

## Chapter 2 Thermodynamics

### 2.1 The first law of thermodynamics

#### Learning objectives:

- What is the difference between heat and work?
- How can we change the internal energy of a system?
- What is an adiabatic change?

#### Heat transfer

Heat is energy transferred due to temperature difference. Heat transferred to an object either causes the internal energy of the object to rise or allows the object to do work, or both. Energy transformations often involve heating the surroundings. For example, a falling mass loses potential energy. What happens to the potential energy? If the mass is accelerating as it falls, it speeds up so it gains kinetic energy. Assuming air resistance is negligible, the loss of potential energy equals the gain of kinetic energy of the object. If the mass then hits the floor, what happens to its kinetic energy? Sound energy is produced and the atoms of the floor at the impact point gain some energy as well. So the floor becomes just a little warmer as a result of the impact.

Another example is when a battery is used to run an electric motor to raise a mass, as in Figure 1. The battery supplies electrical energy which is partly converted into potential energy of the raised mass. Some of the electrical energy is used for heating the motor windings because the windings have resistance. Also, frictional forces cause heating at the motor bearings. So not all the electrical energy is converted to potential energy. The difference is lost through heating of the motor and its surroundings. The surroundings gain energy through heat transfer.

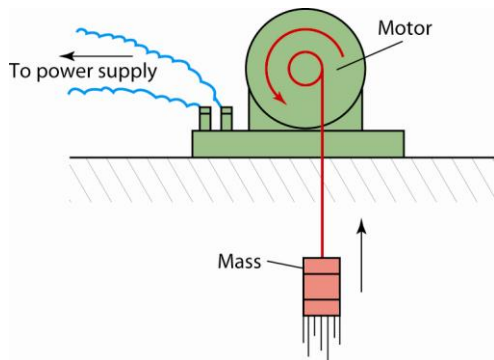


Figure 1 *Energy on the move*

#### Why does heat transfer always seem to make energy spread out?

Is it possible to draw energy from the surroundings and concentrate it in some form?

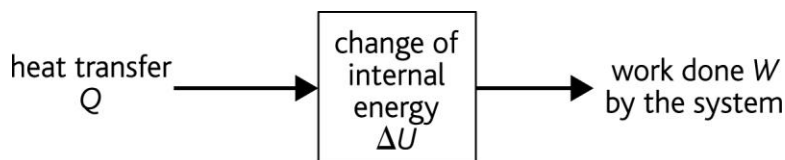
Thermodynamics is the study of energy transformations, and it gives the answers to these important questions. The laws of thermodynamics were established by scientists in the 19th century. These laws are based on experiments involving energy transformations, and scientists put great faith in them because no experiment has ever disproved them.

**The first law of thermodynamics states that heat energy supplied to a system either increases the internal energy of the system or enables it to do work, or both.**

### Link

See A2 Physics A Topic 11.1 for internal energy.

If the internal energy of the object increases by  $\Delta U$ , and the work done **by** the system is  $W$ , then the heat transfer into the system,  $Q = \Delta U + W$



**Figure 2** Change of internal energy

For example, if a system's internal energy rises by 20 J and the work done by the system is 14 J, the heat transferred into the system must be 34 J.

The first law is based on conservation of energy. There are two ways of changing the internal energy of any system. One way is to use forces to do work. The other way is by heat transfer. So the first law tells us that the change of internal energy,  $\Delta U = Q - W$  where  $Q$  is the heat transfer into the system and  $W$  is the work done by the system.

When the equation for the first law is used, sometimes the values are negative.

- A negative value for  $\Delta U$  means the internal energy has decreased.
- A negative value for  $Q$  means that a temperature difference causes energy to be transferred out of the system.
- A negative value for  $W$  means that work is done on the system by outside forces, energy is transferred into the system.

For example, if the heat transferred  $Q$  to the system is 18 J and its gain of internal energy  $\Delta U$  is 25 J, then rearranging the equation for the first law of thermodynamics  $Q = \Delta U + W$  gives:

$$W = Q - \Delta U = 18 - 25 = -7 \text{ J}$$

In other words, 7 J of work is done on the system and 18 J of heat is transferred into it so its gain of internal energy is 25 J.

### Worked example

When a gas expands, it does 180 J of work and loses 120 J of internal energy. Calculate the heat transfer to or from the gas.

#### Solution

Work done by the gas,  $W = 180 \text{ J}$

Change of internal energy of the gas,  $\Delta U = -120 \text{ J}$  (minus because its internal energy decreases)

Heat transfer  $Q = \Delta U + W = -120 + 180 = +60 \text{ J}$

Therefore 60 J of heat is transferred into the gas.

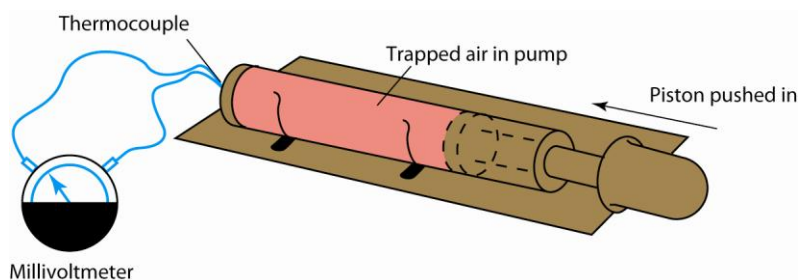
## Thermodynamics at work

### Stretching a rubber band

- 1 Stretch a rubber band quickly then hold it, still stretched, against your lower lip. It feels warm. The internal energy of the rubber increases because you have done work on the rubber band when you stretched it. The stretching was so rapid that heat transfer **to** the surroundings was negligible. An energy transfer where no heat transfer occurs is called an **adiabatic change**.
- 2 Keep the rubber band stretched against your lip and you should find that it cools to 'lip' temperature. Its internal energy falls due to heat transfer **to** the surroundings. No work is done because there is no movement.
- 3 Now release the rubber band quickly, and put it unstretched against your lip again. You ought to find that it becomes colder than 'lip' temperature as a result of unstretching. Its internal energy has fallen because internal forces in the rubber do work as it unstretches. No heat transfer occurs because the change is rapid.
- 4 Finally the unstretched rubber warms up to 'lip' temperature because heat transfer **from** the surroundings increases its internal energy.

### Compressing a gas

Figure 3 shows a simple arrangement of a bicycle pump with a blocked outlet. A thermocouple inserted through the blocked outlet allows the air temperature inside the pump to be monitored.



**Figure 3** *Compressing a gas*

- 1 Push the piston in rapidly and hold it in position. The thermocouple thermometer ought to show a temperature rise due to increase of internal energy of the air. The push force does work on the air rapidly, so no heat transfer from the air occurs. Hence the work done increases the internal energy.
- 2 Return the piston to its initial position and allow the temperature to return to its initial value as well. Then push the piston in very gradually. No temperature rise occurs this time. Why? You still do work on the air as you push the piston in. However, any slight rise of temperature causes heat transfer from the air so the temperature returns to the initial value. In other words, heat transfer from the air occurs at the same rate as work is done on it. So no rise of internal energy occurs.

### Summary questions

- 1 When a gas expands, it does 90 J of work and loses 120 J of internal energy. Calculate the heat transfer to or from the gas.
- 2 A model vehicle is driven by a rubber band 'engine' which initially has 12 J of elastic potential energy stored in it.  
When the rubber band is released, the vehicle runs up a slope and gains 5 J of gravitational potential energy before it stops with 3 J of elastic potential energy stored in the rubber band.

- a Calculate the change of internal energy of the rubber band.
- b Estimate:
  - i the work done by the rubber band
  - ii the heat transfer to or from the surroundings.
- 3 In a gas explosion, a gas is suddenly ignited then expands very rapidly.

- a Explain why:
  - i the gas does no work when it ignites
  - ii no heat transfer occurs when the gas expands
- b Complete the table on the right to indicate the energy transfers that take place at each stage.

Stage	$Q$	$\Delta U$	$W$
gas ignites	$>0$	?	0
gas expands	0		

- 4 Each row of the table represents a situation where heat transfer occurs or work is done. For each situation:
  - i complete the table by filling in the empty boxes
  - ii describe the energy changes that take place.

	$Q$	$\Delta U$	$W$
a	+80 J	0	
b		-400 J	+ 180 J
c		+30 J	-30 J
d	-500 J		-240 J

## 2.2 Thermodynamics of ideal gases

### Learning objectives:

- How much work is done by a gas when it expands?
- How can we determine the work done by an ideal gas from a pressure–volume graph?
- How can we calculate the pressure of a gas when it undergoes an adiabatic change?

### Work done by an ideal gas

The internal energy  $U$  of an ideal gas is due to the kinetic energy of its molecules. No internal forces of attraction exist between ‘ideal gas’ molecules so an ideal gas does not possess molecular potential energy. To change the temperature of a gas, its internal energy  $U$  must be changed. To do this, work must be done or heat must be transferred to or from the gas.

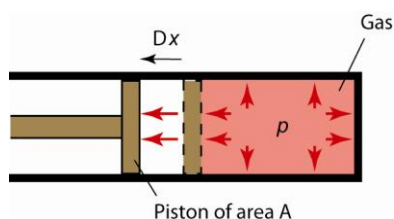
#### Link

See *A2 Physics A* Topic 12.2 to remind yourself about the properties of an ideal gas.

**Work is done on a gas when its volume is reduced** (compressed). At constant volume, no work is done because the forces due to pressure in the gas do not move the container walls. When a gas is compressed, work is done on it by the applied forces which push the pressure forces back. Try compressing the air in a bicycle pump by blocking the outlet and pushing the piston in rapidly.

**Work is done by a gas when it expands.** When a gas expands, the forces due to its pressure pushes back the container walls, as in Figure 1. So the gas does work when it expands. For example, when the valve on a cylinder of carbon dioxide gas is released, the cylinder outlet releases a high-pressure jet of  $\text{CO}_2$  gas into the air. The gas does work to expand and push back the air. The  $\text{CO}_2$  uses up so much of its internal energy to expand that it cools and solidifies. You may have seen solid  $\text{CO}_2$  produced in this way for use in cloud chambers.

Consider a tube of gas at pressure  $p$  trapped by a piston, as in Figure 1.



**Figure 1** Work done by a gas

The force due to pressure on the piston is  $pA$ , where  $A$  is the area of cross-section of the tube. To hold the piston in place, an equal and opposite force  $F$  must be applied to it.

Suppose the gas is allowed to expand a little without any significant change of pressure. The piston moves out by a distance  $\Delta x$ . So the change of gas volume is  $A\Delta x$ .

The work done by the gas is given by force  $\times$  distance moved  $= F\Delta x = pA\Delta x = p\Delta V$  since  $A\Delta x =$  the volume change,  $\Delta V$ . Therefore, when a gas expands and its volume increases by  $\Delta V$  at constant pressure  $p$ .

**work done by the gas,  $W = p\Delta V$**

For example, if the volume increases from  $0.100 \text{ m}^3$  to  $0.102 \text{ m}^3$  at a pressure of  $10^5 \text{ Pa}$ , the work done by the gas is  $10^5 \times 0.002 = 200 \text{ J}$ .

### Note

If the volume had decreased from  $0.100 \text{ m}^3$  to  $0.098 \text{ m}^3$ , the volume change would have been  $-0.002 \text{ m}^3$ , giving work done by the gas equal to  $-200 \text{ J}$ . In this case, the  $-$  sign indicates energy transferred to the gas.

## Adiabatic changes of an ideal gas

An **adiabatic change** is defined as one which occurs without heat transfer. A gas can be compressed adiabatically by reducing its volume rapidly. The change is then too fast for measurable heat transfer. Another way is to insulate the gas container so heat transfer cannot occur.

For any adiabatic change of an ideal gas, it can be shown that the gas pressure  $p$  and the gas volume  $V$  are given by the following equation which is referred to as the **adiabatic equation** for an ideal gas.

$$pV^\gamma = \text{constant}$$

where  $\gamma$ , referred to as the adiabatic constant, has a value that depends on the number of atoms in each molecule of the gas.

### Notes

- 1 Proof of this equation is **not** required. You do need to be able to use the equation. An outline of the proof is provided in Appendix B to give a deeper understanding of the principles that underlie the equation.
- 2 You will be given the value of  $\gamma$  in an examination question. For a monatomic gas,  $\gamma = 1.67$ ; for a diatomic gas,  $\gamma = 1.40$ ; for a polyatomic gas,  $\gamma = 1.33$ .
- 3 Adiabatic changes of an ideal gas can be plotted on a graph of pressure against volume. The curves are steeper than isotherms (which are constant temperature curves in accordance with  $pV = \text{constant}$ ), as Figure 2 shows. For example, if a gas is compressed from volume  $V_1$  at temperature  $T_1$  to volume  $V_2$ , the final temperature is higher than  $T_1$  (which is the temperature it would have stayed at if it had been compressed isothermally). This is because heat transfer from the gas cannot occur in an adiabatic change. So the internal energy of the gas and hence the final temperature and pressure are higher than they would have been be if the compression had been isothermal.

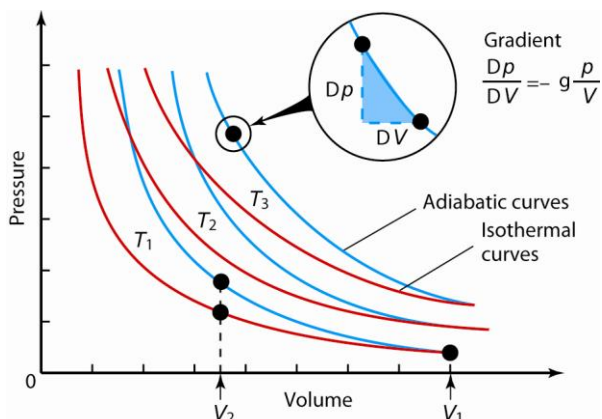


Figure 2 Adiabatic curves

### Worked example

A cylinder contains  $0.24 \text{ m}^3$  of air at a pressure of  $100 \text{ kPa}$  and a temperature of  $290 \text{ K}$ . The air in the cylinder is compressed adiabatically by means of a piston to a volume of  $0.11 \text{ m}^3$ . Calculate the pressure and the temperature of the air immediately after it has been compressed.

$\gamma = 1.40$  for air

### Solution

To find the final pressure  $p$ , applying  $pV^\gamma = \text{constant}$  to the initial and final pressures and volumes gives

$$p \times 0.11^{1.4} = 100 \times 10^3 \times 0.24^{1.4} \text{ where } p \text{ is the final pressure.}$$

$$\text{Therefore } p = 100 \times 10^3 \times \left(\frac{0.24}{0.11}\right)^{1.4} = 300 \times 10^3 \text{ Pa} = 300 \text{ kPa}$$

To find the final temperature  $T$ , applying  $pV = nRT$  in the form  $\frac{pV}{T} = \text{constant} (= nR)$  gives

$$\frac{300 \times 10^3 \times 0.11}{T} = \frac{100 \times 10^3 \times 0.24}{290}$$

$$\text{Rearranging this equation gives } T = \frac{300 \times 10^3 \times 0.11 \times 290}{100 \times 10^3 \times 0.24} = 400 \text{ K}$$

## Pressure against volume curves

For a fixed mass of an ideal gas, the pressure, volume and temperature must always conform to the ideal gas law  $pV = nRT$ . Pressure against volume curves are a helpful way to show the changes in a gas.

### Link

See A2 Physics A Topic 12.2 for the ideal gas law.

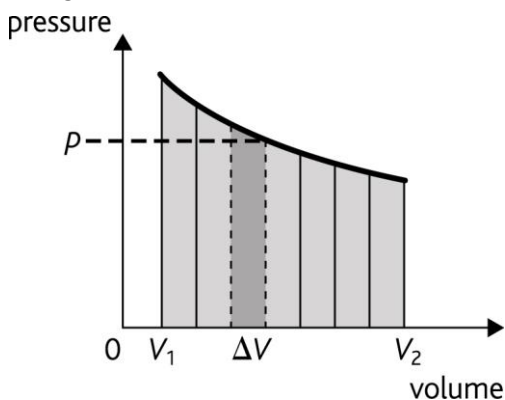
Figure 2 shows three particular changes in which a fixed mass of gas is compressed from volume  $V_1$  to a smaller volume  $V_2$ :

**At constant pressure**, the change is represented by a horizontal line from  $V_1$  to  $V_2$ .

**At constant temperature** (i.e. an isothermal change), the change is represented by a curve in accordance with Boyle's law (i.e. pressure  $p$  is inversely proportional to the volume  $V$ ). Thus the equation for the curve is  $p = \frac{k}{V}$  where  $k = nRT$ .

For no heat transfer (i.e. an adiabatic change), the change is a curve that is steeper than the isothermal curve. The equation for an adiabatic curve is the adiabatic equation  $pV^\gamma = \text{constant}$ .

**The area under any  $p$ - $V$  curve represents the work done on or by the gas.** To understand this, consider the graph in Figure 3 which shows one small step in the process of changing the volume of a gas from volume  $V_1$  to volume  $V_2$ .



**Figure 3** The area under a  $p$ - $V$  curve

In this small step, the volume changes by  $\Delta V$  at pressure  $p$ . Therefore, as explained previously, the work done in this small step  $= p\Delta V$ .

This is represented on the graph by a narrow strip (shown in blue) of width  $\Delta V$  and of height  $p$  under the curve. The area of this strip  $=$  its height  $\times$  its width  $= p\Delta V$ . Therefore, it follows that the area of the strip represents the work done for this small change of volume.

Hence the work done in the overall change of volume from  $V_1$  to  $V_2$  is represented by the total area under the curve from  $V_1$  to  $V_2$ .

**Note**

- If the change is an expansion, the gas does work equal to the area under the curve in expanding.
- If the change is a compression, an external force does work equal to the area under the curve on the gas in compressing it.

If we now look at Figure 4, which of the three processes involves most work being done to compress the gas? The area under the top line (which represents an adiabatic change i.e. no heat transfer) is greater than under the other two lines (constant temperature and constant pressure) so more work is done on the gas when it is compressed adiabatically than in the other two processes.



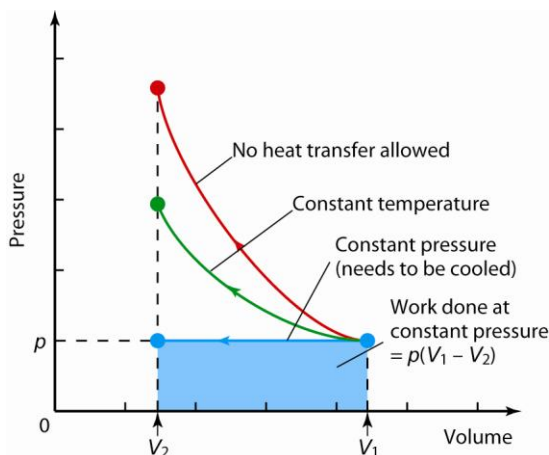


Figure 4 Area under  $p$ - $V$  curves

### Summary questions

$R = 8.32 \text{ J mol}^{-1} \text{ K}^{-1}$

- 1 An ideal gas at a constant pressure of 100 kPa expands from a volume of  $0.0045 \text{ m}^3$  and a temperature of 300 K to a volume of  $0.0060 \text{ m}^3$ .
  - a Calculate:
    - i the number of moles of the gas
    - ii the final temperature of the gas
    - iii the change of its internal energy.
  - b Show that the work done by the gas is 150 J and calculate the heat transfer to or from the gas.
- 2 1.2 moles of gas in a cylinder is compressed isothermally from an initial pressure of 120 kPa and a volume of  $0.025 \text{ m}^3$  to a final volume of  $0.040 \text{ m}^3$ .
  - a Calculate:
    - i the temperature of the gas
    - ii the final pressure of the gas.
  - b Sketch a graph to show how the pressure and volume changed during this compression and estimate the work done on the gas.
- 3 a A cylinder contains  $0.036 \text{ m}^3$  of air at a pressure of 150 kPa and a temperature of 300 K. The air in the cylinder expands adiabatically to a volume of  $0.052 \text{ m}^3$ . Calculate the pressure and the temperature of the air immediately after it has been compressed.
 

$\gamma = 1.40$  for air

  - b Sketch a graph to show how the pressure and volume changed during this compression and estimate the work done by the gas.
- 4 A sealed can of volume  $8.0 \times 10^{-5} \text{ m}^3$  contained gas at an initial pressure of 190 kPa. When the nozzle of the can was pressed for a short time, 5.0% of the gas in the can escaped.
  - a Explain why the expansion of the gas into the atmosphere may be considered to be adiabatic.
  - b Calculate the volume of the gas that escaped after it escaped, assuming it expanded until its pressure fell to atmospheric pressure at 100 kPa.
 

$\gamma = 1.40$  for the gas in the can

## 2.3 Heat engines

### Learning objectives:

- Why does a heat engine always waste energy?
- What is an indicator diagram and how do we use it to find work done?
- What do we mean by overall efficiency?

### The second law of thermodynamics

A **heat engine** is designed to do useful work which is supplied to it as a result of heat transfer. A steam engine or a car engine does work as long as it has fuel. Heat transfer from the burning fuel to the engine takes place, allowing the engine to do work. The internal energy of the engine does not change, provided its temperature is steady. So heat transfer to the engine is equal to the work done by the engine + the heat losses to the surroundings.

$$Q_{in} = W + Q_{out}$$

where:

$Q_{in}$  = heat transfer to the engine

$W$  = work done by the engine

$Q_{out}$  = heat losses to the surroundings.

### The steam turbine

Jets of high-pressure steam from a boiler are directed at the blades of a turbine wheel. The force of the steam makes the turbine wheel turn. After impact, the steam is at much lower pressure. It passes into a condenser where it turns back to water which is returned to the boiler. So heat energy from the boiler produces steam which does useful work. Then the steam transfers heat energy to the condenser at a much lower temperature. Figure 1 shows the process. The energy gained by the condenser is waste energy in the sense that it has not been used for work.

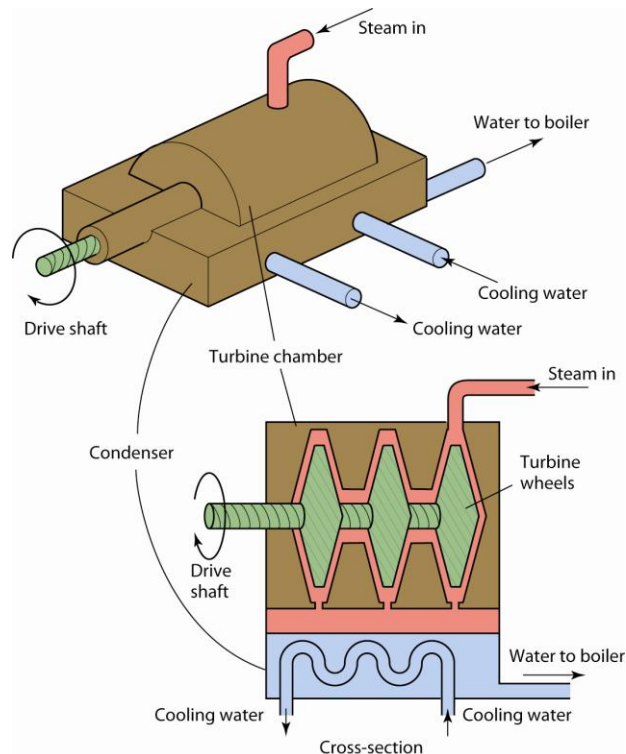


Figure 1 Steam turbines

Is it possible to design an engine where no energy is wasted? In the turbine of Figure 1 what would happen if the condenser were bypassed? The low pressure after the turbines would rise and stop the flow of steam into the turbines. So the condenser, a low temperature sink, is essential if heat transfer is to be produced by a high temperature source. This is true of any heat engine.

## The turbojet engine

Consider the turbojet engine shown in Figure 2. The air intake is compressed then heated strongly by the burning fuel so a jet of very hot air and combustion gases is forced from the exhaust; this pushes the engine in the opposite direction. The fuel supplies the energy for the work to be done by the engine. But the hot air from the exhaust transfers some of that energy to the atmosphere. The fuel burning supplies a high temperature source. The atmosphere provides a low temperature sink.

For an engine to do work as a result of heat transfer from a high temperature source, we need a low temperature sink to draw the energy from the source. So the work done can never be equal to the heat transfer from the source.

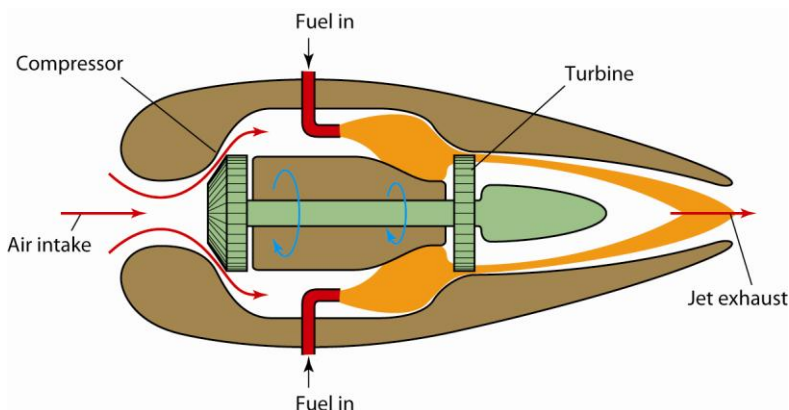


Figure 2 A turbojet engine

**The second law of thermodynamics states that it is impossible for heat transfer from a high temperature source to produce an equal amount of work.**

The first law of thermodynamics tells us that the energy put into a machine is equal to the energy from the machine. You can't get more energy out than you put in. The second law goes further; the useful energy from any engine is always less than the energy put into the machine.

**The efficiency of an engine is defined as**  $\frac{\text{the work done by the engine}}{\text{energy supplied to the engine}}$

The **percentage efficiency** of an engine is defined as:

$$\frac{\text{the work done by the engine}}{\text{energy supplied to the engine}} \times 100\%$$

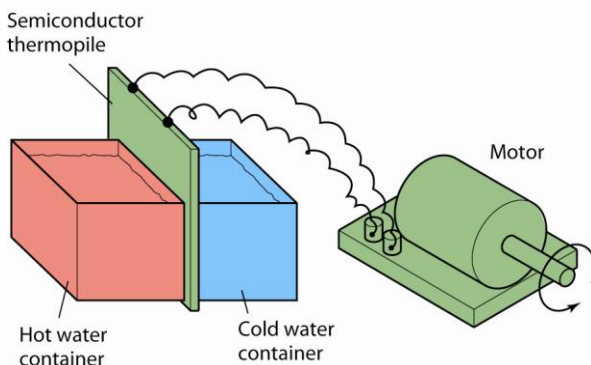
So a heat engine has a percentage efficiency of  $\left(\frac{W}{Q_{\text{in}}}\right) \times 100\%$  where  $W$  is the work done by the engine, and  $Q_{\text{in}}$  is the heat transferred to it. Since  $W$  is always less than  $Q_{\text{in}}$ , no engine can have an efficiency of 100% or more.

### Application

#### Investigating heat engines

An engine at work due to heat transfer is shown in Figure 3. It consists of a thermopile unit in between a container of hot water and a container of cold water. The thermopile unit consists of two different metals joined together in a repeated series such that the junctions, referred to as thermocouples, are on opposite faces of the thermopile unit. When one face of the thermopile unit is hotter than the other face, a potential difference is produced by each thermocouple to give a

total pd equal to the sum of the thermocouple pds. The thermopile unit in Figure 3 is being used to run an electric motor. If the containers are swapped round, the motor reverses because the thermopile pd is reversed.



**Figure 3** Investigating heat engines

In operation as shown in Figure 3, the thermopile unit acts as a heat engine which accepts energy  $Q_{in}$  from the hot water to do work  $W$  on the motor. The thermopile loses energy  $Q_{out}$ , equal to  $Q_{in} - W$ , to the cold water tank. The cold water tank needs to stay colder than the hot water tank to keep energy transferring through the thermopile unit so it does work on the motor. To keep the motor running, the temperature of the hot water could be maintained using a low voltage immersion heater and an ice pack could be used to keep the cold water cold.

## Engine cycles

In the turbojet, cold air is drawn in by the compressor and hot air is forced out from the exhaust. The working substance is air which is compressed and expelled continuously. In the steam turbine, the turbine wheels are forced round by steam from a boiler. After forcing the turbines round, the steam condenses and is returned to the boiler for re-use. The working substance is the water in the boiler which is used in a continuous cycle.

All heat engines work in a continuous cycle of operations. In each cycle, the working substance goes through a sequence of being heated, used to do work then cooled before being reheated to enable it to do more work.

### The petrol engine

The petrol engine uses air as the working substance, heating it by internal combustion. Figure 4 shows the pressure changes in one cylinder of a four-cylinder petrol engine in one complete cycle. The cylinders 'fire' sequentially, thus maintaining the motion of each cylinder.

The graph in Figure 4 is referred to as an **indicator diagram**. It is based on the assumption that there is a fixed mass of air in the cylinder throughout each cycle obeying the gas laws. This idealisation, called the **Otto cycle**, is not strictly correct since oxygen from the air and petrol vapour burn in the cylinder. Also, the pressure and temperature in the cylinder at any instant vary within the cylinder. Nevertheless, the idealised diagram is very useful to understand the thermodynamics of the petrol engine and can be compared with similar diagrams for other types of engines to compare their efficiencies.

Consider one cycle starting at A after the induction stroke which draws the petrol–air mixture into the cylinder at constant pressure:

Figure 4 The four-stroke petrol engine

**A → B Compression** The air in the cylinder is now trapped and compressed adiabatically by the incoming piston. No heat enters or leaves the cylinder.

**B → C Ignition** A spark ignites the mixture, raising the pressure and temperature in the cylinder very rapidly.

**C → D Power** High pressure in the cylinder forces the piston outward. Work is done adiabatically by the expanding gas.

**D → A Cooling** The gas is cooled rapidly by contact with the internal walls of the cylinder block, reducing the pressure to the initial pressure. This may be assumed to be at constant volume.

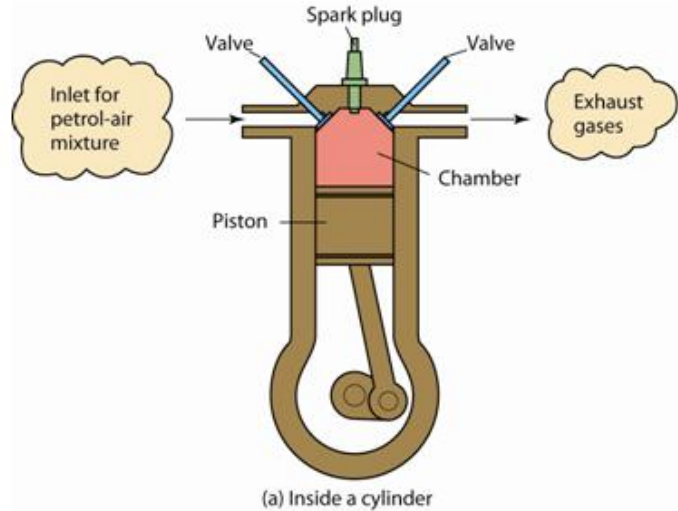
Before starting the next cycle ABCD, the piston moves in from A to E to force the exhaust gases out (the exhaust stroke) and then returns to draw more fuel into the cylinder (the induction stroke). The next cycle ABCD then starts.

In one cycle ABCD, the working substance:

- absorbs heat  $Q_{in}$  during ignition (i.e. from B to C) and rejects heat  $Q_{out}$  during cooling (i.e. from D to A). These changes are assumed to take place at constant volume
- does work during the power stroke (i.e. from C to D) and has work done on it during compression (i.e. from A to B). The net work done by the working substance is represented in Figure 4 by the area enclosed by ABCD and is referred to as the **indicator loop**.

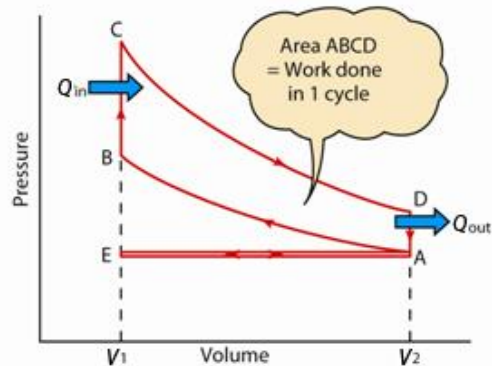
Since the temperature of the working substance is the same at the end of a cycle as it was at the start, there is no change in the internal energy of the engine.

Therefore, the net work done  $W$  by one engine cylinder in one cycle =  $Q_{in} - Q_{out}$  = the area of the indicator loop.



	A → B Compression	B → C Ignition	C → D Power	D → A Cooling	A → E Exhaust	E → A Induction
Inlet	Closed	Closed	Closed	Closed	Closed	Open
Outlet	Closed	Closed	Closed	Closed	Open	Closed
Piston moves	Up	-	Down	-	Up	Down

(b) A complete cycle



(c) Indicator diagram for the Otto cycle

**The indicated power of the engine**

- = the rate of net work done by the engine
- = area of indicator loop  $\times$  number of cycles per second  $\times$  number of cylinders

**The input power to the engine**

= the calorific value of the fuel (in  $\text{J kg}^{-1}$ )  $\times$  the fuel flow rate (in  $\text{kg s}^{-1}$ )

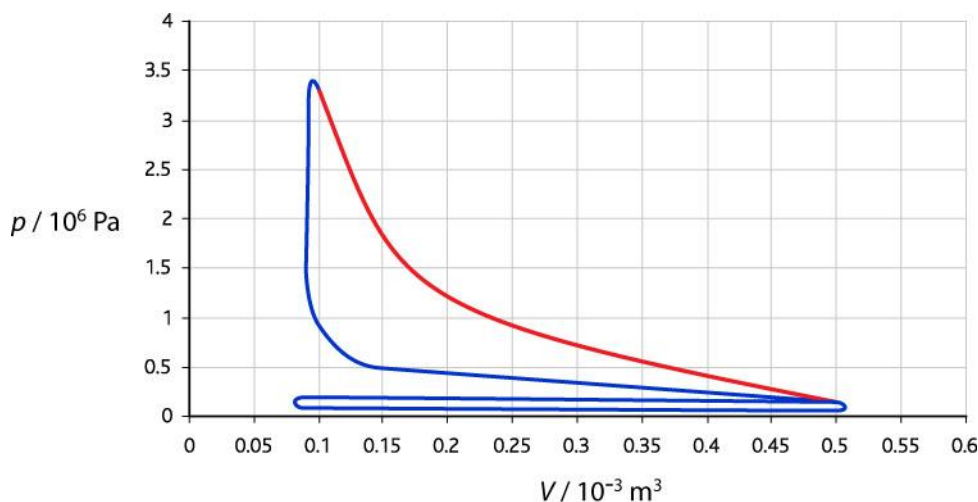
where the **calorific value** of the fuel is the energy released per unit mass of fuel when the fuel is burned.

Therefore, as the thermal efficiency of the engine =  $\frac{\text{work done per second}}{\text{energy supplied per second}}$

$$\text{the thermal efficiency} = \frac{\text{indicated power}}{\text{input power from fuel combustion}}$$

**Worked example**

A vehicle engine with four cylinders uses fuel of calorific value  $47 \text{ MJ kg}^{-1}$  at a rate of  $3.4 \times 10^{-3} \text{ kg s}^{-1}$  when it operates at 40 cycles per second. The  $p$ - $V$  diagram for one cylinder is shown in Figure 5 below.



**Figure 5**

- a** Show that the input power to the engine from fuel combustion is 160 kW.
- b** Estimate
  - i** the indicated power of the engine
  - ii** its thermal efficiency.

**Solution**

**a** Input power to the engine = calorific value of the fuel (in  $\text{J kg}^{-1}$ )  $\times$  the fuel flow rate (in  $\text{kg s}^{-1}$ )  
 =  $47 \text{ MJ kg}^{-1} \times 3.4 \times 10^{-3} \text{ kg s}^{-1}$   
 = 160 kW

**b i** The area of a single 'grid block' of the graph represents work done =  $p\Delta V$   
 =  $(0.50 \times 10^6) \times (0.05 \times 10^{-3})$   
 = 25 J

Therefore, the indicated power of the engine



$$\begin{aligned}
 &= \text{indicator loop area} \times \text{number of cycles per second} \times \text{number of cylinders} \\
 &= 12 \text{ blocks} \times 25 \text{ J per block} \times 40 \text{ s}^{-1} \times 4 \text{ cylinders} \\
 &= 48 \text{ kW}
 \end{aligned}$$

ii Thermal efficiency =  $\frac{\text{indicated power}}{\text{input power from fuel combustion}} = \frac{48}{160} = 0.3$

## The diesel engine

No spark plug is needed in the diesel engine. Diesel oil is sprayed into the cylinder at maximum compression. Because the air is at high temperature immediately after compression, the fuel mixture ignites on contact with the air in the cylinder, pushing the piston down at constant pressure.

Figure 6 shows the indicator diagram for a diesel engine. Note that:

- the compression stroke compresses the working substance to about  $\frac{1}{20}$  th of its volume before compression, in comparison with about  $\frac{1}{9}$  th for a petrol engine
- fuel injection occurs at constant pressure because the fuel ignites as it is injected and pushes the piston out.

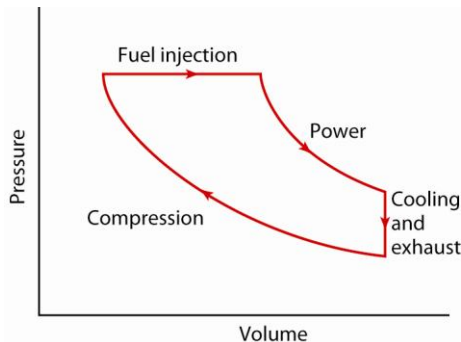


Figure 6 Indicator diagram for a diesel engine

## More about efficiency

Some of the work done by an engine is wasted due to friction between such moving parts as the transmission gears or between the engine drive shaft and its bearings. As a result, the output power of the engine is less than its indicated power. As explained below, the output power of an engine is usually referred to as the **brake power**.

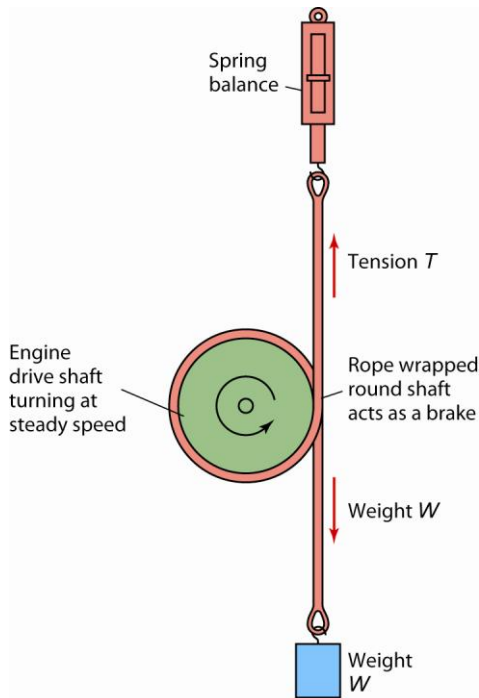
The **mechanical efficiency** of an engine is the fraction of the indicated power it delivers as brake power. Therefore,

$$\text{the mechanical efficiency of an engine} = \frac{\text{brake power}}{\text{indicated power}}$$

The difference between the indicated power and the brake power is referred to as the **friction power**. This is the power wasted by the engine due to friction.

Figure 7 shows how the brake power of an engine can be measured. The method requires the engine torque to be measured when the drive shaft is turning at a constant angular speed. As explained in Topic 1.3.

**the output or brake power = the engine torque  $\times$  the angular speed of the drive shaft**



Frictional torque =  $(W - T)r$  where  $r$  = shaft radius  
 At steady angular speed  $v$ ,  
 engine torque = frictional torque  $(W - T)r$   
 hence output power =  $(W - T)rv$

**Figure 7** *Measuring brake power*

The **overall efficiency** of an engine is the fraction of the input power from fuel combustion that is delivered as brake power. Therefore,

$$\text{the overall efficiency of an engine} = \frac{\text{brake power}}{\text{input power from fuel combustion}}$$

Because the mechanical efficiency =  $\frac{\text{brake power}}{\text{indicated power}}$

and the thermal efficiency =  $\frac{\text{indicated power}}{\text{input power from fuel combustion}}$

**the overall efficiency of an engine = its mechanical efficiency  $\times$  its thermal efficiency**

**Note**

percentage efficiency = efficiency  $\times$  100%



**Summary questions**

- 1** Explain qualitatively why the efficiency of a heat engine is less than 100%.
- 2** A vehicle engine with 4 cylinders uses fuel of calorific value  $43 \text{ MJ kg}^{-1}$  at a rate of  $3.7 \times 10^{-3} \text{ kg s}^{-1}$  when it operates at 48 cycles per second. In this situation, the work done by each cylinder in each engine cycle is 0.23 kW.  
Calculate:
  - a** the input power from the engine
  - b** the percentage thermal efficiency of the engine.
- 3** With the aid of indicator diagrams, describe how an engine cycle of a diesel engine differs from that of a petrol engine.
- 4** A certain jet engine does 500 kJ of work for every kilogram of air that passes through it. When the mass flow rate through the engine is  $9.6 \text{ kg s}^{-1}$ , frictional heating in the engine causes a total power loss of 400 kW.
  - a** Calculate:
    - i** the work done per second by the engine
    - ii** the output power of the engine.
  - b** The engine uses fuel of calorific value  $42 \text{ MJ kg}^{-1}$  at a rate of  $0.31 \text{ kg s}^{-1}$ . Calculate:
    - i** the input power to the engine
    - ii** the overall efficiency of the engine.

## 2.4 Efficiency and thermodynamics

### Learning objectives:

- What is a reversible heat engine?
- How efficient can a heat engine be in theory?
- What are the main reasons why an engine does not achieve its theoretical efficiency?

### The reversible engine

The most efficient type of engine is a theoretical engine called a reversible engine. Its ‘inventor’ was a 19th century scientist called Carnot. A reversible engine is defined as one which can be operated in reverse, so that work  $W$  done on it causes maximum heat transfer from cold to hot, as in Figure 1. The heat transfer to the high temperature ‘reservoir’ is given by  $Q_{in} = W + Q_{out}$ , where  $Q_{out}$  is the heat transfer from the cold reservoir. In other words, a reversible heat engine doesn’t waste energy.

If you could design an even more efficient engine, you could disprove the second law of thermodynamics. Your engine could do work on a reversible engine to make heat transfer from cold to hot without any help! Unfortunately, such an engine is impossible. Heat always tends to spread out.

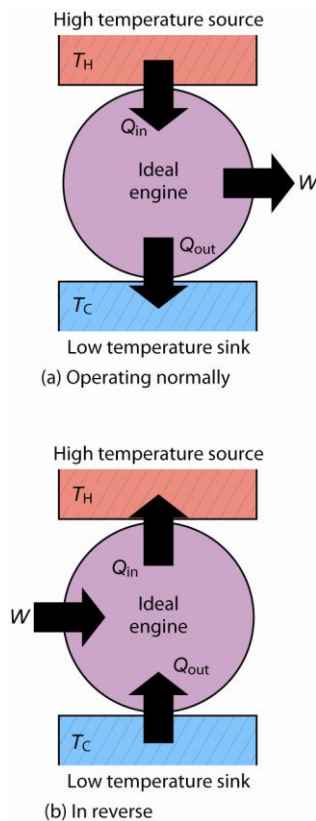


Figure 1 A reversible engine

### The efficiency of a reversible engine

The efficiency of a reversible engine depends on the temperature of the high temperature source or ‘hot space’ and the temperature of the low temperature sink or ‘cold space’. This is because the absolute scale of temperatures is defined in terms of a reversible engine. As explained above, a reversible engine is a theoretical engine which can be operated in reverse so that work done on it causes maximum heat transfer from cold to hot, as in Figure 1. We define the source temperature  $T_H$  and the sink temperature  $T_C$  as the ratio  $Q_{in}/Q_{out}$ , as shown in Figure 2.

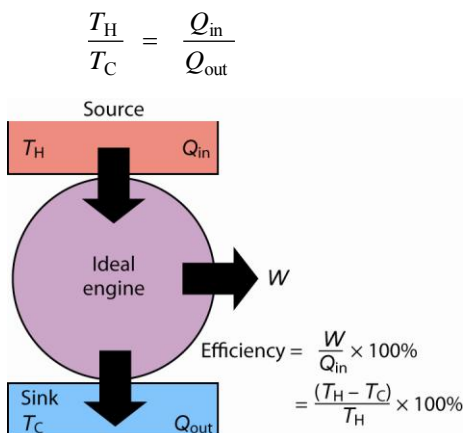


Figure 2 Efficiency of a reversible engine

**Note**

For an ideal gas, the ideal gas equation  $pV = nRT$  gives  $\frac{T_1}{T_2} = \frac{p_1V_1}{p_2V_2}$ . This equation is used to

measure absolute temperatures using a gas thermometer. For example, for such a thermometer operating at constant volume, the absolute temperature  $T$  of the gas at pressure  $p$  is given by

$$T = \frac{273.16 \times p}{p_{Tr}}$$

where  $p_{Tr}$  is the pressure of the gas at the triple point of water. This is the fixed

point defined as 273.16 K at which ice, water and water vapour are in thermal equilibrium. The information given here is not part of the specification and is provided to give a deeper understanding of the relationship between the definition of the absolute scale of temperatures and the measurement of absolute temperature as outlined in Topics 11.1 and 12.1 of the *A2 Physics A* book.

If the heat transfer from the low temperature ‘sink’ is  $Q_{out}$ , the heat transfer to the high temperature ‘source’ is given by  $Q_{in} = W + Q_{out}$  where  $W$  is the work done on the engine when it operates in reverse.

Since the % efficiency of a heat engine =  $\frac{W}{Q_{in}} \times 100\%$ , and  $W = Q_{in} - Q_{out}$ , then the efficiency

$$\text{formula can be written as efficiency} = \frac{(Q_{in} - Q_{out})}{Q_{in}} \times 100\% = \left(1 - \frac{Q_{out}}{Q_{in}}\right) \times 100\%$$

For a reversible engine,  $\frac{Q_{out}}{Q_{in}} = \frac{T_C}{T_H}$

Therefore, the % efficiency of a reversible engine  $= \left(1 - \frac{T_C}{T_H}\right) \times 100\% = \frac{(T_H - T_C)}{T_H} \times 100\%$

Because a reversible engine is the most efficient engine possible, the maximum possible efficiency of any heat engine operating between a heat source and a heat sink at temperatures  $T_H$  and  $T_C$  respectively is given by

$$\text{maximum possible efficiency} = \frac{(T_H - T_C)}{T_H} \times 100\%$$

For example:

- a thermopile engine operating between 373 K and 273 K can never have an efficiency greater than 27%
- an engine operating between temperatures of 700 K and 300 K can never have an efficiency greater than 56%.

The only way any engine could be made 100% efficient would be to make the sink at absolute zero ( $T_C = 0$ ), and that is impossible.

**Energy becomes less and less useful when it is transferred** because real engines operate at less than maximum efficiency.

- A reversible engine does work  $W$  by taking energy  $Q_{in}$  as heat transfer from a high temperature source  $T_H$ . The engine has to supply energy  $Q_{out}$  to a low temperature sink  $T_C$  to enable it to do work. Suppose we use the engine to raise a weight so the work done is stored as gravitational potential energy of the weight. We can then use the stored energy to do work  $W$  on the heat engine, operating it in reverse. The reversible engine would then transfer energy  $Q_{in}$  back to  $T_H$ , taking energy  $Q_{out}$  from  $T_C$ . Our operations have taken us back to the starting point, as if nothing had happened. No energy has been wasted. This can only happen with a reversible engine.
- Now suppose we use a real engine instead. It does less work for the same heat transfer  $Q_{in}$  from the high temperature source as its efficiency is less than the reversible engine. Therefore, when it operates in reverse, it cannot return energy  $Q_{in}$  to the high temperature source. Its efficiency must be less than that of a reversible engine under the same conditions.

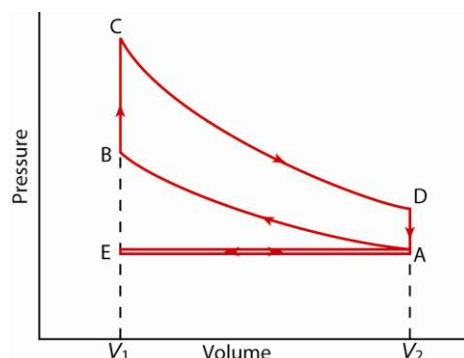
## Efficiency in practice

As explained in the previous topic, the overall efficiency of an engine = its mechanical efficiency  $\times$  its thermal efficiency. Mechanical efficiency and thermal efficiency are both lower in practice than in theory for the following reasons:

**Thermal efficiency** in theory depends on the temperatures of the heat ‘source’ and the heat ‘sink’ as explained in the next topic. Typical theoretical values of thermal efficiency are 58% for a petrol engine and 65% for a diesel engine. In practice, a typical petrol engine has a thermal efficiency of about 30% in practice in comparison with about 35% for a typical diesel engine. The main reasons why thermal efficiency values are much lower in practice than in theory are:

- the indicator loop area in practice does not have sharp corners as the theoretical indicator diagram in Figure 3 has. Therefore, less work is done than if it had sharp corners (which are due to the valves not opening and closing instantly)
- the maximum temperature is not reached as in perfect combustion
- the expansion and compression strokes are not perfectly adiabatic as assumed in calculating the theoretical thermal efficiency

- work is done by the engine to bring about the induction and exhaust strokes.



A→B	B→C	C→D	D→A	A→E	E→A
compression	ignition	power	cooling	exhaust	induction

Figure 3 The indicator diagram for the Otto cycle

Mechanical efficiency is less than 100% because:

- friction between the moving parts in the engine cannot be eliminated
- oil used to lubricate the bearings in an engine are viscous which means they cause some resistance to the motion of the moving parts.

### Application

#### More about the thermodynamics of a petrol engine

In a petrol engine, energy from the fuel in a cylinder is released as heat when the fuel is ignited by a spark.

The heat source is the ignited fuel in the cylinder hence the engine is referred to as an internal combustion engine. The heat transfer  $Q_{in}$  from the ignited fuel causes an increase of temperature in the cylinder when ignition occurs in proportion to the heat transferred. Hence, with reference to the indicator diagram in Figure 3 (where  $T_A, T_B, T_C, T_D$  = temperatures at A, B, C, D).

$$Q_{in} \propto (T_C - T_B)$$

The heat sink is the engine casing which transfers heat from the cylinders to the surroundings. This heat transfer  $Q_{out}$  occurs during cooling after the power stroke and before the exhaust stroke. The heat transfer  $Q_{out}$  causes a decrease of temperature in the cylinder in proportion to the heat transferred. Hence, with reference to the indicator diagram in Figure 3

$$Q_{out} \propto (T_D - T_A)$$

$$\text{Since the \% efficiency of a heat engine} = \frac{Q_{in} - Q_{out}}{Q_{in}} \times 100\%$$

$$= \left( 1 - \frac{Q_{out}}{Q_{in}} \right) \times 100\%$$

$$\text{Therefore the theoretical \% efficiency of a petrol engine} = \left( 1 - \frac{T_D - T_A}{T_C - T_B} \right) \times 100\%$$

Using the adiabatic equation for an ideal gas, it can be shown that the temperature difference ratio

$\frac{T_D - T_A}{T_C - T_B}$  is equal to  $r^{-0.4}$ , where  $r$  is the ratio of the  $\frac{\text{maximum volume}}{\text{minimum volume}}$  of the cylinder, referred to as the compression ratio.

A typical value of  $r = 9$  gives a value of  $r^{-0.4}$  equal to 0.42 and therefore a theoretical thermal efficiency of 58%. In comparison, for the reasons listed in Topic 2.3, the % efficiency of a real petrol engine is about 30%.

### Application

#### Combined heat and power (CHP) stations

In an electricity power station, turbine engines are used to drive electricity generators. The engines need to be cooled to keep them working otherwise the steam used to drive them would not condense. Cooling is achieved by pumping water through the engine block. The water is heated as it passes through the engine block. In CHP stations, this hot water is used to heat local buildings. Taking account of this use of heat that would otherwise be wasted, CHP stations are more efficient than non-CHP stations that generate equal electrical power.

### Summary questions

- 1** A thermopile engine operating between water tanks at temperatures of 20°C and 80°C delivers 4 W of electrical power to an electric motor and supplies heat to the cold water tank at a rate of 36 J s<sup>-1</sup>.
  - a** Calculate:
    - i** the heat per second transferred from the hot water tank
    - ii** the efficiency of the engine.
  - b** Calculate the maximum possible efficiency of the engine in this situation.
- 2** In a geothermal power station, a heat engine uses hot water from below the surface as a heat source and cold sea water as a heat sink. The temperature of the hot water is 79°C. When the sea water is at 9°C, the heat engine has a power output of 3.2 MW. Calculate:
  - a** the theoretical efficiency of the heat engine
  - b** the rate at which the heat engine extracts heat from the hot water, assuming the engine operates at its theoretical efficiency.
- 3** A gas-powered engine operates between temperatures of 1200 K and 350 K. When the engine uses gas of calorific value 32 MJ kg<sup>-1</sup> supplied at a rate of 8.4 kg h<sup>-1</sup>, the mechanical power output of the engine is 40 kW.
  - a** Calculate the theoretical efficiency of the engine.
  - b**
    - i** Show that the input power to the engine is 75 kW.
    - ii** Calculate the heat transferred by the heat engine to its low-temperature sink.
- 4**
  - a** Give two reasons why a petrol engine cannot achieve:
    - i** its maximum thermal efficiency
    - ii** its maximum mechanical efficiency.
  - b** Discuss whether a petrol engine is more efficient in summer than in winter.

## 2.5 Heat pumps

### Learning objectives:

- What is a heat pump?
- How efficient can a heat pump be?
- What do we mean by coefficient of performance?

### Heat pumps at work

A heat pump is a device that transfers energy from a cold space to a hot space. For example, the thermopile unit operating in reverse is a heat pump because it transfers energy from the cold 'tank' to the hot 'tank'.

Heat pumps may be used to heat buildings economically. Such a heat pump is effectively a refrigerator in reverse where the 'ice box' is outside the building and the condenser coils and pump are inside. Figure 3 shows how a refrigerator works. The heat released by the vapour condensing in the condenser coils keep the inside of the building warm.



Figure 1 A heat pump at work

The **coefficient of performance** of a heat pump is defined as

$$\frac{\text{the heat transferred, } Q_{\text{in}}, \text{ into the hot 'space'}}{\text{the work done, } W, \text{ to bring about the heat transfer}}$$

Figure 2 shows a generalised heat pump operating between a cold space at temperature  $T_C$  and a hot space at temperature  $T_H$ .

For work done  $W$  and heat transfer  $Q_{\text{out}}$  from the cold space, the heat transferred to the hot space  $Q_{\text{in}} = Q_{\text{out}} + W$ .

Therefore

$$\text{the coefficient of performance of a heat pump} = \frac{Q_{\text{in}}}{W} = \frac{Q_{\text{in}}}{(Q_{\text{in}} - Q_{\text{out}})}$$

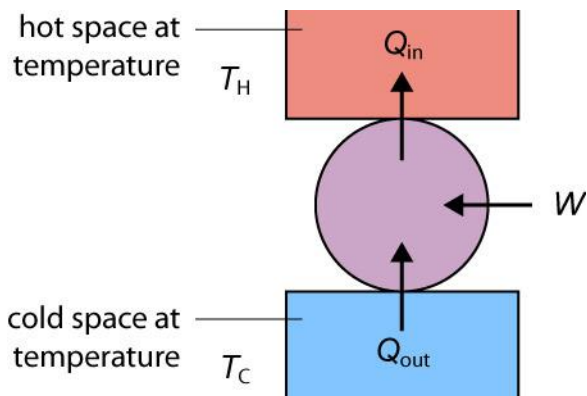


Figure 2 A heat pump

Suppose a heat pump fitted with a 150 W electric pump transfers 900 W of heat from the outside of a building to the inside.  $W = 150 \text{ W}$  and  $Q_{\text{out}} = 900 \text{ W}$ .

- The heat transfer  $Q_{\text{in}}$  into the building is 1050 W (= 150 W + 900 W).
- The coefficient of performance of the heat pump is therefore 7 (= 1050 W ÷ 150 W).

Clearly, heating using a heat pump is seven times cheaper than using electric heaters.

### Application

#### How much heat can a heat pump transfer?

The coefficient of performance of a heat pump cannot exceed a certain value, determined by the temperatures between which the pump works. The maximum value is possible only for a reversible heat pump (i.e. a reversible engine operating as a heat pump).

In this instance,  $\frac{Q_{\text{in}}}{Q_{\text{out}}} = \frac{T_{\text{H}}}{T_{\text{C}}}$  where  $T_{\text{H}}$  and  $T_{\text{C}}$  are the temperatures of the hot ‘space’ and the cold ‘space’ respectively. Therefore:

- for a reversible heat pump,

$$\text{the coefficient of performance} = \frac{Q_{\text{in}}}{(Q_{\text{in}} - Q_{\text{out}})} = \frac{T_{\text{H}}}{(T_{\text{H}} - T_{\text{C}})}$$

- for a real heat pump,

$$\frac{Q_{\text{in}}}{Q_{\text{out}}} < \frac{T_{\text{H}}}{T_{\text{C}}}, \text{ the coefficient of performance of a real heat pump} < \frac{T_{\text{H}}}{T_{\text{H}} - T_{\text{C}}}$$

The formula for coefficient of performance of a ‘reversible’ heat pump in terms of temperature is not part of the specification and is given here to show that there is a theoretical limit to the performance of a heat pump that depends on the temperatures of the hot space and the cold space, just as there is for an engine.

## Refrigerators at work

A heat pump transfers heat energy from a cold space to a hot space whereas a refrigerator transfers heat from a hot object to cool it down. In other words, in a heat pump we are interested in how much energy is delivered to the surroundings (a hot space) for the work that is done. In a refrigerator we are interested in how much energy is removed from the food and drink (the cold space) for the work done. We are not really concerned with how good the fridge is at heating the kitchen!



This difference in function between a heat pump and a refrigerator means the coefficient of performance of a refrigerator is defined as  $\frac{Q_{out}}{W}$  (in comparison with  $\frac{Q_{in}}{W}$  for a heat pump).

Therefore:

$$\text{the coefficient of performance of a refrigerator} = \frac{Q_{out}}{W} = \frac{Q_{out}}{(Q_{in} - Q_{out})}$$

For example, a refrigerator with a coefficient of performance of 5 and fitted with a 20 W electric pump extracts heat from the cold space at a rate of 100 W (= 5 × 20 W) and transfers 120 W of heat to the hot space.

### Application

#### The refrigerator

Figure 3 shows how a refrigerator works. The following notes on the working of a refrigerator are not on the specification and are provided here to give deeper understanding of basic ideas to do with heat transfer from a cold space to a hot space. The working substance in a refrigerator, the refrigerant, is forced continuously through a cycle in which it absorbs energy at low temperature and loses energy at high temperature. This is a heat engine cycle in reverse, as shown by the indicator diagram in Figure 3. Examples of refrigerants include ammonia, freon and sulphur dioxide.

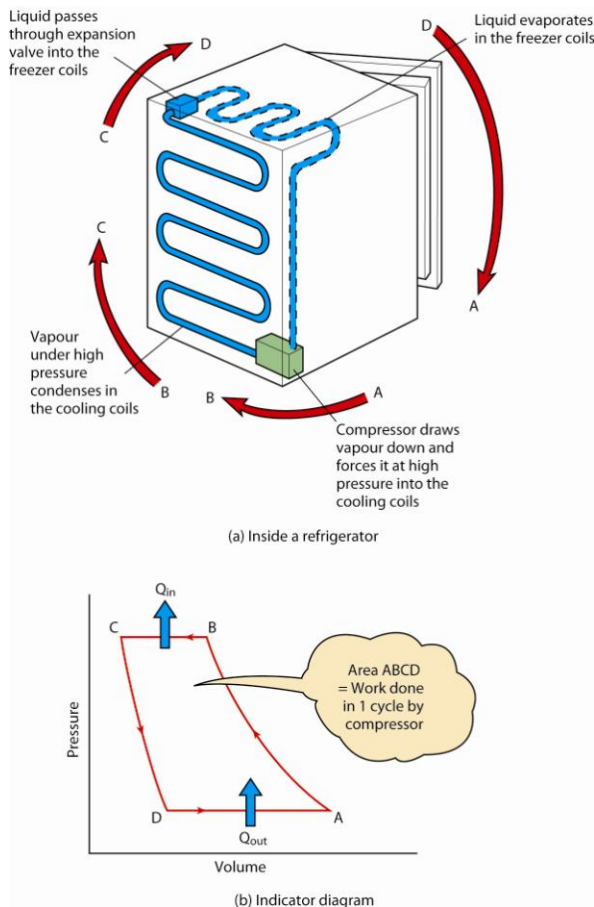


Figure 3 The refrigerator

For one complete cycle ABCD:

**A → B** The refrigerant is in the vapour state and is compressed adiabatically to high pressure.

**B → C** The high pressure vapour condenses in tubes outside the refrigerator, releasing internal energy to the surroundings.

**C → D** The condensed liquid passes through an expansion valve, causing its pressure and temperature to fall.

**D → A** The liquid evaporates in tubes inside the ice box, gaining internal energy from objects in the ice box. The low pressure vapour is then drawn into the compressor for the next cycle.

Thus the refrigerant gains energy  $Q_{\text{out}}$  from the ice box between D and A and loses energy  $Q_{\text{in}}$  to the surroundings between B and C. Hence,  $Q_{\text{in}} = Q_{\text{out}} + W$  where  $W$  is the work done by the pump.

Thus a refrigerator which uses a working substance that vaporises at low pressure at  $-15^{\circ}\text{C}$  ( $= 258\text{ K}$ ) and condenses at high pressure at  $30^{\circ}\text{C}$  ( $= 303\text{ K}$ ) has a maximum coefficient of

performance of  $\frac{303}{303 - 258} = 6.7$ . In practice, irreversible effects such as fluid turbulence and friction would reduce the coefficient of performance from the maximum possible value.

### Summary questions

- 1 A heat pump fitted with a 120 W electric motor supplies 600 W of heat per second to a room from outdoors. Calculate:
  - a the coefficient of performance of the heat pump
  - b the heat gained from outdoors.
- 2 A thermopile unit operating as a heat pump between a hot water tank and a cold water tank transfers energy to the hot water at a rate of 150 W when the unit is supplied steadily with 50 W of electrical power.
  - a Calculate the rate at which heat is supplied by the cold water tank.
  - b Show that the thermopile unit has a coefficient of performance of 3.
- 3 Explain why the coefficient of performance of a refrigerator is less than that of a heat pump operating between the same temperatures.
- 4 A refrigerator with a 5 W pump cools a plastic beaker containing 0.15 kg of water from  $15^{\circ}\text{C}$  to  $0^{\circ}\text{C}$  in 940 s. Calculate:
  - a the heat energy extracted from the water
  - b the heat energy per second released to the surroundings
  - c the coefficient of performance of the refrigerator.Specific heat capacity of water =  $4200\text{ J kg}^{-1}\text{ K}^{-1}$

## Appendices

The notes in the two appendices are **not** required for the specification. They are provided to enable a deeper understanding of the topics.

### Appendix A Heat capacities of an ideal gas

**Heat transfer**,  $Q$ , to a gas to change its temperature must increase its internal energy. But if the gas expands when it is heated, it uses energy to do work. So the heat transfer  $Q$  to the gas is equal to its increase of internal energy + the work done by the gas to expand.

$$Q = \Delta U + p\Delta V$$

where

$\Delta V$  = change of volume at constant pressure  $p$

$Q$  = heat transfer to the gas

$\Delta U$  = change of its internal energy.

The **molar heat capacity at constant volume**,  $C_V$ , is defined as the heat transfer required to raise the temperature of 1 mole by 1 K at constant volume.

For 1 mole of gas heated at constant volume so its temperature increases by  $\Delta T$ , the heat transfer  $Q = C_V\Delta T$ .

Since  $\Delta V = 0$ , then no work is done hence the increase of internal energy  $\Delta U = Q = C_V\Delta T$ .

The **molar heat capacity at constant pressure**,  $C_P$ , is defined as the heat transfer to the gas required to raise the temperature of 1 mole by 1 K at constant pressure.

For 1 mole of gas heated at constant pressure so its temperature increases by  $\Delta T$ , the heat transfer  $Q = C_P\Delta T$ .

The internal energy increases by the same amount as at constant volume because the temperature increase is the same. So  $\Delta U = C_V\Delta T$ . However, the gas expands to keep the pressure constant. So it does work equal to  $p\Delta V$  where  $\Delta V$  is the increase of volume.

Substituting the above terms into  $Q = \Delta U + W$  therefore gives  $C_P\Delta T = C_V\Delta T + p\Delta V$  which means that  $C_P$  is greater than  $C_V$ .

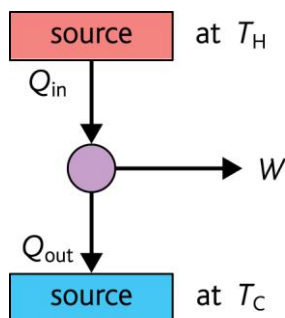


Figure 1 Heat capacities

#### Link

See A2 Physics A Topic 11.2 for specific heat capacity.

The **adiabatic constant**,  $\gamma$ , of a gas is defined as the ratio of its molar specific heats  $\frac{C_p}{C_v}$

It can be shown that:

- for a monatomic gas,  $\gamma = \frac{5}{3} = 1.67$
- for a diatomic gas (i.e. two atoms per molecule),  $\gamma = \frac{7}{5} = 1.40$
- for a polyatomic gas (i.e. more than two atoms per molecule),  $\gamma = \frac{4}{3} = 1.33$

## More about the adiabatic constant

The notes below are not required for the specification and are intended to give a deeper understanding of the thermodynamics of an ideal gas.

**For 1 mole of an ideal gas heated by a temperature increase  $\Delta T$  at constant pressure  $p$ ,** as explained above,  $C_p \Delta T = C_v \Delta T + p \Delta V$

- Before heating the gas, its pressure, volume and temperature are such that  $pV = RT$
- After heating the gas, its pressure, volume and temperature are such that  $p(V + \Delta V) = R(T + \Delta T)$

Hence, by subtraction,  $p \Delta V = R \Delta T$

Hence  $C_p \Delta T = C_v \Delta T + p \Delta V = C_v \Delta T + R \Delta T$

Cancelling  $\Delta T$  and rearranging therefore gives:

$$C_p - C_v = R$$

**The internal energy  $U$  of an ideal gas is due entirely to the kinetic energy of its molecules.**

Using the kinetic theory equation  $pV = \frac{1}{3} N m c_{\text{rms}}^2$ , we can show that the total kinetic energy of 1 mole of its molecules at temperature  $T$  is  $\frac{3}{2} RT$ . Hence its internal energy  $U = \frac{3}{2} RT$  at temperature  $T$ .

- If the gas is **monatomic** and its temperature rises by 1 K, its internal energy rises by  $\frac{3}{2} R$ . Because no work is done by the gas, this increase in internal energy is equal to the heat transfer at constant volume,  $C_v \Delta T$ . In other words  $C_v \Delta T = \frac{3}{2} R$ .

Also, since  $C_p - C_v = R$ ,  $C_p = C_v + R = \frac{5}{2} R$ .

Therefore, for a monatomic gas,  $\gamma = \frac{C_p}{C_v} = \frac{\frac{5}{2} R}{\frac{3}{2} R} = \frac{5}{3} = 1.67$ .

- Each molecule of a monatomic gas can move in three independent ‘translational’ directions so it is said to have three ‘degrees of freedom. We can associate a third of the internal energy of the gas molecules with each degree of freedom. A **diatomic** gas has two further degrees of freedom due to it having two independent directions of rotation perpendicular to its ‘diatomic’ axis.

- The internal energy of 1 mole of a diatomic gas at temperature  $T$  is therefore  $\frac{5}{2} RT$ .

Hence  $C_V \Delta T = \frac{5}{2} R \Delta T$  and  $C_P \Delta T = \frac{7}{2} R \Delta T$ . Therefore  $\gamma = \frac{C_P}{C_V} = \frac{7}{5} = 1.40$

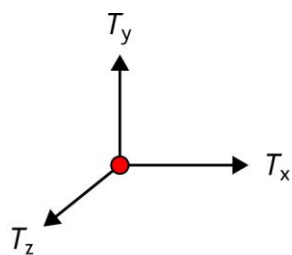
- Each molecule of a **polyatomic** gas has six degrees of freedom, three as for a monoatomic gas and three due to it having three independent directions of rotation.

The internal energy of 1 mole of such a gas at temperature  $T$  is therefore  $\frac{6}{2} RT$ .

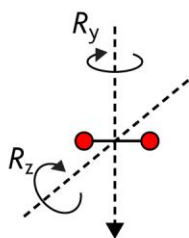
Hence  $C_V \Delta T = \frac{6}{2} R \Delta T$  and  $C_P \Delta T = \frac{8}{2} R \Delta T$ . Therefore  $\gamma = \frac{C_P}{C_V} = \frac{8}{6} = 1.33$

**Key for Figure 2:**

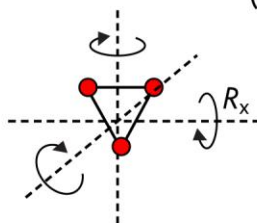
- Translational degrees of freedom  $T_x, T_y, T_z$
- Rotational degrees of freedom  $R_x, R_y, R_z$



a) A monoatomic gas molecule



b) A diatomic gas molecule  
( $T_x, T_y, T_z$  not shown)



c) A polyatomic gas molecule  
( $T_x, T_y, T_z, R_y, R_z$  not shown)

**Figure 2** Degrees of freedom

**Link**

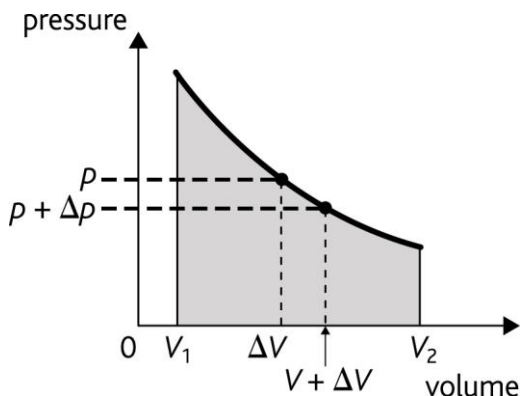
See A2 Physics A Topic 12.3 for the kinetic theory of gases equation.

## Appendix B Outline proof of $pV^\gamma = \text{constant}$

Consider the adiabatic expansion of an ideal gas in a little more detail.

Work is done by the gas as its volume increases, but heat transfer does not occur. So the internal energy decreases, and therefore the temperature falls.

Consider Figure 1 which represents an adiabatic expansion from volume  $V_1$  to volume  $V_2$ . Suppose a small adiabatic change of volume  $\Delta V$  as in Figure 1 causes the temperature to change by  $\Delta T$  for 1 mole of gas.



**Figure 1** The area under a  $p$ - $V$  curve

Before the change, the pressure and volume are linked by

$$pV = RT \tag{1}$$

After the change, the pressure changes to  $p + \Delta p$  and the volume to  $V + \Delta V$ . So the new values are linked by

$$(p + \Delta p)(V + \Delta V) = R(T + \Delta T) \tag{2}$$

Subtracting equation 1 from equation 2 and neglecting the second order term  $\Delta p \Delta V$  gives

$$p\Delta V + V\Delta p = R\Delta T \tag{3}$$

Using  $Q = \Delta U + p\Delta V$ , we obtain  $0 = C_V\Delta T + p\Delta V$  since  $Q = 0$  and  $\Delta U = C_V\Delta T$  from Appendix 1.

Rearranging this equation gives

$$p\Delta V = -C_V\Delta T \tag{4}$$

Substituting  $-C_V\Delta T$  for  $p\Delta V$  from equation 4 into equation 3 gives

$$-C_V\Delta T + V\Delta p = R\Delta T$$

Rearranging this equation gives

$$V\Delta p = C_V\Delta T + R\Delta T = C_P\Delta T \text{ (since } C_P = C_V + R \text{)}$$

$$\text{Therefore } V\Delta p = C_P\Delta T \tag{5}$$

Dividing equation 5 by equation 4 and cancelling  $\Delta T$  therefore gives

$$\frac{V\Delta p}{p\Delta V} = \frac{C_P}{-C_V}$$

Rearranging this equation and substituting  $\gamma$  for  $\frac{C_p}{C_v}$  therefore gives

$$\frac{\Delta p}{\Delta V} = -\gamma \frac{p}{V}$$

In the limit  $\Delta V \rightarrow 0$ , we write this equation as  $\frac{dp}{dV} = -\gamma \frac{p}{V}$

This equation can be solved by integration to give  $pV^\gamma = \text{constant}$ , the adiabatic equation for an ideal gas.

## Answers

### 1.1

- 1 a**  $0.20 \text{ rad s}^{-2}$       **b i**  $360 \text{ rad}$       **ii**  $57$   
**2 a**  $75 \text{ rad s}^{-1}$       **b**  $1.5 \text{ rad s}^{-2}$       **c**  $300$   
**3 a**  $1.4 \times 10^5 \text{ rad}$       **b**  $22\,000$   
**4 a**  $107 \text{ rad s}^{-1}$       **b i**  $5.0 \text{ s}$       **ii**  $42.4$       **c**  $21 \text{ rad s}^{-1}$

### 1.2

- 1**  $0.27 \text{ N m}$   
**2 b i**  $21 \text{ rad s}^{-2}$       **ii**  $376 \text{ turns}$   
**3** Disc Y  
**4 b ii**  $17 \text{ kg m}^2$

### 1.3

- 1 a**  $9.6 \text{ J}$       **b i**  $19 \text{ N m}$       **ii**  $50 \text{ rad}$   
**2 a**  $12.1 \text{ J}$       **b**  $0.2 \text{ J}$       **c i**  $11.9 \text{ J}$       **ii**  $6.3 \times 10^{-4} \text{ kg m}^2$   
**4 a i**  $47 \text{ kg}$       **ii**  $0.57 \text{ kg m}^2$       **b i**  $28 \text{ kJ}$       **ii**  $2.0 \text{ kW}$

### 1.4

- 1 a**  $4.5 \text{ N m s}$       **b**  $0.50 \text{ N m}$   
**2 a**  $1.1 \text{ N m s}$       **b**  $0.022 \text{ kg m}^2$   
**4**  $2.6 \times 10^{-2} \text{ kg m}^2$

### 2.1

- 1**  $30 \text{ J}$  transferred to the gas  
**2 a**  $9 \text{ J}$       **b i**  $5 \text{ J}$       **ii**  $4 \text{ J}$  transferred to the surroundings  
**3 b**

Stage	$Q$	$\Delta U$	$W$
gas ignites	$>0$	$>0$	$0$
gas expands	$0$	$<0$	$>0$

- 4 a**  $W = 80 \text{ J}$   
 Heat transfer into the system = work done by the system  
**b**  $Q = -220 \text{ J}$   
 Heat transfer from the system + work done by the system lowers the internal energy by  $400 \text{ J}$   
**c**  $Q = 0$   
 Work done on the system is equal to its gain of internal energy



**d**  $\Delta U = -260 \text{ J}$

The internal energy of the system is lowered by 260 J as a result of the difference between 500 J of heat transfer from the system and 240 J of work done on the system.

## 2.2

- 1 a i** 0.18 moles      **ii** 400 K      **iii** + 225 J      **b** 380 J (to the gas)  
**2 a i** 300 K      **ii** 75 kPa      **b** 1.4 kJ  
**3 a** 90 kPa, 260 K      **b** 1.1 J  
**4 b**  $6.3 \times 10^{-6} \text{ m}^3$

## 2.3

- 2 a** 160 kW      **b** 28%  
**4 a i** 4.8 MW      **ii** 4.4 MW      **b i** 13 MW      **ii** 0.37

## 2.4

- 1 a i**  $40 \text{ J s}^{-1}$       **ii** 0.10      **b** 0.17  
**2 a** 0.20      **b** 16 MW  
**3 a** 0.71      **b ii** 35 kW

## 2.5

- 1 a** 5      **b** 480 W  
**2 a** 100 W  
**4 a** 9.5 kJ      **b** 15 W      **c** 2

## Image acknowledgements

Section 1.4 figure 3: NASA