

Answers to examination-style questions

Answers	Marks	Examiner's tips
<p>1 (a) (i) torque = force \times diameter $= 0.12 \times 0.34 = 4.1 \times 10^{-2}$ N m</p> <p>(ii) $T = I\alpha$ $\alpha = \frac{4.1 \times 10^{-2}}{0.17} = 0.24$ rad s⁻²</p> <p>(b) (i) $\omega_2 = \omega_1 + \alpha t$ $0 = 0.92 - 0.24t$ $t = 3.8$ s</p> <p>(ii) $\omega_2^2 = \omega_1^2 - 2\alpha\theta$ $0 = 0.92^2 - (2 \times 0.24 \times \theta)$</p> <p>$\theta = 1.7(6)$ rad $= 1.76 \times \frac{360}{2\pi} = 101^\circ$</p>	<p>1</p> <p>1</p> <p>1</p> <p>1</p> <p>1</p> <p>1</p>	<p>The two equal forces provide a couple. The torque of the couple is one of the forces \times the distance between them, which is the diameter.</p> <p>Simply rearrange the equation for torque and substitute values for T and I.</p> <p>Alternatively, you could use the equation $\theta = \omega_1 t - \frac{1}{2}\alpha t^2$ but for both equations you must use the minus sign to show that the camera is decelerating.</p> <p>The mark here is for the final answer in degrees. Remember that 360° is 2π radians.</p>
<p>2 (a) Angular speed $= 22\,000$ rev min⁻¹ $\times \frac{2\pi}{60}$ $= 2300$ rad s⁻¹ Energy stored $E_K = \frac{1}{2}I\omega^2$ $= 0.5 \times 0.60 \times 2300^2$ $= 1.6$ MJ</p> <p>(b) (i) Time = $\frac{E_K}{P} = \frac{1.6 \times 10^6}{8.7} = 1.84 \times 10^5$ s</p> <p>(ii) Torque = $\frac{\text{power}}{\text{average speed}}$ $= \frac{8.7}{(2300/2)} = 7.5(6) \times 10^{-3}$ N m</p> <p>or: $T = I\alpha$ $= \frac{0.60 \times 2300}{1.84 \times 10^5} = 7.5 \times 10^{-3}$ N m</p> <p>(c) In B more of the mass is at a greater radius than in A ... so I is greater and so energy stored is greater.</p>	<p>1</p> <p>1</p> <p>1</p> <p>1</p> <p>1</p> <p>1</p>	<p>When using $E_K = \frac{1}{2}I\omega^2$, ω must be in rad s⁻¹. To convert rev min⁻¹ to rad s⁻¹ multiply by 2π and divide by 60. Because the question asks you to 'Show that....' you get the mark for showing the calculation correctly. Do not write $E_K = \frac{1}{2}I\omega^2 = 1.6$ MJ.</p> <p>Note that this is about 51 hours! Very low friction bearings have been designed for high energy flywheels.</p> <p>Note that if you use $T = \frac{P}{\omega}$ you must use the average angular speed. Alternatively, use $T = I\alpha$ where $\alpha = \frac{\text{change in angular speed}}{\text{time taken for wheel to come to rest}}$</p> <p>To score the full 2 marks in part (c), you must discuss the way the mass is distributed about the axis. It is not enough simply to write that B must have a larger moment of inertia because its radius is greater.</p>

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<p>3 (a) (i) area under graph = $\frac{\pi}{2}$ $(0.5 \times 25 \times \omega_{\max}) + (25 \times \omega_{\max}) +$ $(0.5 \times 30 \times \omega_{\max}) = \frac{\pi}{2}$ $(12.5 + 25 + 15) \omega_{\max} = \frac{\pi}{2}$ $52.5 \omega_{\max} = 1.57$ giving $\omega_{\max} = 0.030 \text{ rad s}^{-1}$ <i>or</i> area = $(0.5 \times 25 \times 0.030) +$ $(25 \times 0.030) +$ $(0.5 \times 30 \times 0.030)$ $= 1.57 \text{ rad}$ which is $\frac{\pi}{2}$ or 90°</p> <p>(ii) $\alpha = \frac{0.030}{25}$ $= 1.2 \times 10^{-3} \text{ rad s}^{-2}$</p> <p>(b) (i) Using $T = I \alpha$ gives $T = 9.1 \times 10^8 \times 1.2 \times 10^{-3}$ $= 1.09 \times 10^6 \text{ N m}$ Motor torque needed $= (1.09 \times 10^6) + (3.5 \times 10^5)$ $= 1.44 \times 10^6 \text{ N m}$</p> <p>(ii) Power $P = T\omega$ $= 3.5 \times 10^5 \times 0.030$ $= 10.5 \text{ kW}$</p> <p>(iii) Rotational kinetic energy E_k $= 0.5 I \omega^2 = 0.5 \times 9.1 \times 10^8 \times 0.030^2$ $= 4.1 \times 10^5 \text{ J}$</p>	<p>1</p> <p>1</p> <p>1</p> <p>1</p> <p>1</p> <p>1</p> <p>1</p> <p>1</p> <p>1</p> <p>1</p>	<p>Remember that the area under a speed–time graph gives distance travelled, which in this case is 90°, or $\frac{\pi}{2}$ radians. The area has been split into two triangles and a rectangle. The area of a triangle is $\frac{1}{2} \times \text{base} \times \text{height}$.</p> <p>In a question where you are asked to ‘show that ...’ it is vital that you do show all the steps. In the alternative method the ‘answer’ of 0.030 rad s^{-1} has been used to calculate the angular distance, which comes out to the $\frac{\pi}{2}$ rad that the bridge turns through.</p> <p>This is found from the slope of the first part of the graph (up to 25 s).</p> <p>Note the unit for torque. Don’t write $\text{kg m}^2 \text{ rad s}^{-2}$.</p> <p>The motor has to accelerate the bridge and overcome the friction at the bearing, so add the friction torque to the accelerating torque.</p> <p>Once it is going at constant speed the motor only has to overcome the friction torque.</p> <p>Don’t forget to square the maximum angular speed.</p> <p>You must first change the rotational speed in rev min^{-1} to angular speed in rad s^{-1}. One rev is 2π rad and there are 60 s in one minute. N m s is the preferred unit, but the alternatives would be accepted.</p>
<p>4 (a) (i) $1500 (\text{rev min}^{-1}) \times \frac{2\pi}{60} = 157 \text{ rad s}^{-1}$ angular momentum = $I \omega = 0.56 \times 157$ $= 87.9 \text{ N m s}$ (or $\text{kg m}^2 \text{ s}^{-1}$ or $\text{kg m}^2 \text{ rad s}^{-1}$)</p>	<p>1</p> <p>1</p>	<p>You must first change the rotational speed in rev min^{-1} to angular speed in rad s^{-1}. One rev is 2π rad and there are 60 s in one minute. N m s is the preferred unit, but the alternatives would be accepted.</p>

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<p>(b) (i) Angular momentum (or $I\omega$) is constant. I increases when clutch engaged, so ω falls.</p> <p>(ii) I after engagement = $0.56 + 0.94$ = $1.5 \text{ (kg m}^2\text{)}$ $87.9 = 1.5 \times \omega_2$ $\omega_2 = 58.6 \text{ rad s}^{-1}$</p>	<p>1</p> <p>1</p> <p>1</p>	<p>Think of this as a 'collision' between the electric motor shaft and the flywheel. In collisions momentum is always conserved, but kinetic energy isn't. Do not just write 'momentum is conserved' – it is important to mention that <i>angular</i> momentum is conserved here. The total angular momentum before the clutch is engaged must equal the total angular momentum after the engagement. $I_{\text{motor}} \omega_1 = (I_{\text{motor}} + I_{\text{flywheel}}) \omega_2$</p>
<p>5 (a) (i) Use of $\theta = \frac{1}{2}\alpha t^2$ $30 \times 2\pi = 0.5 \alpha 2.1^2$ $\alpha = 85 \text{ rad s}^{-2}$.</p> <p>(ii) $\omega_2 = \omega_1 + \alpha t$ = $0 + 85 \times 2.1$ = 179 rad s^{-1}. (or 180 rad s^{-1})</p> <p>(iii) $T = I \alpha$ = $1.8 \times 10^{-4} \times 85$ = 0.015 N m $F = \frac{0.015(3)}{0.002(0)} = 7.7 \text{ N}$</p> <p>(b) $T = I\alpha = 1.8 \times 10^{-4} \times \frac{179}{72}$ = $4.5 \times 10^{-4} \text{ N m}$</p>	<p>1</p> <p>1</p> <p>1</p> <p>1</p> <p>1</p>	<p>If the string is wrapped 30 times around the axle, the axle must turn 30 full revolutions before the string completely unwinds from the axle. Each revolution is an angular displacement of 2π radians.</p> <p>Alternatively you could use $\omega_2^2 = \omega_1^2 + 2\alpha\theta$ or $\theta = \frac{1}{2}(\omega_1 + \omega_2) t$</p> <p><i>torque = force × radius</i>. Do not forget to halve the axle diameter and convert to metres.</p> <p>The same formula as in (a)(iii) is used, but in this case α is the angular deceleration, as the disc slows from 179 rad s^{-1} to zero in 72 s.</p>
<p>6 (a) (i) Relevant points include:</p> <ul style="list-style-type: none"> • flywheel speeds up (absorbs energy) when driving torque is greater than load torque • slows down (gives up energy) when load torque is high • makes rotational motion smoother (reduces fluctuations in speed) • acts as a reservoir of rotational energy • and so enables machine to run whilst operator uses both hands. 	<p>any 2</p>	<p>You are told at the beginning of the question that the load torque varies, so if there was no flywheel the motion of the driveshaft would be jerky. The flywheel stores the energy put in by the person turning the handle as rotational kinetic energy.</p> <p>An alternative way of approaching this is to use the idea of angular momentum ($I\omega$). The torque on the driveshaft from the sewing will alter the value of $I\omega$, but if I is large ω will not vary very much.</p>

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<p>(b) $E_K = \frac{1}{2}I \omega^2$ $= 0.5 \times 3.7 \times 10^{-3} \times 14^2$ $= 0.36(3) \text{ J}$</p>	1	This is simply a case of putting the numbers into the formula. Don't forget the 0.5 and remember to square ω .
<p>(c) Change in $E_K = \frac{1}{2}I(\omega_1^2 - \omega_2^2)$ $0.082 = 0.5 \times 3.7 \times 10^{-3} \times (14^2 - \omega_2^2)$ $\omega_2 = 12.3 \text{ rad s}^{-1}$</p>	1 1 1	It is very important that you read the question carefully. The kinetic energy of the flywheel is reduced <i>by</i> 0.082 J, (and not <i>to</i> 0.082 J). Alternatively, you could use final $E_K = 0.36 - 0.082 = \frac{1}{2}I \omega_2^2$.
<p>7 (a) Use of $p_1 V_1^\gamma = p_2 V_2^\gamma$ gives $p_2 = p_1 \left(\frac{V_1}{V_2} \right)^\gamma$ $p_2 = 1.0 \times 10^5 \left(\frac{1.2 \times 10^{-5}}{3.1 \times 10^{-7}} \right)^\gamma$ $= 1.6(7) \times 10^7 \text{ Pa}$</p>	1 1	Be careful when using the equation, in noting where the superscript goes: it is only on the Vs. You must include the substitution of the correct values in the rearranged equation because this is a "show that..." type of question and you are given the answer.
<p>(b) (i) $n = \frac{p_1 V_1}{RT_1}$ $= \frac{1.0 \times 10^5 \times 1.2 \times 10^{-5}}{8.31 \times 290}$ $= 5.0 \times 10^{-4} \text{ mol}$</p>	1 1	This revises the ideal gas equation from thermal physics in Unit 5.
<p>(ii) $\frac{p_1 V_1}{T_1} = \frac{p_2 V_2}{T_2}$ $T_2 = \frac{p_2 V_2 T_1}{p_1 V_1}$ $T_2 = \frac{(1.7 \times 10^7) \times (3.1 \times 10^{-7}) \times 290}{(1.0 \times 10^5) \times (1.2 \times 10^{-5})}$ $= 1300 \text{ K}$</p>	1	Alternatively you can use $T_2 = \frac{p_2 V_2}{Rn}$ using your answer to (b)(i) for n .
<p>(c) Relevant points include:</p> <ul style="list-style-type: none"> • $\Delta Q = \Delta U + \Delta W$ with symbols explained • if plunger pushed in slowly, there is sufficient time for heat transfer • most of the work done (ΔW) goes to heat transfer (ΔQ) from tube or metal plug • any increase in ΔU will be zero or very small [or $\Delta U = 0$] • relates ΔU to temperature increase. 	any 3	If the plunger is pushed in slowly, the compression of the gas will not be adiabatic, and there will be time for heat to transfer out of the tube and so there will not be such a high rise in internal energy ΔU . Since the temperature is dependant on the internal energy, the temperature rise will not be enough to ignite the cotton wool. It is important you know the differences between the terms <i>heat transfer</i> , <i>internal energy</i> and <i>temperature</i> .

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8 (a) (i) Work done per kg = area enclosed by loop Suitable method for finding area (such as counting squares) used correctly. Correct scaling factor Leading to 500 kJ.	1 1 1	Do not worry if you have never met a question on a ram jet engine before. You calculate the work done in the same way as for any engine cycle except that here the volume axis is in m ³ per kg, so the area gives the work done per kg of air that passes through the engine. If you count 1 cm × 1 cm squares the work done per square (the scaling factor) is 0.25 × (0.5 × 10 ⁵) J kg ⁻¹ . You must show all the steps in your method.
(ii) Power P = work done per kg × fuel flow rate = 500 (kJ kg ⁻¹) × 9.9 (kg s ⁻¹) = 4950 kW	1	You cannot use the formula for indicated power in the data and formula booklet because this engine has no cylinders. The units give a hint here: $\frac{\text{J}}{\text{s}} = \frac{\text{J}}{\text{kg}} \times \frac{\text{kg}}{\text{s}}$
(iii) Output power = indicated power – friction power $P_{\text{out}} = 4950 - 430$ = 4520 kW	1	There are a lot of ‘powers’ to learn: <i>input</i> power, <i>indicated</i> power, <i>friction</i> power and <i>output</i> power (also called <i>brake</i> power for some engines).
(b) (i) $P_{\text{in}} = \text{fuel flow rate} \times \text{calorific value}$ = 0.30 × 44 × 10 ⁶ = 13(.2) × 10 ⁶ W.	1	
(ii) efficiency = $\frac{4520 \times 10^3}{13.2 \times 10^6} = 0.34$ or 34%	1	overall efficiency = $\frac{\text{output power}}{\text{input power}}$
9 (a) 0.24 × input energy = 1.3 × 10 ¹¹ input energy = $\frac{1.3 \times 10^{11}}{0.24}$ = 5.4 × 10 ¹¹ J	1	This revises efficiency = $\frac{\text{useful output energy}}{\text{input energy}}$ from unit 2.
(b) Mass of fuel = $\frac{5.4 \times 10^{11}}{10.4 \times 10^6}$ = 52 100 kg	1	You can use <i>input power</i> = calorific value × fuel flow rate but note that here we are dealing with <i>energy</i> , not power. In other words input energy = calorific value × mass of fuel.
(c) (i) Changes temperatures to 693 K and 283 K Maximum efficiency $\eta = \frac{T_H - T_C}{T_H}$ $\eta = \frac{693 - 283}{693} = 0.59$ (59%)	1 1	When you use the equation $\frac{T_H - T_C}{T_H}$, the temperatures must be in Kelvin. Remember that absolute temperature in K = temperature in °C + 273

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<p>(ii) Relevant valid reasons include:</p> <ul style="list-style-type: none"> • friction in moving parts • heat losses from boiler, or turbine • energy losses in generator ($I^2 R$) • processes in real steam cycle not truly adiabatic or isothermal. 	any 1	In your answer you should refer to heat loss <i>from</i> part of the plant or <i>to</i> somewhere, usually 'the surroundings', or friction between moving parts (or friction in the turbine or generator). Simply writing 'heat loss' or 'friction' is not enough to gain the mark.
<p>(iii) Relevant points include:</p> <ul style="list-style-type: none"> • does not use an irreplaceable fuel source. • trees will absorb the CO₂ produced in combustion so less contribution to global warming (or 'carbon neutral'). • may not require imports of fossil fuel if wood is grown locally. • wood does not produce acid rain (or no SO₂ produced). 	any 1	Do not assume that wood must be cheaper than oil or coal.
<p>10 (a) (i) Realisation that ΔW (or $p \Delta V$) = 0 So $\Delta U = \Delta Q = 700 \text{ J}$</p>	1	No work is done in process B → C
	1	because the volume does not change. The first law is $\Delta Q = \Delta U + \Delta W$ and since $\Delta W = 0$, ΔU must equal ΔQ .
<p>(ii) $n = \frac{pV}{RT} = \frac{1.0 \times 10^5 \times 0.5 \times 10^{-3} \text{ J}}{8.3 \times 293}$ $n = 0.021 \text{ mol}$</p>	1	The only way to do this is to find somewhere on the cycle where you know the pressure, volume and temperature. This is point A. You have to read the pressure and volume from the graph.
<p>(iii) $W =$ area enclosed (by loop) Appropriate method for finding area (such as counting squares).</p>	1	Counting squares is probably the easiest method. You have a choice – do you count the small (approx 2 mm × 2 mm) squares or the large (approx 1 cm × 1 cm) squares? If you have time, counting small squares will give you a more accurate answer, but counting large squares is fine, provided you end up with an answer in the range 13 to 15 squares.
<p>Correct scaling factor used (to give answer of 350 J ± 20J), for example: 14 squares × (0.5 × 10⁶) × (0.05 × 10⁻³) = 350 J 350 squares × (0.1 × 10⁶) × (0.01 × 10⁻³) = 350 J</p>	1	Multiply the number of squares by the number of joules each square represents. One large square represents (0.5 × 10 ⁶) × (0.05 × 10 ⁻³) J and one small square represents (0.1 × 10 ⁶) × (0.01 × 10 ⁻³) J. Make sure you show all the steps in your working.

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- (b) Relevant points include:
- piston always moving so heating not at constant volume in real cycle
 - no sharp corners on real cycle because valves take time to open and close
 - maximum temperature not realized because of imperfect combustion
 - expansion and compression strokes not truly adiabatic in real cycle – heat losses occur
 - real cycle needs induction and exhaust strokes.

any 2 You are asked to **explain** ‘differences between the ... cycles as illustrated by Figures 9 and 10.’ So in your answer, you must give **reasons** for the differences – you can easily state two differences just by looking at the graphs, but you would not be awarded any marks for just doing this.

- 11 (a) (i) Compression (or decrease in volume) with *no heat transfer* from the gas
- (ii) The stroke occurs so quickly that there is *no time* for heat transfer.

1 You will need to learn the definitions of both adiabatic and isothermal processes. In an adiabatic process there can be no heat transfer.

(b) (i) $p_1 V_1^{1.4} = p_2 V_2^{1.4}$
 $p_2 = p_1 (V_1/V_2)^{1.4}$
 $= 1.0 \times 10^5 \times \left(\frac{4.5 \times 10^{-4}}{0.23 \times 10^{-4}}\right)^{1.4}$
 $= 6.4 \times 10^6 \text{ Pa}$

1 You get the mark here for rearranging the formula.

1 Read the values from the graph carefully, not forgetting the $\times 10^{-4}$ on the volume axis and the $\times 10^6$ on the pressure axis.

(ii)

	$\Delta W/\text{J}$	$\Delta Q/\text{J}$	$\Delta U/\text{J}$
Process A \rightarrow B	-255	0	255
Process B \rightarrow C	813	2860	2047

You need to be careful when using the first law formula. A negative sign for ΔW means that work is done **on** the gas.

1 A \rightarrow B
Heat transfer ΔQ is 0 because it is an adiabatic process. Substitute -255 J and 0 J in $\Delta Q = \Delta U + \Delta W$ to get $\Delta U = (+)255 \text{ J}$

2 B \rightarrow C
This is a constant pressure process so first find ΔW using $\Delta W = p \Delta V$ and then substitute in $\Delta U = \Delta Q - \Delta W$

In process B \rightarrow C
 $\Delta W = p \Delta V$
 $= 6.4 \times 10^6 \times (1.5 - 0.23) \times 10^{-4}$
 $= 813 \text{ J}$
 $\Delta U = \Delta Q - \Delta W$
 $= 2860 - 813 = 2047 \text{ J.}$

- 12 (a) (i) Work done = area of loop
 Suitable method for finding area (such as counting squares) used correctly
 Scaling factor calculated for example
 419 small squares $\times (0.05 \times 10^5) \times 0.1$
 or 17 1 cm squares $\times (0.25 \times 10^5) \times 0.5$
 Leading to 210 kJ.

1 Do not be put off by the diagrams
1 looking complicated, or the fact that you might not have covered atmospheric engines in class. Read the question carefully – it is really a very simple form of engine and all the information you need is given.

This is another case of finding the area of the cycle by counting squares and converting to joules by calculating the area of each square in joules.

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<p>(ii) $\frac{210}{6.0} = 35 \text{ kW}$ or $\frac{[\text{ans to (a)}]}{6.0}$</p>	1	The indicated power is the power developed in the cylinder of the engine so is calculated using power $= \frac{\text{indicated work per cycle}}{\text{time of one cycle}}$
<p>(b) (i) Output power = $\frac{mgh}{t}$ $= \frac{(7600 \times 9.8 \times 1.8)}{6.0}$ $= 22 \text{ kW}$</p> <p>(ii) Mechanical efficiency $= \frac{\text{output power}}{\text{indicated power}}$ $= \frac{22}{35}$ or $\frac{22}{[\text{ans (a)(ii)}]}$ $= 0.63 \text{ (63\%)}$</p>	1	This revises work on gravitational potential energy in Unit 2. The engine is having to increase the gravitational potential energy of 7600 kg of water raised through 1.8 m in 6.0 seconds.
<p>(c) (i) Maximum efficiency $\eta_{\text{max}} = \frac{T_H - T_C}{T_H}$ $= \frac{376 - 288}{376} = 0.23 \text{ (23\%)}$</p>	1	Remember that the maximum theoretical efficiency depends on the highest and lowest temperatures in the cycle. The temperature of the cooling water is the lowest temperature. Don't forget to use Kelvin temperatures.
<p>(ii) Relevant points include:</p> <ul style="list-style-type: none"> • Heat loss through cylinder wall or piston • Cylinder and piston have to be heated from cold every cycle • Cold cylinder walls will condense some of the input steam • Leakage of steam or air past piston • Friction (max 1 mark) – but must say where, for example: piston/cylinder pivot/beam in pump in operating valves 	any 2	The examiner wants to know if you have understood why this particular engine was very inefficient (without expecting you to be an expert in atmospheric engines!) You might be tempted to write “friction between moving parts” and “heat loss to the surroundings” but these apply to all heat engines. You would only score marks by stating where friction acts and/or why the heat losses are so great.