

M1. (a) (i) $8.3 \text{ rev} = 8.3 \times 2\pi \text{ rad}$ ✓ (= 52 rad)

Use of $\omega_2^2 = \omega_1^2 + 2\alpha\theta$

$0 = 6.4^2 + 2 \times \alpha \times 52$ ✓

If eqtn(s) of motion used correctly with $\theta = 8.3$ (giving $\alpha = 2.5$), give 2 out of first 3 marks.

OR use of $\theta = \frac{1}{2}(\omega_1 + \omega_2)t$ leading to $t = 16.25 \text{ s}$ and $\omega_2 = \omega_1 + \alpha t$

$\alpha = (-) 0.39$ ✓ rad s⁻² ✓

Accept: s⁻²

Unit mark is an independent mark

4

(ii) $T = I\alpha$

$= 8.2 \times 10^{-3} \times 0.39 = 3.2 \times 10^{-3} \text{ N m}$ ✓

Give CE from a i

1

(b) (i) ($W = T\theta$ or $W = T\omega t$) where $\theta = 0.78 \times 270$ ✓ (= 210 rad)

$= 3.2 \times 10^{-3} \times 210 = 0.67 \text{ J}$ ✓

Give CE from a ii

2

(b) (ii) ratio = $\frac{900 \times 270}{0.67}$ or $\frac{2.4(3) \times 10^5}{0.67}$ ✓

$= 3.6 \times 10^5$ ✓

CE from b i. Must be in the form: number $\times 10^5$ with number calculated correctly.

900 \times 270 or 2.4(3) $\times 10^5$ or equivalent must be seen for 1st mark

1 mark for only writing 3.6×10^5

2

(Total 9 marks)

M2. (a) Use of $I = \Sigma mr^2$ or expressed in words ✓

With legs close to chest, more mass at smaller r , so I smaller ✓

2

(b) (i) Angular momentum is conserved / must remain constant OR no external torque acts ✓

WTTE

as I decreases, ω increases and vice versa to maintain $I\omega$ constant ✓
OR as I varies, ω must vary to maintain $I\omega$ constant

2

- (ii) (Angular velocity increases initially then decreases (as he straightens up to enter the water)).

No mark for just ang. vel starts low then increases then decreases, i.e. for describing ω only at positions 1,2 and 3.

With one detail point e.g. ✓

- Angular velocity when entering water is greater than at time $t = 0$ s.
- Angular velocity increases, decreases, increases, decreases
- Maximum angular velocity at $t = 0.4$ s
- Greatest rate of change of ang. vel. is near the start
- Angular velocity will vary as inverse of M of I graph

1

- (c) angular. momentum = $10.9 \times 4.4 = 48$ (N m s) ✓

(ω_{\max} occurs at minimum I)

Allow 6.3 to 6.5. If out of tolerance e.g. 6.2 give AE for final answer

minimum $I = 6.4 \text{ kg m}^2$ (at 0.4 s) ✓

$6.4 \times \omega_{\max} = 48$ leading to

$\omega_{\max} = 7.5 \text{ rad s}^{-1}$ ✓

3

(Total 8 marks)

M3. (a) (i) $T = Fr = 32 \times 0.15$
 $= 4.8 \text{ N m}$ ✓

1

(ii) $\omega = 2600 \times 2\pi/60$ ($= 270 \text{ rad s}^{-1}$) ✓ accept 272 rad s^{-1}

total torque = $4.8 + 1.2 = 6.0 \text{ N m}$ ✓

$P = T\omega$

$= 6.0 \times 270 = 1620 \text{ W}$ ✓

3

(b) $\alpha = \frac{270 - 0}{8.5} = 32 \text{ rad s}^{-2}$ ✓

$I = T/\alpha = \frac{1.2}{32} = 0.038$ ✓ kg m² ✓

OR use of $\Theta = \frac{1}{2}(\omega_2 + \omega_1)t$ (= 1150 rad) ✓

and $\frac{1}{2}I\omega^2 = T\Theta$ leading to $I = 0.038$ ✓ kg m² ✓

3

(c) $E = \frac{1}{2}I\omega^2$

$= 0.5 \times 0.038 \times 270^2 = 1400 \text{ J}$ ✓

$P = E/t = 1400/0.005 = 280 \text{ kW}$ ✓

2

[9]

M4. (a) the (total) angular momentum (of a system) remains constant provided no external torque acts (on the system) ✓

1

(b) (i) (as core radius decreases M of I of core decreases) $I\omega$ must remain constant ✓

I decreases so ω increases and greater ω means shorter period of rotation or less time for one revolution ✓

2

(ii) $0.4 mR_1^2 \times 2\pi/T_1 = 0.4 mR_2^2 \times 2\pi/T_2$ ✓

$T_2/T_1 = R_2^2/R_1^2$ ✓

$T_2 = \frac{(12 \times 10^3)^2}{(4.1 \times 10^7)^2} 3.8 \times 10^6$

$= 0.33 \text{ s}$ ✓ 2 sf ✓

or

$$0.4 \text{ m} \times (4.1 \times 10^7)^2 \times \omega_1$$

$$= 0.4 \text{ m} \times (12 \times 10^3)^2 \times \omega_2 \checkmark$$

$$\omega_1 = 2\pi/T = 2\pi/(3.8 \times 10^6) = 1.7 \times 10^{-6} \text{ rad s}^{-1} \checkmark (1.65 \times 10^{-6})$$

leading to

$$\omega_2 = 20 \text{ rad s}^{-1} [19.3 \text{ if } 1.65 \times 10^{-6} \text{ used}]$$

$$T_2 = 2\pi/\omega_2 = 0.31 \text{ s (2 sf throughout)} \checkmark 2 \text{ sf} \checkmark$$

$$[0.33 \text{ s if } 1.65 \times 10^{-6} \text{ rad s}^{-1} \text{ and } 19.3 \text{ rad s}^{-1} \text{ used}]$$

4

[7]

- M5.** (a) **The candidate's writing should be legible and the spelling, punctuation and grammar should be sufficiently accurate for the meaning to be clear.**

The candidate's answer will be assessed holistically. The answer will be assigned to one of three levels according to the following criteria.

High Level (Good to excellent): 5 or 6 marks

The information conveyed by the answer is clearly organised, logical and coherent, using appropriate specialist vocabulary correctly. The form and style of writing is appropriate to answer the question.

The candidate provides a comprehensive and logical explanation which recognises that energy stored depends on moment of inertia and ω^2 and that moment of inertia itself depends on the mass and how the mass is distributed about the axis of rotation, quoting and explaining Σmr^2 . They will appreciate that a high mass will result from using high density material and they will realise that increasing the radius alone will not necessarily increase M or I for a given mass, but the shape (eg spoked or not) might. They will refer to means of promoting high speed and may appreciate why there is a limit to the maximum speed depending on tensile strength.

Intermediate Level (Modest to adequate): 3 or 4 marks

The information conveyed by the answer may be less well organised and not fully coherent. There is less use of specialist vocabulary, or specialist vocabulary may be used incorrectly. The form and style of writing is less appropriate.

The candidate provides a comprehensive and logical explanation which links moment of inertia and ω^2 to energy stored and will discuss the factors that affect the moment of inertia, but there may be errors in their understanding. They will probably refer to the need to reduce friction but they may not state how this may be done. There may be some reference to the density or strength of the materials used, but the links with energy storage or M of I will be vague. The answer should be adequately or well presented in terms of spelling, punctuation and grammar.

Low Level (Poor to limited): 1 or 2 marks

The information conveyed by the answer is poorly organised and may not be relevant or coherent. There is little correct use of specialist vocabulary. The form and style of writing may be only partly appropriate.

The candidate recognises that energy storage depends on speed and M of I, but may not link M of I adequately to radius or mass or the distribution of the mass. They may confuse power or angular momentum with energy. The answer may lack coherence and may contain a significant number of errors in terms of spelling and punctuation.

Incorrect, inappropriate or no response: 0 marks

No answer or answer refers to unrelated, incorrect or inappropriate physics.

Statements expected in a competent answer should include some of the following marking points.

Linking E to ω^2

Linking E to Σmr^2

Importance of shape; put more m at greater r , use thin axle

High density material promotes high m for given size

Friction reduced by low-friction (eg magnetic- or air-) bearings and smooth outer surface

Rotational speed limited by tensile strength of material

Reference to centripetal force

Frictional torque increases with rotational speed

Some answers might refer to adverse gyroscopic effects or need for perfect balance

max 6

(b) (i) $\Delta E = \frac{1}{2} I (\omega_1^2 - \omega_2^2) = 0.5 \times 0.036 \times (6400^2 - 3100^2)$
 $= 560 \times 10^3 \text{ J}$ ✓

1

(ii) $P = E/t = \frac{560 \times 10^3}{6.6} = 85000 \text{ W}$ ✓

accept 85 kW if unit is changed in answer line from W to kW

1

(iii) $T = P/\omega_{\text{ave}} = \frac{85 \times 10^3}{4750}$ ✓ N m ✓

or $T = I\alpha = 0.036 \times \frac{(6400 - 3100)}{6.6} = 18$ ✓ N m ✓

2

(iv) $T\theta = \Delta E$

$\theta = 560 \times 10^3 / 18 = 31 \times 10^3 \text{ rad}$ ✓

$\frac{31 \times 10^3}{2\pi} = 4900 \text{ rev}$ ✓

or

$\theta = \frac{1}{2} (6400 + 3100) \times 6.6 = 31 \times 10^3 \text{ rad}$ ✓

$= 4900 \text{ rev}$ ✓

or $\theta = \omega_1 t - \frac{1}{2} \alpha t^2$

$= 6400 \times 6.6 - \frac{1}{2} 500 \times 6.6^2 = 31 \times 10^3 \text{ rad}$ ✓

$= 4900 \text{ rev}$ ✓

2

[12]

M6. (a) (i) $I = 82 + 39 \times 0.90^2 + 28 \times 0.50^2$ (1)

$= 120 \text{ kg m}^2$ (1) to 2 sig figs (1)

3

(ii) $\omega = 35 \times 2\pi/60$ (1) = 3.7 rad s⁻¹

$E = \frac{1}{2} I \omega^2 = 0.5 \times 120 \times 3.7^2 = 820 \text{ J}$ (1)

(accept 800 to 821 J depending on sf carried through)

2

- (b) (i) angular momentum must be conserved **(1)**
so if I decreases ω must increase **(1)** 2
- (ii) $120 \times 3.7 = 91 \times \omega_2$ **(1)**
 $\omega_2 = 4.9 \text{ rad s}^{-1}$ **(1)** 2
- (iii) $E = 0.5 \times 91 \times 4.9^2 = 1100 \text{ J}$ (1090 J) **(1)**
(give CE only if correct I value used)
accept 1050 – 1100 J 1
- (c) work done or energy transferred as children move towards
the centre **(1)**
or work done as centripetal force moves inwards **(1)** 1

[11]

- M7.** (a) (i) $T = Fr = 7.0 \times 0.075$
 $= 0.53$ **(1)** N m **(1)** 2
- (ii) $P = T\omega$
 $= 0.53 \times 120 = 64 \text{ W}$ **(1)** 1
- (b) use of equation(s) of motion:
 $\theta = \frac{1}{2}(120 + 0) \times 6.2 = 370 \text{ rad}$ **(1)**
 $370/2\pi = 59 \text{ rotations}$ **(1)** 2

[5]

- M8.** (a) $22\,000 \text{ (rev min}^{-1}) \times 2\pi/60$ **(1)** (= 2300 rad s⁻¹)
energy stored (= $\frac{1}{2} I\omega^2$) = $\frac{1}{2} \times 0.60 \times 2300^2$ **(1)** (= 1.6 MJ) 2

(b) (i) $t = E/P = \frac{1.6 \times 10^6}{8.7} \text{ (1)} = 1.84 \times 10^5 \text{ s (1)}$ (51 hours)

(ii) torque = $\frac{\text{power}}{\text{average speed}} = \frac{8.7}{(2300/2)} = 7.5(6) \times 10^{-3} \text{ (1) N m (1)}$

or $T = I a = 0.60 \times 2300 / (1.84 \times 10^5) = 7.5 \times 10^{-3} \text{ (1) N m (1)}$

max 3

(c) in **B** more of the mass is at a greater radius than in **A** (1)

so I greater and so energy stored greater (for same speed) (1)

2

[7]

M9. (a) (i) energy: kinetic energy = $\frac{1}{2}I\omega^2$, \rightarrow small stored energy
 [or less work/energy needed to produce change]
 power = rate of energy change, fast change \rightarrow high power
 torque: $T = I\alpha$, α large so large torques needed unless I small,
 momentum, impulse: $L = I\omega$, impulse = ΔL so unless I small, large
 angular impulses are needed
marking: for any **one** of the above:
 for correct consideration (1)
 for mathematical justification (1)

(ii) explanations based on $I = mr^2$ (1)
 low **mass**, small diameter (1)

4

(b) (i) $\alpha = \frac{\omega_1 - \omega_2}{t} = \frac{120 + 120}{50 \times 10^{-3}} = 4.8 \times 10^3 \text{ rad s}^{-2}$

(ii) $T = I\alpha = 4.4 \times 10^{-5} \times 4.8 \times 10^3 = 0.21(1) \text{ N m (1)}$
 (allow C.E. from incorrect value of α from (i))

(iii) $\theta = \left(\frac{\omega_1 + \omega_2}{2} \right) t = \left(\frac{120 + 0}{2} \right) 25 \times 10^{-3} = 1.5 \text{ rad (1)}$

3

[7]

M10. (a) (i) torque = force \times diameter **(1)**
 $= 0.12 \times 0.34 = 4.1 \times 10^{-2} \text{Nm}$ **(1)**

(ii) (use $T = I\alpha$ gives) $\alpha = \frac{4.1 \times 10^{-2}}{0.17}$ **(1)**
 $(= 0.24 \text{ rads}^{-2})$

3

(b) (i) (use of $w_2 = w_1 + \alpha t$ gives) $0 = 0.92 - 0.24 t$ and $t = 3.8(3)\text{s}$ **(1)**

(ii) (use of $w_2^2 = w_1^2 - 2a\theta$ gives) $0 = 0.92^2 - (2 \times 0.24 \times \theta)$ **(1)**

$$\theta = (1.7(6) \text{ rad}) = \left(1.76 \times \frac{360}{2\pi} \right) 101^{(o)}$$

$$\text{[or } \theta = w_1 t - \frac{1}{2} \alpha t^2 \text{]}$$

3

[6]

M11. (a) (angular speed $=$) $22\,000 \text{ (rev min}^{-1}) \times \frac{2\pi}{60}$ **(1)**
 $(= 2300 \text{ rad s}^{-1})$

energy stored $(= \frac{1}{2} I w^2) = \frac{1}{2} \times 0.60 \times 2300^2$ **(1)**
 $(= 1.6 \text{ MJ})$

2

(b) (i) $t \left(= \frac{E}{P} \right) = \frac{1.6 \times 10^6}{8.7} = 1.84 \times 10^5 \text{ s}$ **(1)**
 (51 hours)

(ii) torque $= \frac{\text{power}}{\text{average speed}} = \frac{8.7}{(2300/2)} = 7.5(6) \times 10^{-3} \text{ Nm}$ **(1)**

$$\text{[or } T = I\alpha = \frac{0.6 \times 2300}{1.84 \times 10^5} = 7.5 \text{ N m]} \text{ (1)}$$

2

(c) in B more of the mass is at greater **radius** than in A **(1)**
 so I greater and so energy stored greater **(1)**

2

[6]

- M12.** (a) (i) energy: kinetic energy = $\frac{1}{2}I\omega^2$, \rightarrow small stored energy
 [or less work/energy needed to produce change]
 power = rate of energy change, fast change \rightarrow high power
 torque: $T = I\alpha$, α large so large torques needed unless I small,
 momentum, impulse: $L = I\omega$, impulse = ΔL so unless I small, large
 angular impulses are needed
marking: for any **one** of the above:
 for correct consideration **(1)**
 for mathematical justification **(1)**

4

- (ii) explanations based on $I = mr^2$ **(1)**
 low **mass**, small diameter **(1)**

(b) (i) $\alpha = \frac{\omega_1 - \omega_2}{t} = \frac{120 + 120}{50 \times 10^{-3}} = 4.8 \times 10^3 \text{ rad s}^{-2}$ **(1)**

(ii) $T = I\alpha = 4.4 \times 10^{-5} \times 4.8 \times 10^3 = 0.21(1) \text{ N m}$ **(1)**

(allow C.E. from incorrect value of α from (i))

(iii) impulse = torque \times time = $0.21 \times 50 \times 10^{-3} = 1.1 \times 10^{-2} \text{ N m s}$ **(1)**

($1.05 \times 10^{-2} \text{ N m s}$)

(allow C.E. for value of T from (ii))

[or $\Delta L = I(\omega_2 - \omega_1) = 4.4 \times 10^{-5} \times 240 = 1.1 \times 10^{-2} \text{ N m s}$]

(iv) $\theta = \left(\frac{\omega_1 + \omega_2}{2} \right) t = \left(\frac{120 + 0}{2} \right) 25 \times 10^{-3} = 1.5 \text{ rad}$ **(1)**

4

[8]

- M13.** (a) (i) $110 \text{ rpm} = \left(\frac{110 \times 2\pi}{60} \right) = 11.5 \text{ (rad s}^{-1}\text{)}$ **(1)**
 kinetic energy = $\frac{1}{2}I\omega^2 = 0.5 \times 150 \times 11.5^2 = 9.9(2) \text{ kJ}$ **(1)**
 (use of 12 for conversion above gives 10.8 kJ)

(ii) average useful $P_{\text{out}} = \frac{9.92 \times 10^3}{15} = 660 \text{ W}$ **(1)** (661 W)

(use of k.e. = 10.8 kJ gives 720 W)

(iii) $P_{\text{av}} = T_{\text{acc}} \omega_{\text{av}}$ gives $T = \left(\frac{661}{11.5/2} \right) = 115 \text{ N m}$ **(1)** **(1)** (for ω_{av})

(use of $P_{\text{out}} = 720 \text{ W}$ gives 125 N m)

5

(b) work done against friction = $T_r \theta$ and $T_r = \frac{9.95 \times 10^3}{35 \times 2\pi}$ **(1)**

= 45(.2) N m **(1)**

[or use of $\omega_2^2 = \omega_1^2 + 2\alpha\theta$, $T = I\alpha$

i.e. $0 = 11.5^2 + (2\alpha \times 2\pi \times 35)$ gives $\alpha = 0.301$ (rad s⁻²)

$T (= I\alpha) = 150 \times 0.301 = 45(.1)$ N m]

2

[7]

M14. (a) moment of inertia of the rockets

= $(2 \times 0.54 \times (0.80)^2) + (2 \times 0.54 \times (0.50)^2) = 0.96$ (kg m²) **(1)**

total moment of inertia = $0.96 + 0.14$ (kg m²) **(1)** (= 1.10 kg m²)

2

(b) (i) torque = $(2 \times 3.5 \times 0.80) + (2 \times 3.5 \times 0.50) = 9.1$ N m **(1)**

(ii) $\alpha \left(= \frac{T}{I} \right) = \frac{9.1}{1.1} = 8.3$ rad s⁻² **(1)** (8.27 rad s⁻²)

(allow C.E. for value of torque from (i))

(iii) one turn = 6.28 rad **(1)**

$\theta = \omega_1 t + \frac{1}{2} \alpha t^2$ gives $6.28 = 0.5 \times 8.3 \times t^2$ and $t = 1.2(3)$ s **(1)**

(allow C.E. for value of α from (ii))

4

(c) frictional couple (due to air resistance) increases as angular speed increases **(1)**
when frictional couple = driving torque [or when no resultant torque], then no acceleration **(1)**

2

[8]

M15. (a) energy supplied = $15 \times 10^3 \times 3 \times 60 = 2.7$ MJ **(1)**

(use of $E_k = \frac{1}{2} I \omega^2$ gives) $2.7 \times 10^6 = 0.5 \times 9.5 \times \omega^2$ **(1)**

(gives $\omega = 754 \approx 750$ rad s⁻¹)

2

- (b) (i) impulse ($= \Delta l \omega$) = $(-9.5 \times 754$
 $= (-7.2 \times 10^3 \text{ N m s (or kg m}^2 \text{ rad s}^{-1})$ **(1)** ($7.16 \times 10^3 \text{ N m s}$)
 (use of $\omega = 750$ gives impulse = $7.1(3) \times 10^3 \text{ N m s}$)

(ii) average torque $\left(= \frac{\text{angular impulse}}{\text{time}} \right) = \frac{716 \times 10^3}{4.5}$
 $= 1600 \text{ N m}$ **(1)**

[or $T_{\text{av}} = I\alpha$, where $\alpha = \frac{750}{4.5} = 167 \text{ (rad s}^{-2})$

$T_{\text{av}} = 9.5 \times 167 = 1600 \text{ N m}$ **(1)]**

2

- (c) area under curve = angular impulse = $T_{\text{av}} \times t$ **(1)**
 area found by counting squares (or correct alternatives) **(1)**

$T_{\text{av}} = \frac{\text{angular impulse}}{t}$, where t is obtained from graph **(1)**

3

[7]

M16. (a) (i) $I = 5.3 \times 10^8 + (9.4 \times 10^4 \times 100^2) = 1.47 \times 10^9 \text{ (kg m}^2)$ **(1)**

(ii) $\omega = \frac{2\pi}{2.5 \times 60} = 4.2 \times 10^{-2} \text{ (rad s}^{-1})$ **(1)**

(iii) (use of $E_k = \frac{1}{2} I \omega^2$ gives) $E_k = 0.5 \times 1.47 \times 10^9 \times (4.2 \times 10^{-2})^2$ **(1)**
 $(= 1.3 \times 10^6 \text{ J})$

3

(b) $P \left(= \frac{E}{t} \right) = \frac{1.3 \times 10^6}{25} = 52 \text{ kW}$ **(1)**

1

(c) (i) (use of $P = T\omega$ gives) $T = \frac{3000}{4.2 \times 10^{-2}} = 7.1(4) \times 10^4 \text{ N m}$ **(1)**
 (allow C.E. for value of ω from (a)(ii))

(ii) (use of $W = T\theta$ gives) $W = 7.1(4) \times 10^4 \times 2\pi$
 $= 4.5 \times 10^5 \text{ J}$ **(1)** ($4.49 \times 10^5 \text{ J}$)
 (allow C.E. for value of T from (c)(i))

2

[6]

M17. (a) (i) $\alpha \left(= \frac{\omega_2 - \omega_1}{t} \right) = \frac{1100 - 0}{4.2} = 260 \text{ rad s}^{-2}$ **(1)** (262 rad s⁻²)

(ii) $T (= I\alpha) = 7.6 \times 10^{-4} \times 262 = 0.20 \text{ N m}$ **(1)**

2

(b) $I_{\text{liquid}} = 8 \times (3.0 \times 10^{-3} (84 \times 10^{-3})^2) = 1.7 \times 10^{-4} \text{ (kg m}^2\text{)}$ **(1)**

$I_{\text{total}} = 7.6 \times 10^{-4} + 1.7 \times 10^{-4} = 9.3 \times 10^{-4} \text{ (kg m}^2\text{)}$ **(1)**

$\alpha \left(= \frac{T}{I} \right) = \frac{0.20}{9.3 \times 10^{-4}} = 215 \text{ (rad s}^2\text{)}$ **(1)**

$t \left(= \frac{\omega_2 - \omega_1}{\alpha} \right) = \frac{1100}{215}$ **(1)** (= 5.1 s)

(allow C.E for value of I_{liquid})

3

(c) $\theta_1 \left(= \frac{(\omega_1 + \omega_2)}{2} t \right) = \frac{1100}{2} \times 5.0 = 2750 \text{ (rad)}$

$\theta_3 = \frac{1100}{2} \times 6.0 = 3300 \text{ (rad)}$ **(1)** (for both θ_1 and θ_3)

$\theta_2 (= \omega_2 t) = 1100 \times (60 - 11) = 53900 \text{ (rad)}$ **(1)**

total angle turned = $\theta_1 + \theta_2 + \theta_3 = 60 \times 10^3 \text{ rad}$ **(1)**

3

[8]

M18. (a) (i) $\omega = \frac{480 \times 2\pi}{60} = 50 \text{ rad s}^{-1}$ **(1)** (50.3 rad s⁻¹)

(ii) (use of $E_k = \frac{1}{2} I \omega^2$ gives) $E_k = 0.5 \times 38 (50.3)^2$ **(1)** (48 kJ)

2

- (b) (i) new kinetic energy = $(48 - 12) = 36$ (kJ)

$$\omega = \sqrt{\frac{20 \times 36 \times 10^3}{38}} = 44 \text{ rad s}^{-1} \text{ (1)}$$

- (ii) angular impulse = change of angular momentum (= $I \Delta\omega$)
 $= 38 \times (50 - 44) = 230 \text{ kg m}^2 \text{ rad s}^{-1}$ (or N m s) (1)

- (iii) torque $\left(\frac{\text{angular impulse}}{\text{time}} \right) = \frac{230}{0.15} = 1.5 \times 10^3 \text{ N m}$ (1)
 (allow C.E for value of angular impulse)

3

[5]

- M19.** (a) (use of $v = \omega r$ gives $\omega = \frac{3.5}{0.2} = 18 \text{ rad s}^{-1}$ (1)

1

(b) (i) $\alpha = \frac{\omega_1 - \omega_2}{t} = (-) \frac{(17.5 + 17.5)}{4.6} = (-)7.6 \text{ rad s}^{-2}$ (1)

- (ii) (use of $T = I\alpha$ gives) $T = 40 \times 7.6 = 300 \text{ N m}$ (1)
 (allow C.E. for value of α from (i))

- (iii) (use of *angular impulse* = Tt gives)
 angular impulse = $300 \times 4.6 = 1.4 \times 10^3 \text{ kg m}^2 \text{ rad s}^{-1}$ (1)
 (allow C.E. for value of T from (ii))

- (iv) uniform torque therefore uniform acceleration, $\therefore t = 2.3 \text{ s}$ (1)

$$\theta = \frac{(\omega_1 + \omega_2)}{2} t = \frac{17.5}{2} \times 2.3 = 20.13 \text{ (rad)} \text{ (1)}$$

$$\text{number of turns} = \frac{20.13}{2\pi} = 3.2 \text{ (so 3 complete turns)} \text{ (1)}$$

6

[7]

M20. (a) (i) torque = $4 \times 0.60 \times 1.8 = 4.3(2)$ N m **(1)**

(ii) $\omega = \frac{2\pi}{110} = 5.7(1) \times 10^{-2}$ (rad s⁻¹) **(1)**

at steady speed, frictional torque = applied torque **(1)**

(use of $P = T\omega$ gives) $P = 4.32 \times 5.71 \times 10^{-2} = 0.25$ W **(1)**
(allow C.E. for value of T from (i))

4

(b) (i) average power = $0.5 \times 0.25 = 0.125$ (W) **(1)**
energy = average power \times time = 0.125×12 **(1)** (= 1.5 J)
(allow C.E. for value of P from (a)(ii))

(ii) (use of *kinetic energy* = $\frac{1}{2}I\omega^2 = 1.5$ gives)

$$I = \frac{2 \times 1.5}{(5.71 \times 10^{-2})^2} = 910 \text{ kg m}^2 \text{ (1)}$$

(allow C.E. for value of ω from (a)(ii))

3

[7]

M21. (a) (i) angular impulse = change in angular momentum **(1)**
 $I\omega - 0 = 1.2$, $\therefore \Delta(I\omega) = 1.2$ kg m² rad s⁻¹ **(1)**

(ii) $I\omega = 1.2$ gives $\omega = \frac{1.2}{4.8 \times 10^{-2}} = 25$ rad s⁻¹ **(1)**

(iii) (torque \times time = angular impulse gives)

$$\text{torque} = \frac{1.2}{2.8} = 0.43 \text{ N m (1)}$$

4

(b) $\theta = \frac{(\omega_1 + \omega_2)t}{2} = \frac{(30 + 0)14}{2} = 210$ (rad) **(1)**

number of turns = $\frac{210}{2\pi} = 33.4$ i.e. 33 complete turns

(allow C.E. for value of θ)

2

[6]

- M22.** (a) (i) energy = power \times time = $150 \times 10^3 \times 4.4 = 6.6 \times 10^5 \text{ J}$ **(1)**
(ii) increase in kinetic energy = energy supplied during acceleration **(1)**

$$6.6 \times 10^5 = 0.5 I (\omega_2^2 - \omega_1^2) \text{ (1)}$$

(allow C.E. for energy value from (i))

$$I = \frac{6.6 \times 10^5 \times 2}{7.4^2 - 1.6^2} = 2.5(3) \times 10^4 \text{ kg m}^2 \text{ (1)}$$

4

- (b) greater moment of inertia (I) **(1)**
($E_k \propto I$) more kinetic energy must be given in the same time **(1)**

2
QWC 2

[6]

