into the above equation with $I_1 = I_2 = 1$ A and $d = 1$ m therefore gives:

$\mu_0 = 2\pi \times 2 \times 10^{-7} = 4\pi \times 10^{-7}$ T m A^{-1}

QUESTION: Describe how the force between the two wires in Figure 2 would alter if the distance between them was doubled without changing the current.

Hertz's discovery of radio waves

Heinrich Hertz was the first person to discover how to produce and detect radio waves. More than 20 years after Maxwell predicted the existence of radio waves, Hertz showed that such waves are produced when high voltage sparks jump across an air gap and he showed they could be detected using a wire loop with a small gap in it. This is shown in Figure 3.

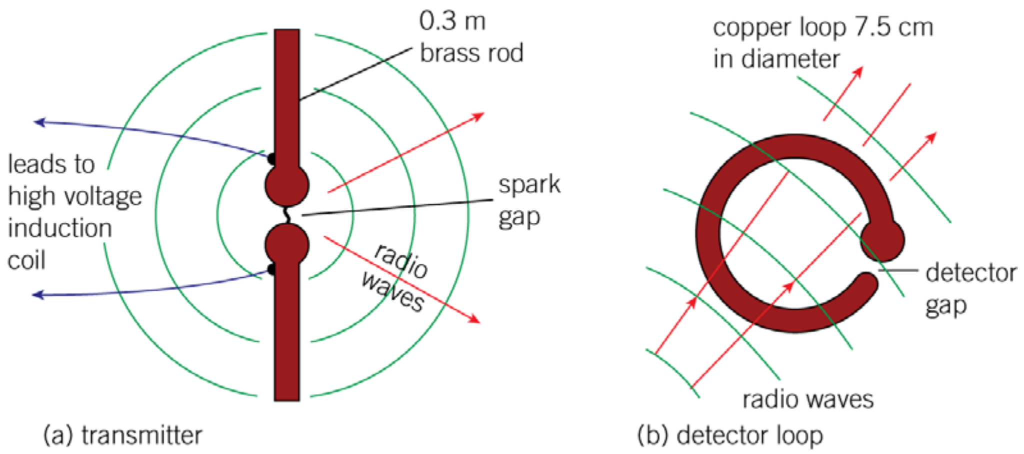

Figure 3 *Hertz's discovery of radio waves*

- The radio waves produced by the spark gap transmitter spread out from the spark gap and pass through the detector loop.

- The waves passing through the detector loop cause a voltage to be induced in the detector loop which makes sparks jump across the detector gap.

Hertz showed that the detector sparks stopped when a metal sheet was placed between the transmitter and the detector thus showing that radio waves do not pass through metal. He found that the radio waves are reflected by a metal sheet and discovered that a concave metal sheet placed behind the transmitter made the detector sparks stronger because it reflected radio waves travelling away from the detector back towards the detector. He also discovered that insulators do not stop radio waves and he showed that the radio waves he produced are polarised.

Note: The induced voltage in the detector loop is due to the oscillating magnetic field of the radio waves. As the waves travel across the loop, the oscillating magnetic field causes oscillating changes in the magnetic flux passing through the loop, which causes an alternating pd to be induced in the loop.

Measuring the wavelength and speed of radio waves

Hertz produced stationary radio waves by using a flat metal sheet to reflect the waves back towards the transmitter, as shown in Figure 4. When he moved the detector along the line between the transmitter and the flat metal sheet, he found that the sparks were not produced at certain detector positions which he realised were the nodes of the stationary wave pattern. He obtained a measurement of 33 cm for the distance between adjacent nodes and so calculated the wavelength of the radio waves as 66 cm (= 2 × the distance between adjacent nodes). Hertz was able to calculate the frequency of the radio waves from the electrical characteristics of the circuit in which the spark gap transmitter was. The charge created by a spark oscillates back and forth across the gap, causing radio waves to be emitted at the same frequency as the spark oscillations. The frequency of the radio waves could be changed by altering the capacitance of the spark gap circuit. Using radio waves of known frequency, Hertz was able to calculate the speed of the radio waves with the equation *speed* = *frequency* × *wavelength*. From his results, he concluded that radio waves travel at the same speed as light and that they are electromagnetic waves.

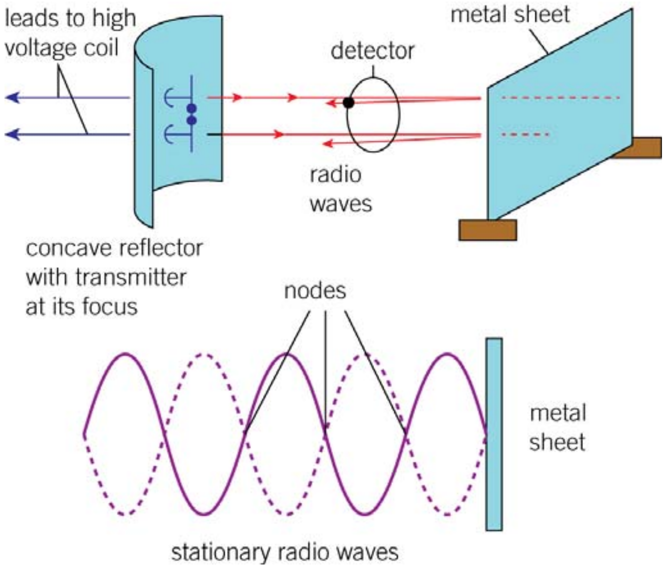

Figure 4 *Measuring the wavelength of radio waves*

Electrical oscillations

Hertz worked out the frequency f of the oscillations in the electrical circuit by using the equation $f = \frac{1}{2\pi\sqrt{LC}}$, where L is the inductance of the spark gap coil and C is the capacitance of the circuit.

- The induced emf in the coil $V_L = L\frac{dI}{dt}$ (from the definition of inductance of a coil which is the induced emf in the coil ÷ the rate of change of current in it), where I is the current in the circuit. The unit of inductance is the henry (H).
- The pd across the capacitor $V_C = \frac{Q}{C}$, where Q is the capacitor charge and C is the capacitance.

The circuit is the electrical equivalent of a mass on a spring in forced oscillation. With negligible resistance, the charge flow in the circuit has a natural frequency of oscillation corresponding to the capacitor pd being equal and opposite to the inductor pd. In other words, at the natural frequency f_0, $\frac{Q}{C} = -L\frac{dI}{dt}$

Rearranging this equation gives $\frac{dI}{dt} = -\frac{Q}{LC}$

Because the current is equal to the rate of change of charge on the capacitor, then $I = \frac{dQ}{dt}$, so $\frac{dI}{dt} = \frac{d^2Q}{dt^2}$

Therefore, $\frac{d^2Q}{dt^2} = -\frac{Q}{LC} = -(2\pi f)^2 Q$, where $(2\pi f)^2 = \frac{1}{LC}$

This is the equation for simple harmonic motion of frequency f. (Look back at Topic 4.2, Measuring waves, in Year 1 of the *AQA Physics* student book.) Because the frequency of the radio waves is equal to the natural frequency of the circuit, rearranging the equation $(2\pi f)^2 = \frac{1}{LC}$ gives the frequency of the radio waves $f = \frac{1}{2\pi\sqrt{LC}}$.

Hertz's circuit produced radio waves of about 450 MHz. He measured the wavelength of the waves at 66 cm. Show that this data gives 3×10^8 m s^{-1} for the speed of electromagnetic waves.

QUESTION: Calculate the capacitance of the transmitter if a 5 μH inductor is used.

Demonstrating the transverse nature of radio waves

Hertz developed a dipole detector consisting of two metal rods aligned with each other at the focal point of a concave reflector, as shown in Figure 5. The reflector focuses the radio waves onto the rods such that the oscillating electric field of the radio waves creates an alternating pd between the two rods (as they are parallel to the electric field), causing sparks at the spark gap.

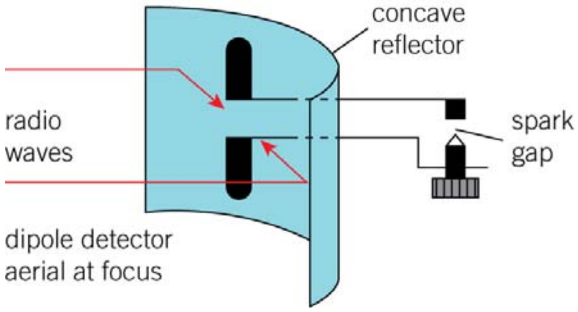

Figure 5 *A dipole detector – construction*

Hertz discovered that when the reflector and the dipole were parallel to the transmitter spark gap, a strong signal was obtained. When the dipole and reflector were rotated gradually from this position, the detector signal decreased and became zero at an angle of rotation of 90° from the initial position. Hertz concluded that the radio waves from the transmitter were polarised and the zero signal was because the dipole had been turned until it was perpendicular to the plane of polarisation of the oscillating electric field which therefore could not produce a pd between the rods.

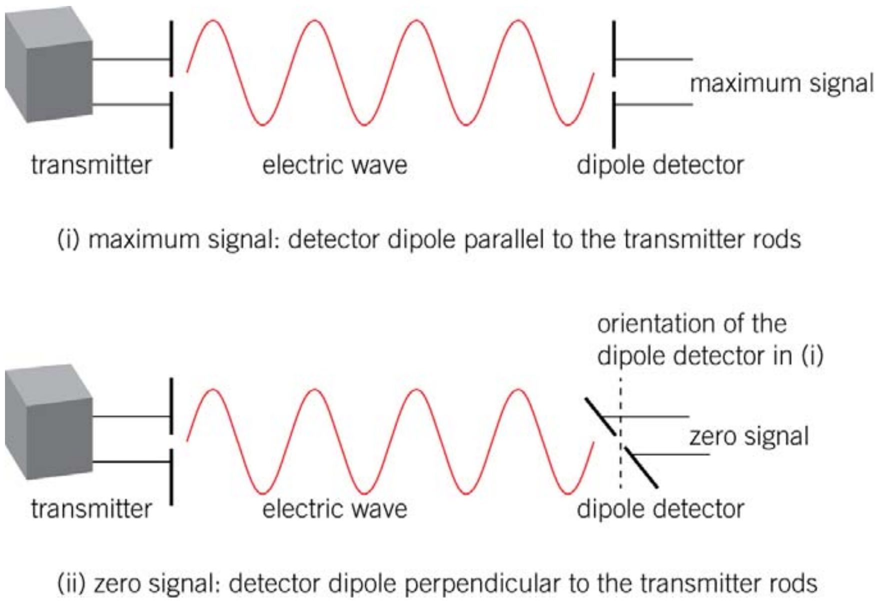

Figure 6 *A dipole detector – polarisation demonstration*

Study tip

The radio waves from the transmitter are polarised when they are created. The dipole detector needs to be parallel to the electric field oscillations for maximum signal strength. The plane of a loop detector needs to be perpendicular to the magnetic field oscillations (which would mean it too is parallel to the electric field oscillations) for maximum signal strength.

Links

Polarisation was looked at in Topic 4.1, Waves and vibrations, in Year 1 of the *AQA Physics* student book.

Stationary waves were looked at in Topic 4.5, Stationary and progressive waves, in Year 1 of the *AQA Physics* student book.

Fizeau's determination of the speed of light

Before Maxwell developed his theory of electromagnetic waves, Armand Fizeau developed the first accurate method of measuring the speed of light in air. The results he obtained helped Maxwell to establish his theory. Fizeau's method involved using a rapidly rotating cog wheel that had 720 teeth around its edge. A narrow beam of light was directed parallel to the axis of the wheel, as shown in Figure 7, so that light could pass through the gaps between the teeth at the edge of the wheel as the wheel rotated. Successive teeth and gaps would therefore chop the beam into pulses of light. The beam was then allowed to travel a long distance to a reflecting mirror, which reflected the light back to where it left the edge of the wheel. The observer, positioned as in Figure 7, will see the reflected light only if a gap is in the path of the returning beam at the moment each pulse of light returns to the wheel. If the gap is replaced by a tooth when the pulse returns, no reflected light will be seen by the observer.

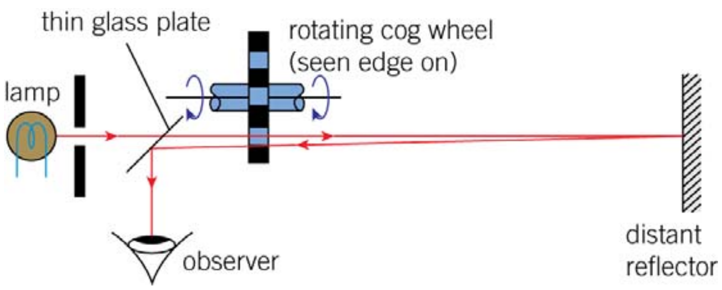

Figure 7 *Fizeau's method*

In the experiment, Fizeau increased the rotation frequency of the cog wheel from rest until the reflected light could not be seen. At this frequency, f_0, the time taken for each pulse to travel from the wheel to the reflector and back is equal to the time taken, t, for a gap to be replaced by a tooth. In this time, each pulse travels a distance equal to $2D$, where D is the distance from the cog wheel to the reflector.

Because the time T for one rotation of the wheel is equal to $\frac{1}{f_0}$, then $t = \frac{T}{2N}$,

where N is the number of teeth on the wheel. So, $t = \frac{1}{2f_0N}$.

Therefore, the speed of light $c = \frac{\text{distance}}{\text{time}} = \frac{2D}{t} = \frac{2D}{1/2f_0N} = 4Df_0N$.

Fizeau also observed that the reflected light could not be seen when the cog wheel was rotating at frequencies $3f_0$, $5f_0$, $7f_0$, and so on. This is because there are $2N$ gaps and teeth around the edge of the wheel, so the wheel must turn though $\frac{n}{2N}$ of a rotation for a gap to be replaced by a tooth, where n is an odd number. Therefore, no reflected light is observed when the wheel is rotating n times faster than the lowest frequency f_0.

Fizeau's first measurements were made with the reflector at a distance of 8633 m from the cog wheel and the light source. He found that the reflected light could not be seen when the lowest rotation frequency of the cog wheel was 12.6 Hz. Prove for yourself that these results give a value of 3.13×10^8 m s^{-1} for the speed of light in air. Further measurements by Fizeau, and separately by Leon Foucault using a different method, gave a value of c of 2.98×10^8 m s^{-1}, which is within 1% of today's accepted value.

Summary questions

1 With the aid of a diagram, state what is meant by an electromagnetic wave.

2 Explain the significance of the equation $c = \frac{1}{\sqrt{\mu_0 \varepsilon_0}}$ derived by Maxwell for the speed of electromagnetic waves in free space.

3 **a** Describe how Hertz set up a stationary wave pattern of radio waves and explain how he measured the wavelength of the radio waves from the stationary wave pattern.

b Hertz discovered that the strength of the radio signal from a radio wave transmitter varies according to the orientation of the detector. Explain this effect and state the conclusion drawn by Hertz about the radio waves from the transmitter.

4 **a** In Fizeau's experiment to measure the speed of light, the reflected light could not be observed when the cog wheel with 720 teeth was rotating at 12.6 Hz. Calculate the time taken for a light pulse to travel from the cog wheel to the reflector and back at this frequency.

b **i** Fizeau also found the reflected light could not be seen for the same distance at a rotation frequency of 37.8 Hz. Explain why the reflected light could not be seen at this frequency.

ii Calculate the next highest frequency at which the reflected light could not be seen.

2.3 The development of the photon theory of light

Learning objectives

→ Explain what the ultraviolet catastrophe is.

→ Discuss why wave theory is unable to explain photoelectricity.

→ Describe the significance of Einstein's explanation of photoelectricity.

The ultraviolet catastrophe

All glowing objects emit electromagnetic radiation, including visible light and infrared radiation. For example, if the current through a torch bulb is increased from zero to its working value, the filament glows dull red then red then orange-yellow as the current increases and the filament becomes hotter. The spectrum of the light emitted shows that there is a continuous spread of colours whose relative intensities change as the temperature is increased. This example shows that:

- the electromagnetic radiation from a hot object at constant temperature consists of a continuous range of wavelengths
- the distribution of intensity with wavelength changes as the temperature of the hot object is increased.

The intensity of the radiation from a hot object at constant temperature was first measured accurately in 1899. Figure 1 shows how the intensity distribution of such radiation varies with wavelength for different temperatures. The curves are called **black body radiation** curves. A black body is defined as a body that is a perfect absorber of radiation (i.e., it absorbs 100% of radiation incident on it at all wavelengths) and therefore emits a continuous spectrum of wavelengths. Remember from GCSE that a matt black surface is the best absorber and emitter of infrared radiation. A small hole in the door of a furnace is an example of a black body; any electromagnetic radiation that enters the hole from outside would be completely absorbed by the inside walls. A star is a black body because any radiation incident on it would be absorbed, and none would be reflected or transmitted by the star.

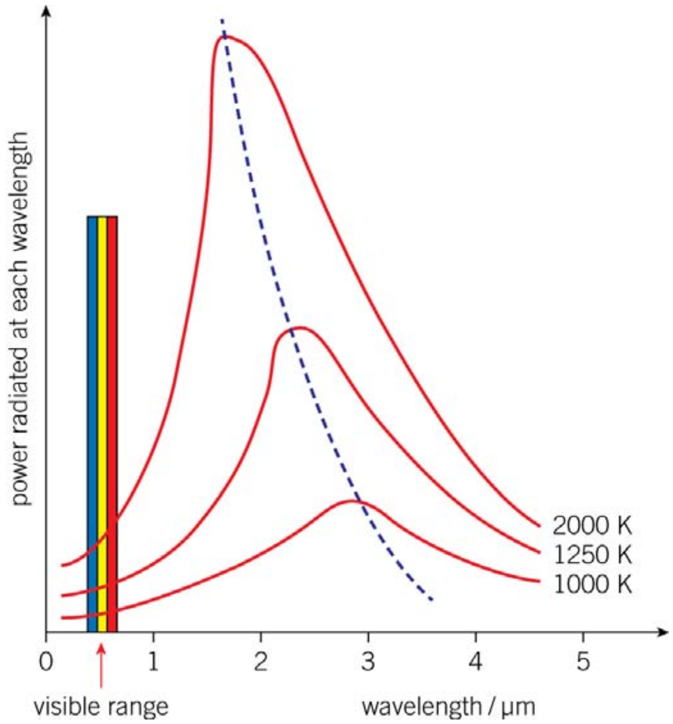

Figure 1 *Black body radiation curves*

Each black body curve in Figure 1 shows an intensity peak at a particular wavelength that depends on the temperature of the black body. However, wave theory cannot explain the existence of such a peak, because wave theory predicts incorrectly that the intensity of the black body radiation from a hot object should become infinite at smaller and smaller wavelengths. This incorrect prediction was called the **ultraviolet catastrophe**. Max Planck solved this problem by introducing the idea that the energy of vibrating atoms can only be in multiples of a basic amount, or quantum of energy. Essentially, the energy levels of vibrating atoms are equally spaced like the rungs of a ladder, as shown in Figure 2. Planck's theory was based on the idea that the quantum of energy of a vibrating atom is proportional to the frequency. He introduced h as the constant of proportionality in the equation $E = hf$ to calculate the least amount (i.e., the quantum) of energy of an atom vibrating at frequency f. Planck used his quantum theory that the energy of a vibrating atom is equal to nhf, where n is a whole number, to develop an equation that fitted black body radiation curves exactly.

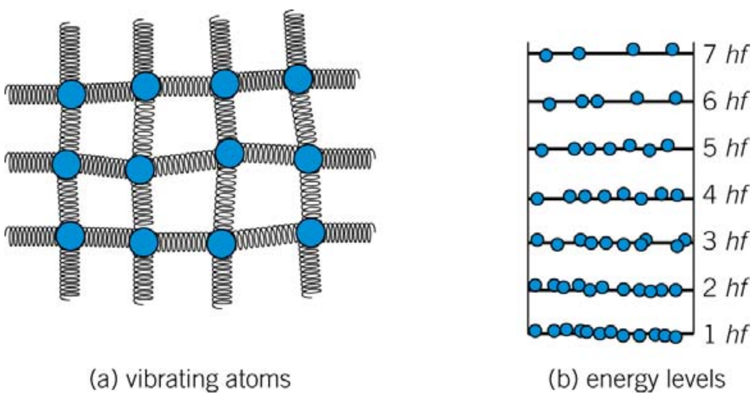

Figure 2 *Energy quanta*

The discovery of photoelectricity

Metals emit electrons when supplied with sufficient energy. Thermionic emission involves supplying the required energy by heating the metal. Another way of supplying the energy is by means of photoelectric emission which involves illuminating the metal with light above a certain frequency. Figure 3 shows a simple demonstration of photoelectric emission from a metal plate. The electroscope is given an initial negative charge which causes the gold leaf to rise. When ultraviolet radiation is directed at the zinc plate, the leaf gradually falls as electrons are emitted from the zinc plate. These emitted electrons are referred to as photoelectrons. The leaf stops falling if the ultraviolet lamp is switched off and resumes its fall when the lamp is switched on again.

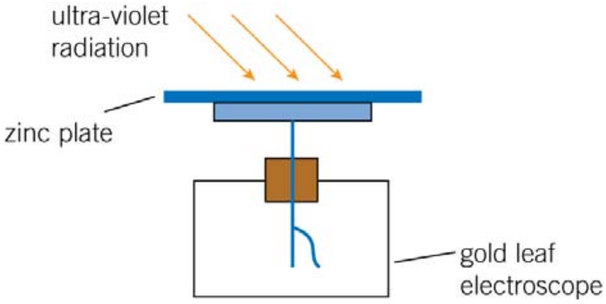

Figure 3 *Demonstrating photoelectricity*

Photoelectric emission was first discovered by Hertz when he was investigating radio waves using a spark gap detector. He observed that the sparks were much stronger when ultraviolet radiation was directed at the spark gap contacts. Further investigations showed that for any given metal:

- No photoelectrons are emitted if the frequency of the incident light is below a certain value known as the **threshold frequency**. This is the minimum frequency of light that can cause photoelectric emission from the metal. For some metals, the threshold frequency is in the frequency range for visible light whereas for other metals it is in the ultraviolet range.
  (Note: references below to light in this context cover visible and ultraviolet light.)
- Photoelectric emission occurs at the instant that light of a suitably high frequency is incident on the metal surface.
- The photoelectrons have a range of kinetic energies from zero up to a maximum value that depends on the type of metal and the frequency of the incident light.
- The number of photoelectrons emitted from the metal surface per second is proportional to the intensity of the incident radiation (i.e., the light energy per second incident on the surface). The more intense the radiation is, the greater the number of photoelectrons leaving the metal each second.

The wave theory of light failed to explain the above observations. According to wave theory, light of any frequency should cause photoelectric emission. Wave theory predicted that the lower the frequency of the light, the longer the time taken by electrons in the metal to gain sufficient kinetic energy to escape from the metal. So the wave theory could not account for the existence of the threshold frequency and it could not explain the instant emission of photoelectrons or their maximum kinetic energy.

Einstein's explanation of photoelectric emission

The photoelectric effect could not be explained for more than 10 years until Einstein, in 1905, put forward the **photon theory** of light and used it to explain photoelectricity. Before Einstein developed the theory, the idea of energy 'quanta' as packets of energy had been used by Planck to explain the continuous spectrum of thermal radiation emitted by an object at a constant temperature. To explain photoelectricity, Einstein applied this 'quantum' idea to electromagnetic radiation which he said consists of wavepackets of electromagnetic energy which he referred to as **photons**, each carrying energy given by $E = hf$, where f is the frequency of the radiation and h is the Planck constant introduced earlier by Planck.

Einstein knew that a conduction electron in a metal needs to have a minimum amount of energy, referred to as the **work function** ϕ of the metal, to escape from the surface of the metal. To explain photoelectricity, he assumed that in order for a conduction electron to escape, it needs to:

- absorb a single photon and therefore gain energy hf
- use energy equal to the work function ϕ of the metal to escape.

Since the mean kinetic energy of a conduction electron in a metal at room temperature is negligible compared with the work function of the metal, it follows that the electron can only escape if the energy it gains from a photon is greater than or equal to the work function of the metal. In other words, $hf \geq \phi$. The frequency of the incident radiation $f > \frac{\phi}{h}$ for photoelectric emission to occur. So, the threshold frequency of the incident radiation, $f_0 = \frac{\phi}{h}$

Notes

1 Einstein assumed a conduction electron absorbs a single photon and that the energy of a photon can only be transferred to one electron, not be spread between several electrons.

2 The threshold frequency is the least frequency of incident radiation on the metal surface that will cause photoelectric emission. The corresponding wavelength, the threshold wavelength $\lambda_0 = \frac{c}{f_0}$, is therefore the maximum wavelength that will cause photoelectric emission from the metal surface.

3 Einstein used the photon theory to explain why, when a metal surface is illuminated by monochromatic radiation of frequency greater than the threshold frequency, the photoelectrons emitted have a maximum amount of kinetic energy that depends on the metal. He assumed that each photoelectron must have absorbed one and only one photon so that its kinetic energy after escaping is equal to the energy it gained, hf, less the energy it used to escape. Because the work function of the metal is the minimum energy an electron needs to escape, the maximum kinetic energy E_{Kmax} of a photoelectron from the metal is given by:

$$E_{Kmax} = hf - \phi$$

Study tip

Remember that photoelectric emission is a one-to-one process in which one electron absorbs one photon.

Stopping potential and threshold frequency

As explained in Topics 3.1 and 3.2 in the *AQA Physics* student book, by making the metal plate or 'cathode' of a vacuum photocell increasingly positive with respect to the metal anode, photoelectric emission is stopped at a certain potential referred to as the **stopping potential**, V_S. At the stopping potential, the maximum kinetic energy of an emitted electron is equal to eV_S (the work that needs to be done to move through potential difference V_S).

Therefore $E_{Kmax} = hf - \phi = eV_S$

The stopping potential is therefore given by

$$eV_S = hf - \phi$$

The stopping potential for different frequencies of incident light can be measured and plotted as a graph of stopping potential against frequency. The result for any metal is a straight line graph in agreement with the equation above rearranged as

$$V_S = \frac{hf}{e} - \frac{\phi}{e}$$

Comparison with the general equation for a straight line graph $y = mx + c$ therefore gives:

- $\frac{h}{e}$ for the gradient m
- $-\frac{\phi}{e}$ for the y-intercept
- $f_0 = \frac{\phi}{h}$ for the x-intercept.

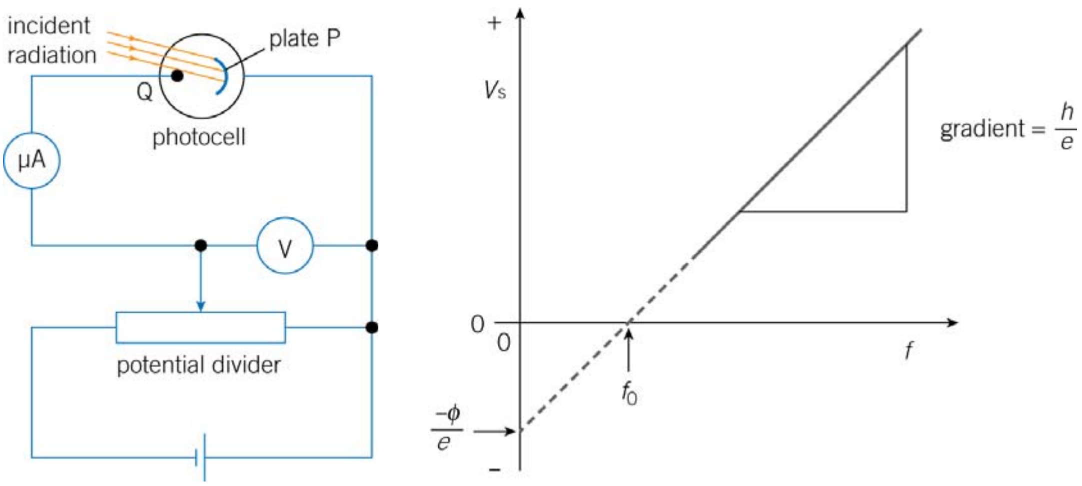

Figure 4 *(a) Investigating photoelectricity (b) Stopping potential graph*

Measurement of stopping potential

Figure 4 (a) shows how the stopping potential can be measured for different frequencies of light using a vacuum photocell. Light of different frequencies can be obtained by passing white light from a filament lamp through different monochromatic filters. With one of the filters in the path of the incident radiation, the potential divider is adjusted initially so the pd across the photocell is zero. The microammeter can be used to measure the current through the photocell due to photoelectric emission from metal plate P (the cathode). When the potential divider is adjusted to make P increasingly positive relating to the collecting terminal Q (the anode), the microammeter reading decreases and becomes zero when the potential of plate P is equal to the stopping potential for that frequency of light. The procedure is then repeated for other values of frequency. A graph of stopping potential against frequency can then be plotted, as shown in Figure 4 (b). The first measurements obtained in this way gave results and a graph as above that confirmed the correctness of Einstein's explanation and thus confirmed Einstein's photon theory of light.

Einstein was awarded the 1921 Nobel prize for physics for the photon theory of light which he put forward in1905 although it was not confirmed experimentally until ten years later.

Link

The vacuum photocell was looked at in Topic 3.2, More about photoelectricity, in Year 1 of the *AQA Physics* student book.

Worked example

$e = 1.6 \times 10^{-19}$ C, $h = 6.63 \times 10^{-34}$ J s, $c = 3.00 \times 10^{8}$ m s^{-1}

Monochromatic light of wavelength 560 nm incident on a metal surface in a vacuum photocell causes a current through the cell due to photoelectric emission from the metal cathode. The emission is stopped by applying a positive potential of 1.30 V to the cathode relative to the anode. Calculate:

a the work function of the metal cathode in electron volts

b the maximum kinetic energy of the emitted photoelectrons when the cathode is at zero potential.

Solution

a The frequency of the incident light $f = \frac{c}{\lambda} = \frac{3.00 \times 10^{8}}{560 \times 10^{-9}} = 5.36 \times 10^{14}$ Hz

Rearranging $eV_S = hf - \phi$ gives

$\phi = hf - eV_S = (6.63 \times 10^{-34} \times 5.36 \times 10^{14}) - (1.6 \times 10^{-19} \times 1.30) = 1.47 \times 10^{-19}$ J $= 0.92$ eV

b At zero potential, the maximum kinetic energy $= hf - \phi = eV_S = 2.08 \times 10^{-19}$ J

Study tip

Remember that the work function is the *minimum* energy an electron needs to escape from the metal.

The significance of Einstein's photon theory

Einstein showed that light consists of photons which are wavepackets of electromagnetic radiation, each carrying energy *hf*, where *f* is the frequency of the radiation. The photon is the least quantity or 'quantum' of electromagnetic radiation and may be considered as a massless particle. It has a dual 'wave–particle' nature in that its particle-like nature is observed in the photoelectric effect and its wave-like nature is observed in diffraction and interference experiments such as Young's double slits experiment.

Summary questions

$e = 1.6 \times 10^{-19}$ C, $h = 6.63 \times 10^{-34}$ J s, $c = 3.00 \times 10^{8}$ m s^{-1}

1 Light of wavelength 535 nm is directed at a metal surface that has a work function of 1.85 eV. Calculate:

- **a** the energy of a photon of wavelength 535 nm
- **b** the maximum kinetic energy of the emitted photoelectrons.

2 When a metal surface is illuminated with light of wavelength 410 nm, photoelectrons are emitted with a maximum kinetic energy of 2.10×10^{-19} J. Calculate:

- **a** the energy of a photon of this wavelength
- **b** the work function of the metal surface
- **c** the stopping potential for this surface illuminated by light of wavelength 410 nm.

3 A certain metal at zero potential emits photoelectrons when it is illuminated by blue light but not when it is illuminated with red light. Explain why photoelectric emission from this metal takes place with blue light but not with red light.

4
- **a** State one experimental observation in the photoelectric effect that cannot be explained using the wave theory of light.
- **b** Describe how the observation in **a** is explained using the photon theory of light.

2.4 Matter waves

Learning objectives

→ Discuss whether matter particles have a dual wave–particle nature.
→ Explain whether matter particles can be diffracted.
→ Explain why an electron microscope is more powerful in terms of magnification than an optical microscope.

Wave–particle duality

If light has a particle-like nature as well as a wave-like nature, do matter particles have a wave-like nature as well as a particle-like nature? In other words, do matter particles have a dual wave–particle nature? Louis de Broglie in 1925 suggested they do. He put forward the hypothesis that all matter particles have a wave-like nature. He said the particle momentum mv is linked to its wavelength λ by the equation

$$mv \times \lambda = h \qquad \text{where } h \text{ is the Planck constant.}$$

De Broglie arrived at this equation after successfully explaining one of the laws of thermal radiation by using the idea of photons as 'atoms of light'. Although photons are massless, in his explanation he supposed a photon of energy hf to have an equivalent mass m given by $mc^2 = hf$ and therefore a momentum $mc = \frac{hf}{c} = \frac{h}{\lambda}$ where λ is its wavelength.

De Broglie's theory of matter waves and his equation 'momentum × wavelength = h' remained a hypothesis for several years until the experimental discovery that electrons in a beam were diffracted when they pass through a very thin metal foil. Figure 1 shows the arrangement.

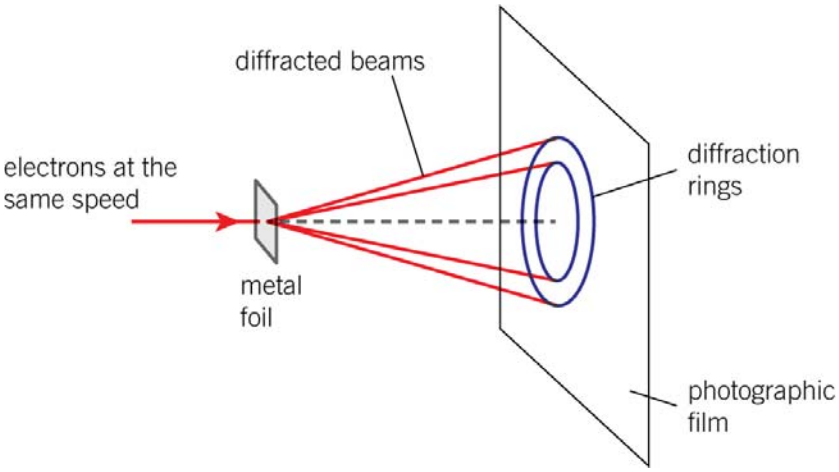

Figure 1 *Diffraction of electrons*

Photographs of the diffraction pattern showed concentric rings, similar to those obtained using X-rays. Since X-ray diffraction was already a well-established experimental technique for investigating crystal structures, it was realised that similar observations with electrons instead of X-rays meant that electrons can also be diffracted and therefore they have a wave-like nature. So de Broglie's hypothesis was thus confirmed by experiment. Particles do have a wave-like nature.

The correctness of de Broglie's equation was also confirmed as the angles of diffraction were observed to:

- increase (i.e., the rings became larger) when the speed of the electrons was decreased, and
- decrease (i.e., the rings became smaller) when the speed was increased.

This is because an increase (or decrease) in the speed of the electrons would increase (or decrease) their momentum. This would therefore reduce (or increase) the de Broglie wavelength in accordance with the rearranged form of the de Broglie equation $\lambda = \frac{h}{mv}$, causing the angles of diffraction of the diffracted electrons to decrease (or increase).

For different electron speeds, the angle of diffraction for each ring was measured and used to calculate the de Broglie wavelength of the electrons. The results showed that the de Broglie wavelength of the electrons is inversely proportional to their speed, in accordance with the de Broglie equation.

Notes

1 From X-ray experiments it was known that a metal consists of lots of tiny crystals called 'grains' packed together. The regular array of atoms in each grain causes the X-rays to be diffracted at certain angles only to the incident beam. Because the grains in a metal are orientated in random directions, the diffracted X-rays form a pattern of rings on the photographic film. The same effect is observed with a beam of monoenergetic electrons, as long as their de Broglie wavelength is of the same order of magnitude as the size of an atom. In addition, with electrons, the diffraction rings can be made smaller or larger by altering the speed of the electrons.

2 For electrons produced by thermionic emission, the speed v of the electrons depends on the anode potential V_A in accordance with the equation $\frac{1}{2}mv^2 = eV_A$, assuming $v << c$, the speed of light in free space. Multipying both sides of this equation by $\frac{1}{2}m$ gives $m^2 v^2 = 2meV_A$.

Taking the square root of both sides of this equation gives an equation for the momentum of each electron in terms of the anode potential: $mv = (2\,meV_A)^{\frac{1}{2}}$

Using the de Broglie equation 'momentum × wavelength = h' therefore gives the de Broglie wavelength λ of an electron in terms of the anode potential V_A,

$$\lambda = \frac{h}{\sqrt{(2meV_A)}}$$

3 Electron diffraction by thin metal foils and other atomic arrangements is called low-energy electron diffraction, to distinguish it from the diffraction of electrons of much higher energies by nuclei. High-energy electron diffraction requires electrons that have de Broglie wavelengths of the order of the size of the nucleus (i.e., ≈ 10^{-15} m). Prove for yourself that the momentum of an electron with a de Broglie wavelength of 3×10^{-15} m must be 2.2×10^{-19} kg m s^{-1}. As explained in Chapter 3 Special relativity, the kinetic energy of an electron with this momentum is about 400 MeV. In comparison, the kinetic energy of an electron with a de Broglie wavelength of the order of 10^{-11} m is of the order of 15 keV.

Worked example

$e = 1.6 \times 10^{-19}$ C, $h = 6.63 \times 10^{-34}$ J s, $m_e = 9.11 \times 10^{-31}$ kg

Calculate the de Broglie wavelength of an electron in a beam produced by thermionic emission and accelerated from rest through a pd of 3600 V.

Solution

$$\lambda = \frac{h}{\sqrt{(2meV_A)}} = \frac{6.63 \times 10^{-34}}{\sqrt{(2 \times 9.11 \times 10^{-31} \times 1.6 \times 10^{-19} \times 3600)}} = 2.05 \times 10^{-11} \text{ m}$$

Study tip

Be careful not to confuse the symbol v for speed (or velocity) and the symbol for potential difference V.

Electron microscopes

An electron microscope makes use of the particle nature of the electron because it uses electric and/or magnetic fields to control electrons and it makes use of the wave nature of the electron to obtain detailed images. To form an image of an atom, the electrons need to have a de Broglie wavelength of 0.1 nm which is an 'order of magnitude' value of the diameter of an atom. The above equation gives a anode potential of about 150 V for electrons to have a de Broglie wavelength of 0.1 nm. However, as you will see below, other factors such as lens aberrations in the transmission electron microscope are more significant in determining the detail in an 'electron microscope' image.

The transmission electron microscope (TEM)

The transmission electron microscope consists of an evacuated tube in which a beam of electrons is directed at a thin sample, as shown in Figure 2. Some of the electrons are scattered by the structures in the sample as they pass through the sample (e.g., grain boundaries in a thin metal sample). Electromagnetic coils acting as 'magnetic lenses' focus the scattered electrons onto a fluorescent screen at the end of the tube to form a magnified image of the sample structure.

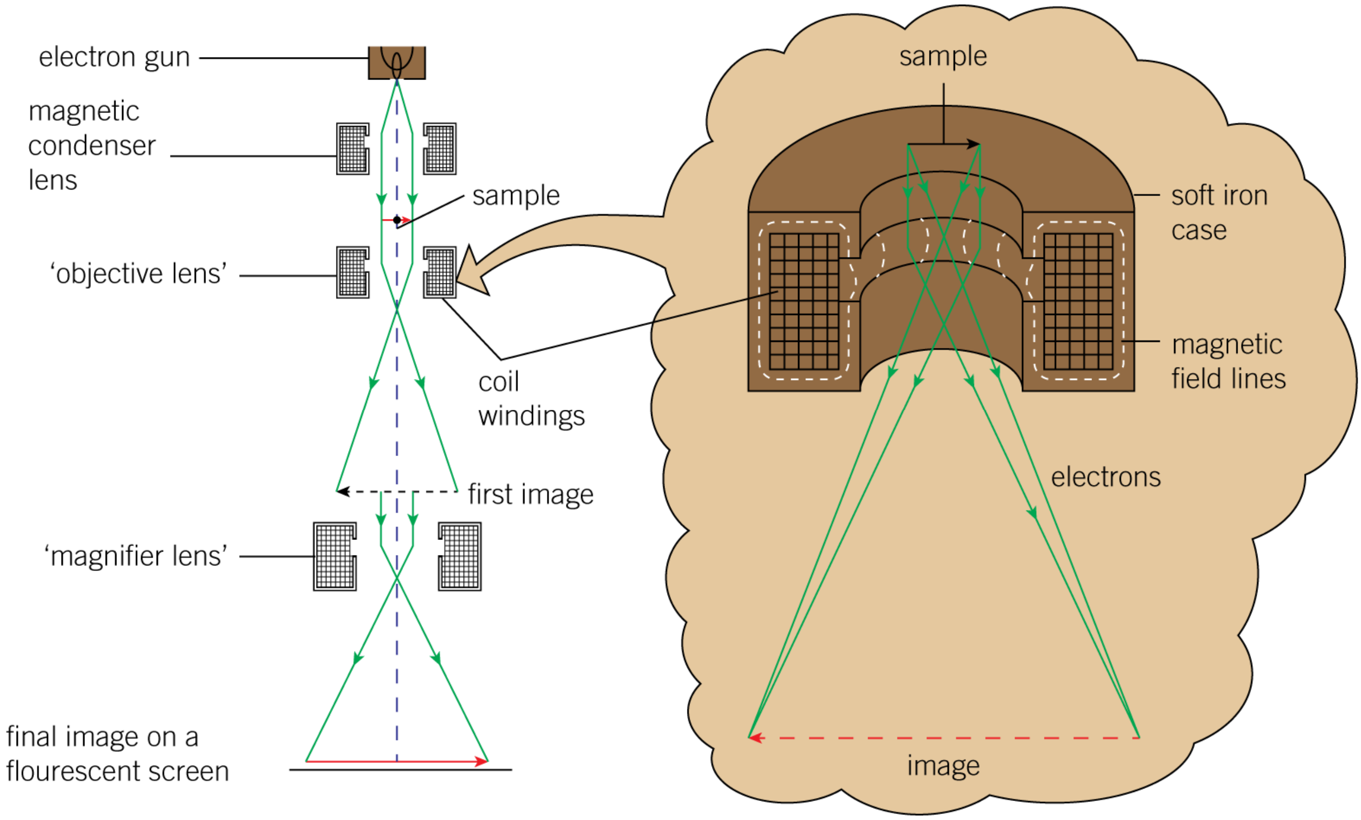

Figure 2 *The transmission electron microscope*

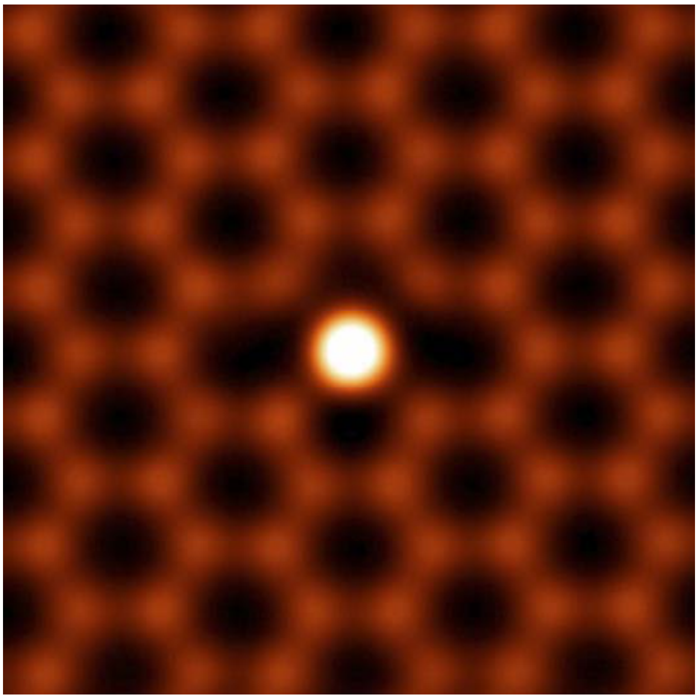

Figure 3 *A coloured TEM image of an individual silicon atom as an impurity in graphene*

1 The 'electron gun' produces electrons by thermionic emission from a heated filament and accelerates them through a hole in a metal anode at constant pd relative to the filament. The electrons emerge through the hole in the anode at the same speed that depends on the anode potential (relative to the filament).

2 The magnetic condenser lens produces a magnetic field that forces the electrons into a parallel beam directed at a very thin sample.

3 The objective lens deflects the scattered electrons so they form an enlarged inverted 'first' image of the sample.

4 The magnifier lens focuses the electrons from the central area of the first image to form a magnified final image on the screen.

The amount of detail in a TEM image (and in an optical microscope image) is determined by the resolving power of the microscope. This is the least separation between two objects in the image that can just be seen apart. The resolving power of a microscope depends on how much diffraction occurs when the electrons (or light in the case of an optical microscope) scattered by the sample pass through the objective lens. As with single slit diffraction, the smaller the wavelength of the waves, the less the amount of diffraction and hence the greater the resolving power.

In an electron microscope, the resolving power can therefore be increased by increasing the anode pd which increases the speed of the electrons and therefore reduces their de Broglie wavelength. Increasing the anode pd also enlarges the image on the screen so a larger and more detailed image is seen.

Note: in an optical microscope, a more detailed image is seen if blue light is used instead of any other colour. This is because blue light has a smaller wavelength than any other colour.

Link

Single slit diffraction was looked at in Topic 5.6, Diffraction, in Year 1 of the *AQA Physics* student book.

The limitations on the amount of detail seen in a TEM image are due to two main factors:

- Sample thickness: electrons passing through the sample suffer a slight loss of speed which increases the de Broglie wavelength slightly and so reduces the resolving power.
- Lens aberrations: the magnetic field in the outer and inner parts of the lens gap may focus electrons from a given point to different positions on the screen instead of to the same position, causing the image to be blurred. Also, the electrons scattered from a given point on the sample may have slightly different speeds due to the process of thermionic emission and also due to passing through different thicknesses of the sample and so they would be focused differently on the screen.

The scanning tunnelling microscope (STM)

In the STM, a fine-tipped metal probe scans across a small area of a surface under investigation at a height of no more than about 1 nm above the surface. The probe is at a constant negative potential of about –1 volt relative to the surface, as shown in Figure 4.

Because the gap between the tip and the surface is so small, there is a small but finite probability that electrons can ‘tunnel’ across the gap. The tip must be at a negative potential relative to the surface to ensure the electrons only tunnel across the gap in one direction (i.e., from the tip to the surface under test).

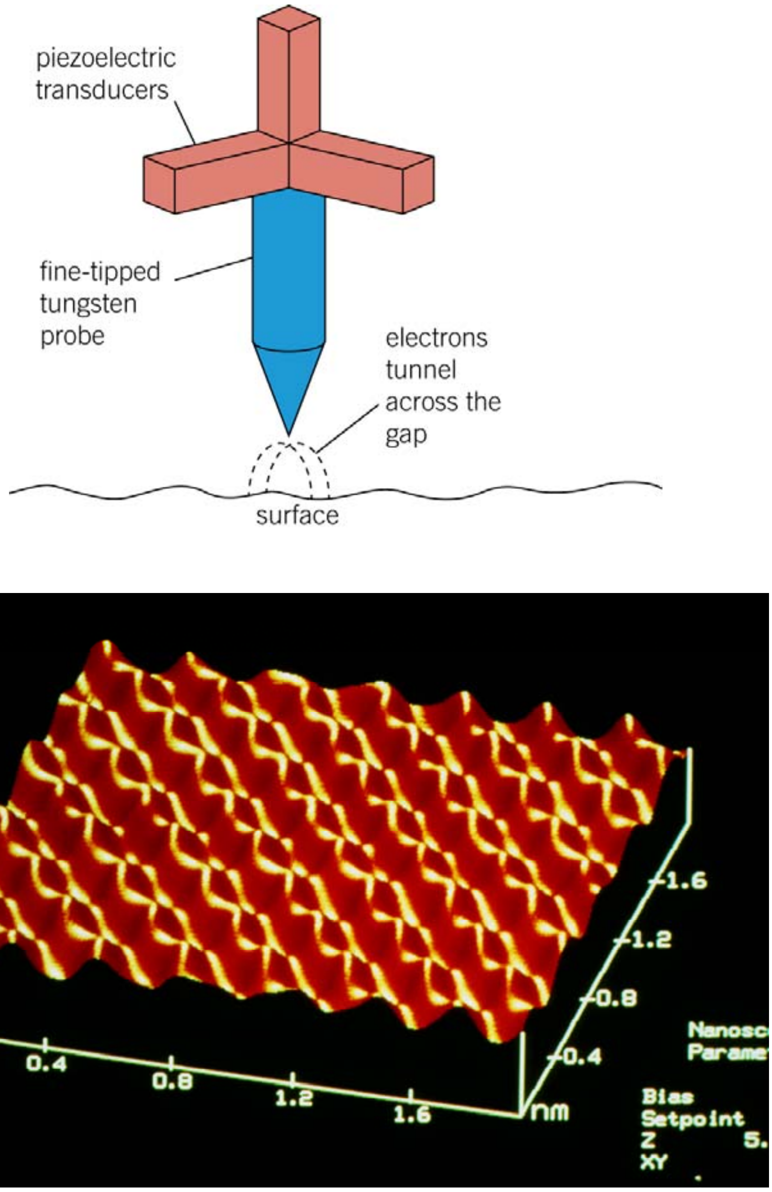

Figure 4 *The scanning tunnelling microscope*

The probe’s scanning movement is controlled by two piezoelectric transducers which move it along successive lines parallel to each other. A third piezoelectric transducer is used to adjust the gap between the tip and the surface. The tunnelling current increases if the gap is made smaller and decreases if the gap is made larger. As the tip scans across the surface, if it moves near a raised atom, the gap width decreases and the tunnelling current increases.

- In constant height mode, the tunnelling current is recorded as the tip scans across the surface in a fixed plane. As it does so, the tunnelling current is recorded and used to map the height of the surface on a computer screen.

- In constant current mode, the gap is kept constant by feeding back changes in the tunnelling current to the piezoelectric transducer that controls the tip height. If the gap between the tip and the surface decreases due to a raised atom, the tunnelling current increases which causes the tip to be raised until the tunnelling current and the gap width are the same as before. The signal to the transducer is recorded and used to map the height of the surface on a computer screen.

In either mode, the image or 'map' of the surface shows the peaks and troughs in the surface due to individual atoms or groups of atoms on the surface. If the initial gap is too large, the tunnelling current will be negligible. If the initial gap is too small, the tip might be damaged by collisions with raised atoms on the surface. The probe and the surface are normally in a vacuum to prevent contamination of the surface under investigation.

The wave nature of the electron is the reason why electrons can cross the gap. They have insufficient kinetic energy to overcome the potential barrier caused by the negative potential and the work function of the metal tip. However, their de Broglie wavelength is sufficiently long to stretch across the narrow gap, giving the electrons a finite probability of crossing the gap. In effect, the amplitude of the electron wave decreases exponentially in the gap as shown in Figure 5. The gap is sufficiently small so:

- the amplitude is finite on the other side of the gap
- small changes in the gap produce measurable changes in the number of electrons per second crossing the gap.

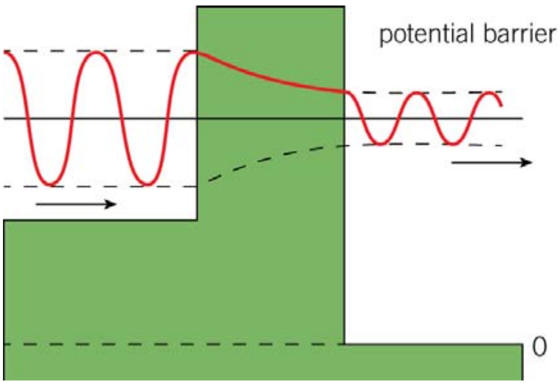

Figure 5 *Electron waves*

Piezoelectricity

Piezoelectricity is a property of certain materials which produce a pd when stretched or compressed and conversely undergo a tiny change of length when a pd is applied to them. This tiny change of length is used to move the tip of an STM by very small distances.

QUESTION: To move the STM tip further down, what must the change be in the pd applied to the sensor that controls the vertical position of the tip?

Summary questions

$e = 1.6 \times 10^{-19}$ C, $h = 6.63 \times 10^{-34}$ J s, $m_e = 9.11 \times 10^{-31}$ kg, $m_p = 1.67 \times 10^{-27}$ kg

1 Calculate the de Broglie wavelength of:

a an electron moving at a speed of 3.2×10^6 ms^{-1}

b a proton moving at the same speed.

2 Calculate the speed and de Broglie wavelength of an electron in an electron beam that has been accelerated from rest through a pd of 2800 V.

3 In a transmission electron microscope, state and explain how the image of a thin sample would change if:

a the anode pd was increased

b the sample was moved so that the beam passed through a thicker part of the sample.

4 a What is meant by the term ‘matter waves’.

b Figure 6 shows a graph of how the tunnelling current in an STM changed when the tip moved along a straight line at constant height. Use the graph to describe and explain how the height of the surface under the tip changed as the tip moved along the line.

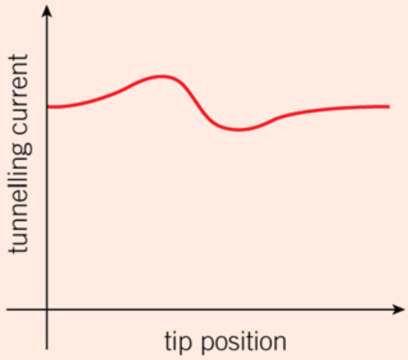

Figure 6