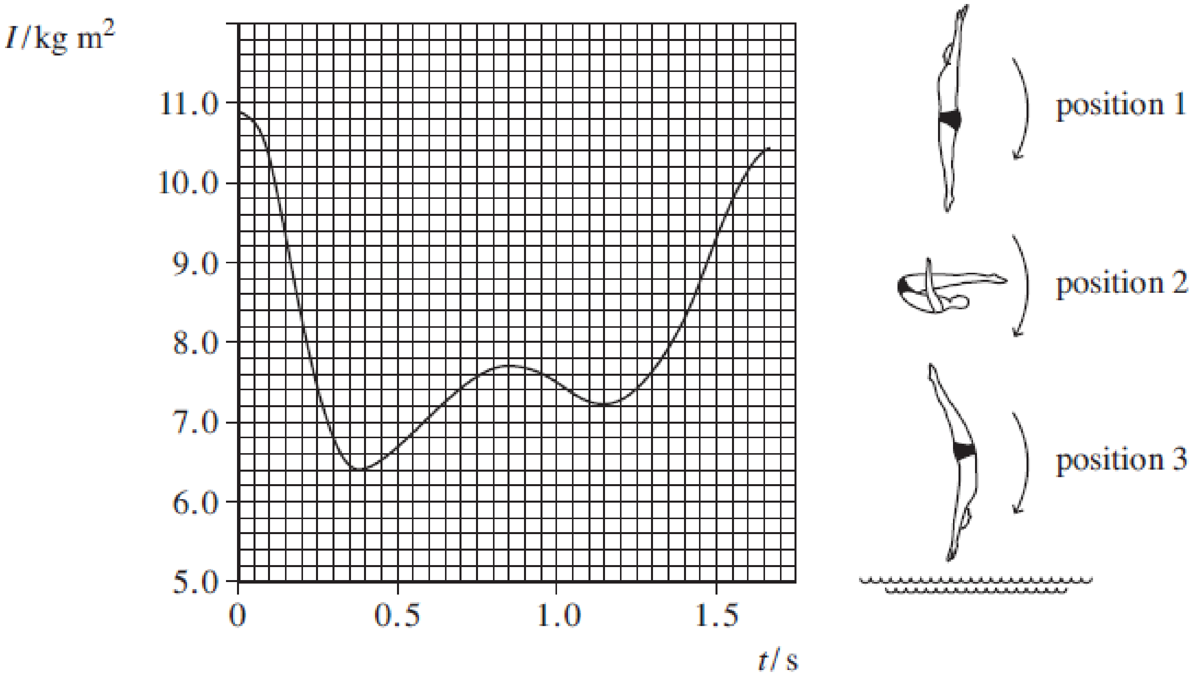
Q1. The graph below shows how the moment of inertia *I* of a diver performing a reverse dive varies with time *t* from just after he has left the springboard until he enters the water.



The diver starts with his arms extended above his head (position 1), and then brings his legs towards his chest as he rotates (position 2). After somersaulting in mid-air, he extends his arms and legs before entering the water (position 3).

(a) Explain how moving the legs towards the chest causes the moment of inertia of the diver about the axis of rotation to decrease.

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(2)

(b) (i) Explain in terms of angular momentum why the angular velocity of the diver varies during the dive.

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....

(2)

(ii) Describe how the angular velocity of the diver varies throughout the dive.

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....

....

(1)

(c) At time *t* = 0 the angular velocity of the diver is 4.4 rad s^{-1} and his moment of inertia about the axis of rotation is 10.9 kg m^2.

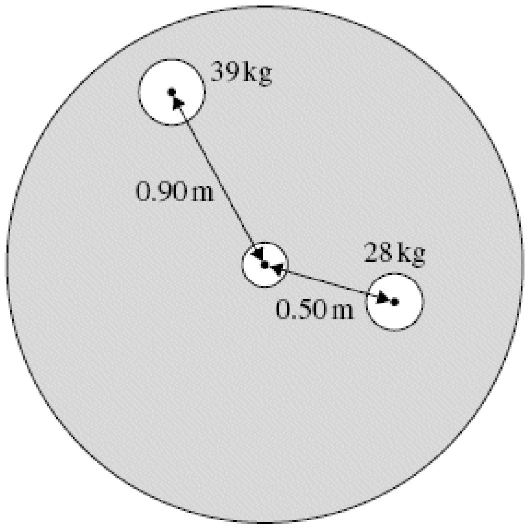
With reference to the graph above calculate the maximum angular velocity of the diver during the dive.

angular velocity .... rad s^{-1}

(3)

(Total 8 marks)

Q2. (a) A playground roundabout has a moment of inertia about its vertical axis of rotation of 82 kg m^2. Two children are standing on the roundabout which is rotating freely at 35 revolutions per minute. The children can be considered to be point masses of 39 kg and 28 kg and their distances from the centre are as shown in the figure below.



(i) Calculate the total moment of inertia of the roundabout and children about the axis of rotation. Give your answer to an appropriate number of significant figures.

answer = ... kg m^2

(3)

(ii) Calculate the total rotational kinetic energy of the roundabout and children.

answer = ... J

(2)

(b) The children move closer to the centre of the roundabout so that they are both at a distance of 0.36 m from the centre. This changes the total moment of inertia to 91 kg m^2.

(i) Explain why the roundabout speeds up as the children move to the centre of the roundabout.

....

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....

(2)

(ii) Calculate the new angular speed of the roundabout. You may assume that the frictional torque at the roundabout bearing is negligible.

answer = ... rad s^{-1}

(2)

(iii) Calculate the new rotational kinetic energy of the roundabout and children.

answer = ... J

(1)

(c) Explain where the increase of rotational kinetic energy of the roundabout and children has come from.

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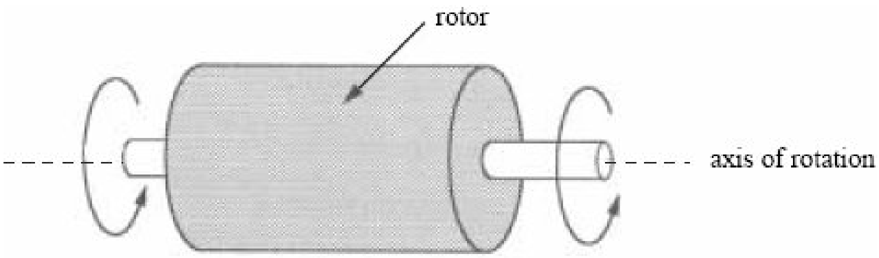
....

....

(1)
(Total 11 marks)

Q3. 'Low inertia' motors are used in applications requiring rapid changes of speed and direction of rotation. These motors are designed so that the rotor has a very low moment of inertia about its axis of rotation.



(a) (i) Explain why a low moment of inertia is desirable when the speed and direction of rotation must be changed quickly.

....

....

....

....

(ii) State, giving a reason in each case, **two** features of rotor design which would lead to a low moment of inertia about the axis of rotation.

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(4)

(b) In one application, a rotor of moment of inertia 4.4×10^{-5} kg m^2 about its axis of rotation is required to reverse direction from an angular speed of 120 rad s^{-1} to the same speed in the opposite direction in a time of 50 ms. Assuming that the torque acting is constant throughout the change, calculate

(i) the angular acceleration of the rotor,

....

....

(ii) the torque needed to achieve this acceleration,

....

....

(iii) the angle turned through by the rotor in coming to rest momentarily before reversing direction.

....

....

(3)
(Total 7 marks)

M1. (a) <u>Use of</u> $I = \Sigma mr^2$ or expressed in words ✓

With legs close to chest, more mass at smaller *r*, so *I* smaller ✓

2

(b) (i) Angular momentum is conserved / must remain constant **OR** no external torque acts √

WTTE

as *I* decreases, *ω* increases and vice versa to maintain *I ω* constant ✓
OR as *I* varies, *ω* must vary to maintain *I ω* constant

2

(ii) (Angular velocity increases initially then decreases (as he straightens up to enter the water)).

No mark for just ang. vel starts low then increases then decreases, i.e. for describing ω only at positions 1,2 and 3.

With one detail point e.g. ✓

- Angular velocity when entering water is greater than at time t = 0 s.
- Angular velocity increases, decreases, increases, decreases
- Maximum angular velocity at t = 0.4 s
- Greatest rate of change of ang. vel. is near the start
- Angular velocity will vary as inverse of M of I graph

1

(c) angular. momentum = 10.9 × 4.4 = 48 (N m s) ✓

(ω_{max} occurs at minimum *I*)

Allow 6.3 to 6.5. If out of tolerance e.g. 6.2 give AE for final answer

minimum *I* = 6.4 kg m^2 (at 0.4 s) ✓

6.4 × ω_{max} = 48 leading to

ω_{max} = 7.5 rad s^{-1} ✓

3

(Total 8 marks)

M2. (a) (i) $I = 82 + 39 \times 0.90^2 + 28 \times 0.50^2$ **(1)**

$= 120$ kg m^2 **(1)** to 2 sig figs **(1)**

3

(ii) $\omega = 35 \times 2\pi/60$ **(1)** $= 3.7$ rad s^{-1}

$E = \frac{1}{2} I \omega^2 = 0.5 \times 120 \times 3.7^2 = 820$ J **(1)**

(accept 800 to 821 J depending on sf carried through)

2

(b) (i) angular momentum must be conserved **(1)**

so if *I* decreases *ω* must increase **(1)**

2

(ii) $120 \times 3.7 = 91 \times \omega_2$ **(1)**

$\omega_2 = 4.9$ rad s^{-1} **(1)**

2

(iii) $E = 0.5 \times 91 \times 4.9^2 = 1100$ J (1090 J) **(1)**

(give CE only if correct *I* value used)

accept 1050 – 1100 J

1

(c) work done or energy transferred as children move towards the centre **(1)**

or work done as centripetal force moves inwards **(1)**

1

[11]

M3. (a) (i) energy: kinetic energy $= \frac{1}{2} I\omega^2$, → small stored energy
[or less work/energy needed to produce change]
power = rate of energy change, fast change → high power
torque: $T = I\alpha$, α large so large torques needed unless *I* small,
momentum , impulse: $L = I\omega$, impulse $= \Delta L$ so unless *I* small, large angular impulses are needed
marking: for any **one** of the above:
for correct consideration **(1)**
for mathematical justification **(1)**

(ii) explanations based on $I = mr^2$ **(1)**
low **mass**, small diameter **(1)**

4

(b) (i) $\alpha = \frac{\omega_1 - \omega_2}{t} = \frac{120 + 120}{50 \times 10^{-3}}$ = 4.8 × 10^3 rad s^{-2}

(ii) $T = I\alpha$ = 4.4 × 10^{-5} × 4.8 × 10^3 = 0.21(1) N m **(1)**
(allow C.E. from incorrect value of α from (i))

(iii) $\theta = \left(\frac{\omega_1 + \omega_2}{2}\right)t = \left(\frac{120 + 0}{2}\right)25 \times 10^{-3}$ = 1.5 rad **(1)**

3

[7]