Q1. The turntable of a microwave oven has a moment of inertia of 8.2×10^{-3} kg m^2 about its vertical axis of rotation.

(a) With the drive disconnected, the turntable is set spinning. Starting at an angular speed of 6.4 rad s^{-1} it makes 8.3 revolutions before coming to rest.

(i) Calculate the angular deceleration of the turntable, assuming that the deceleration is uniform. State an appropriate unit for your answer.

angular deceleration .... unit ....

(4)

(ii) Calculate the magnitude of the frictional torque acting at the turntable bearings.

torque .... N m

(1)

(b) The turntable drive is reconnected. A circular pie is placed centrally on the turntable. The power input to the microwave oven is 900 W, and to cook the pie the oven is switched on for 270 seconds. The turntable reaches its operating speed of 0.78 rad s^{-1} almost immediately, and the friction torque is the same as in part (a)(ii).

(i) Calculate the work done to keep the turntable rotating for 270 s at a constant angular speed of 0.78 rad s^{-1} as the pie cooks.

work done .... J

(2)

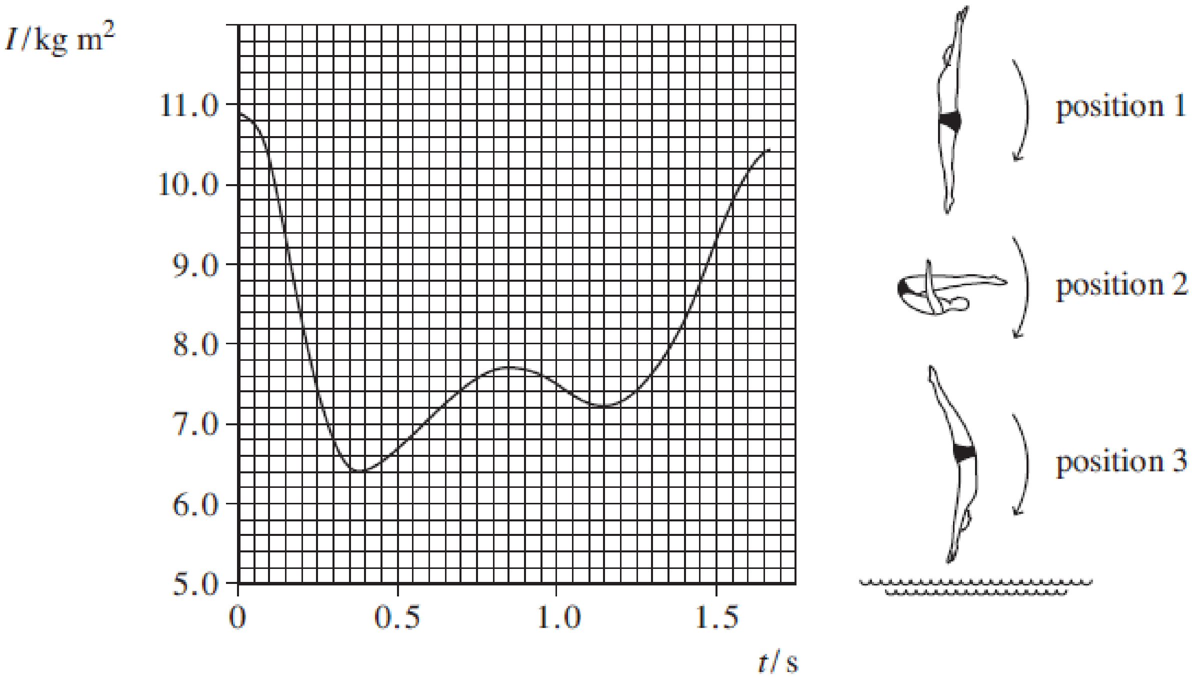
(ii) Show that the ratio

$$\frac{\text{energy supplied to oven}}{\text{work done to drive turntable}}$$

is of the order of 10^5.

(2)
(Total 9 marks)

Q2. The graph below shows how the moment of inertia *I* of a diver performing a reverse dive varies with time *t* from just after he has left the springboard until he enters the water.



The diver starts with his arms extended above his head (position 1), and then brings his legs towards his chest as he rotates (position 2). After somersaulting in mid-air, he extends his arms and legs before entering the water (position 3).

(a) Explain how moving the legs towards the chest causes the moment of inertia of the diver about the axis of rotation to decrease.

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(2)

(b) (i) Explain in terms of angular momentum why the angular velocity of the diver varies during the dive.

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(2)

(ii) Describe how the angular velocity of the diver varies throughout the dive.

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....

(1)

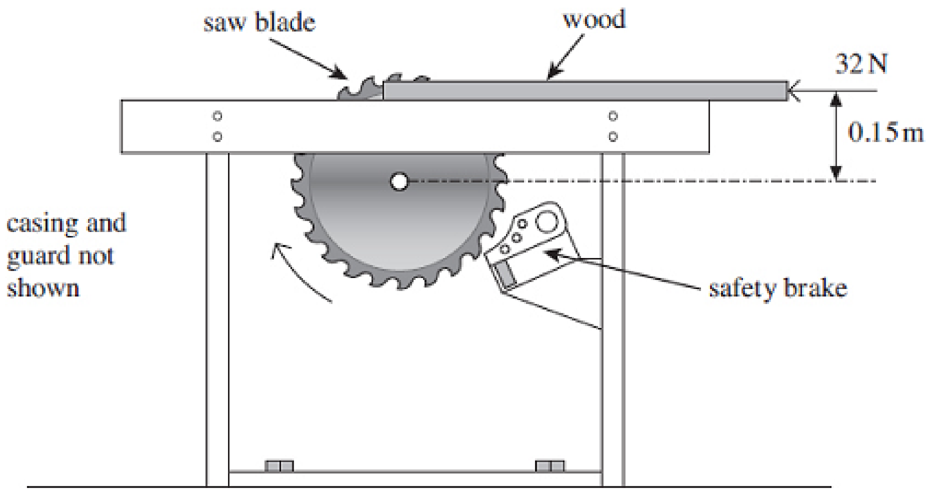
(c) At time $t = 0$ the angular velocity of the diver is 4.4 rad s^{-1} and his moment of inertia about the axis of rotation is 10.9 kg m^2.

With reference to the graph above calculate the maximum angular velocity of the diver during the dive.

angular velocity .... rad s^{-1}

(3)
(Total 8 marks)

Q3. The figure below shows a type of circular saw. The blade is driven by an electric motor and rotates at 2600 rev min^{-1} when cutting a piece of wood. A constant frictional torque of 1.2 Nm acts at the bearings of the motor and axle.



A horizontal force of 32 N is needed to push a piece of wood into the saw. The force acts on the blade at an effective radius of 0.15 m.

(a) (i) Calculate the torque on the saw blade resulting from the horizontal force on the wood.

answer = ... Nm

(1)

(ii) Calculate the output power of the motor when the saw is cutting the wood.

answer = ... W

(3)

(b) Immediately after cutting the wood the motor is switched off. The time taken for the saw blade to come to rest is 8.5 s. Calculate the moment of inertia of the rotating parts (i.e. the motor rotor, axle and blade). State an appropriate unit.

answer = ... unit = ...

(3)

(c) If the blade is accidentally touched when it is rotating, an electronic safety brake stops the blade in 5.0 ms. This is fast enough to prevent serious injury. The safety brake works by forcing a block of aluminium into the saw teeth.

Estimate the rate at which the rotational kinetic energy is dissipated as heat and in deforming the aluminium when the brake operates.

answer = ... W

(2)
(Total 9 marks)

Q4. (a) State the law of conservation of angular momentum.

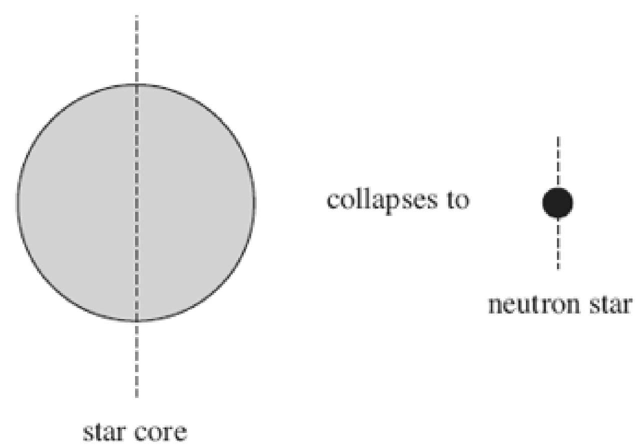
....

....

....

(1)

(b) When a star undergoes a supernova explosion, the star's core collapses into a very much smaller diameter forming an extremely dense *neutron star* as shown in the figure below.



A star has a period of rotation about an axis through its centre of 44 days (3.8×10^6 s) and a core of radius 4.1×10^7 m. The star undergoes a supernova explosion and the core collapses into a neutron star of radius 1.2×10^4 m. You may assume that during the collapse no mass is lost from the core and that the star remains spherical.

Moment of inertia of a sphere of uniform mass m and radius R about an axis through its centre = $0.40mR^2$

(i) Explain why the period of rotation of the star decreases as it becomes a neutron star.

....

....

....

(2)

(ii) Determine the period of rotation of the neutron star. Give your answer to an appropriate number of significant figures.

answer = ... s

(4)

(Total 7 marks)

Q5. Some motor racing cars are fitted with a kinetic energy recovery system (KERS). In this system, as the car brakes approaching a bend, instead of all the lost kinetic energy being dissipated as heat, some of the energy is used to accelerate a flywheel. When the car needs to accelerate out of the bend, the energy in the flywheel assists the engine in providing extra power.

(a) Describe and explain some of the design features of a flywheel in order for it to store maximum energy. Your answer should include consideration of the flywheel's shape, the material from which it is made and its design for high angular speeds.

The quality of your written communication will be assessed in this question.

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(6)

(b) A KERS flywheel has a moment of inertia of 0.036 kgm^2 and rotates at its maximum angular speed of 6400 rad s^{-1}. When the flywheel is used to help accelerate the car, the flywheel's speed reduces uniformly to 3100 rad s^{-1} in a time of 6.6 s. You may assume that frictional losses in the drive mechanism are negligible.

(i) Calculate the energy transferred from the flywheel to the car.

answer = ... J

(1)

(ii) Calculate the average power produced by the decelerating flywheel.

answer = ... W

(1)

(iii) Calculate the decelerating torque on the flywheel, stating an appropriate unit.

answer = ...

(2)

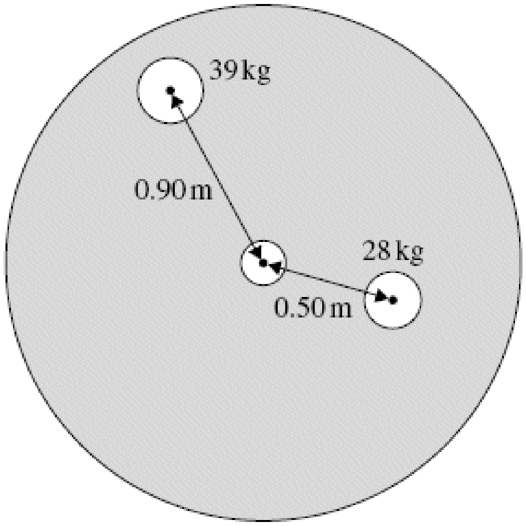
(iv) Calculate the number of revolutions made by the flywheel in the time of 6.6 s.

answer = ... revolutions

(2)

(Total 12 marks)

Q6. (a) A playground roundabout has a moment of inertia about its vertical axis of rotation of 82 kg m^2. Two children are standing on the roundabout which is rotating freely at 35 revolutions per minute. The children can be considered to be point masses of 39 kg and 28 kg and their distances from the centre are as shown in the figure below.



(i) Calculate the total moment of inertia of the roundabout and children about the axis of rotation. Give your answer to an appropriate number of significant figures.

answer = ... kg m^2

(3)

(ii) Calculate the total rotational kinetic energy of the roundabout and children.

answer = ... J

(2)

(b) The children move closer to the centre of the roundabout so that they are both at a distance of 0.36 m from the centre. This changes the total moment of inertia to 91 kg m^2.

(i) Explain why the roundabout speeds up as the children move to the centre of the roundabout.

....

....

....

....

(2)

(ii) Calculate the new angular speed of the roundabout. You may assume that the frictional torque at the roundabout bearing is negligible.

answer = ... rad s^{-1}

(2)

(iii) Calculate the new rotational kinetic energy of the roundabout and children.

answer = ... J

(1)

(c) Explain where the increase of rotational kinetic energy of the roundabout and children has come from.

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(1)
(Total 11 marks)

Q7. A grinding wheel is used to sharpen chisels in a school workshop. A chisel is forced against the edge of the grinding wheel so that the tangential force on the wheel is a steady 7.0 N as the wheel rotates at 120 rad s^{-1}. The diameter of the grinding wheel is 0.15 m.

(a) (i) Calculate the torque on the grinding wheel, giving an appropriate unit.

answer = ...

(2)

(ii) Calculate the power required to keep the wheel rotating at 120 rad s^{-1}.

answer = ... W

(1)

(b) When the chisel is removed and the motor is switched off, it takes 6.2 s for the grinding wheel to come to rest.

Calculate the number of rotations the grinding wheel makes in this time.

answer = ...

(2)

(Total 5 marks)

Q8. Flywheels store energy very efficiently and are being considered as an alternative to battery power.

(a) A flywheel for an energy storage system has a moment of inertia of 0.60 kg m^2 and a maximum safe angular speed of 22 000 rev min^{-1}.

Show that the energy stored in the flywheel when rotating at its maximum safe speed is 1.6 MJ.

(2)

(b) In a test the flywheel was taken up to maximum safe speed then allowed to run freely until it came to rest. The average power dissipated in overcoming friction was 8.7 W.

Calculate

(i) the time taken for the flywheel to come to rest from its maximum speed,

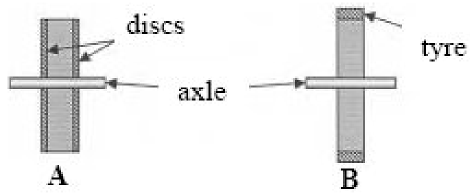
Time taken = ...

(ii) the average frictional torque acting on the flywheel.

Torque = ...

(3)

(c) The energy storage capacity of the flywheel can be improved by adding solid discs to the flywheel as shown in cross-section **A** in the diagram below, or by adding a hoop or tyre to the rim of the flywheel as shown in **B** in the diagram below. The same mass of material is added in each case. State, with reasons, which arrangement stores the more energy when rotating at a given angular speed.



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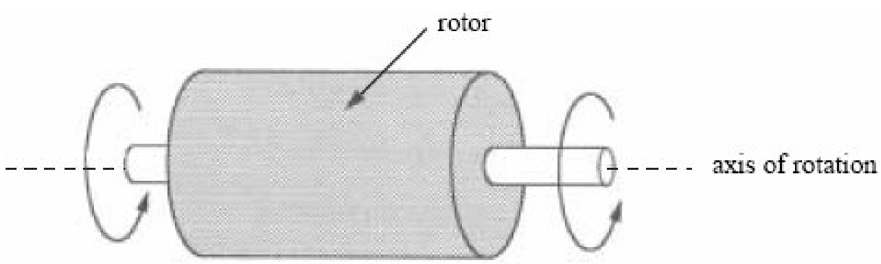
(2)
(Total 7 marks)

Q9. 'Low inertia' motors are used in applications requiring rapid changes of speed and direction of rotation. These motors are designed so that the rotor has a very low moment of inertia about its axis of rotation.



(a) (i) Explain why a low moment of inertia is desirable when the speed and direction of rotation must be changed quickly.

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(ii) State, giving a reason in each case, **two** features of rotor design which would lead to a low moment of inertia about the axis of rotation.

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(4)

(b) In one application, a rotor of moment of inertia 4.4×10^{-5} kg m^2 about its axis of rotation is required to reverse direction from an angular speed of 120 rad s^{-1} to the same speed in the opposite direction in a time of 50 ms. Assuming that the torque acting is constant throughout the change, calculate

(i) the angular acceleration of the rotor,

....

....

(ii) the torque needed to achieve this acceleration,

....

....

(iii) the angle turned through by the rotor in coming to rest momentarily before reversing direction.

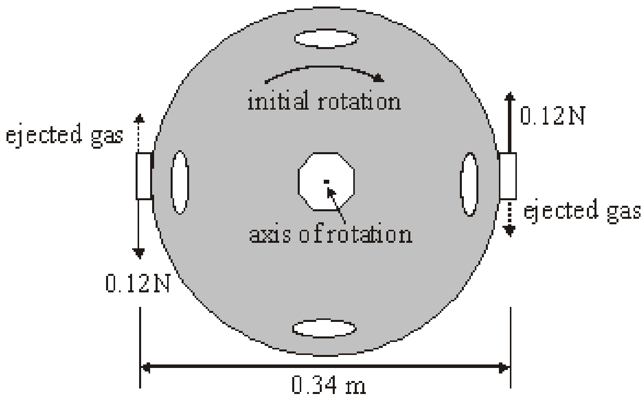
....

....

(3)
(Total 7 marks)

Q10. The figure below shows a remote-control camera used in space for inspecting space stations. The camera can be moved into position and rotated by firing ‘thrusters’ which eject xenon gas at high speed. The camera is spherical with a diameter of 0.34 m.

In use, the camera develops a spin about its axis of rotation. In order to bring it to rest, the thrusters on opposite ends of a diameter are fired, as shown in the figure below.



(a) When fired, each thruster provides a constant force of 0.12 N.

(i) Calculate the torque on the camera provided by the thrusters.

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(ii) The moment of inertia of the camera about its axis of rotation is 0.17 kg m^2. Show that the angular deceleration of the camera whilst the thrusters are firing is 0.24 rad s^{-2}.

....

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(3)

(b) The initial rotational speed of the camera is 0.92 rad s^{-1}. Calculate

(i) the time for which the thrusters have to be fired to bring the camera to rest,

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(ii) the angle turned through by the camera whilst the thrusters are firing. Express your answer in degrees.

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(3)
(Total 6 marks)

Q11. Flywheels store energy very efficiently and are being considered as an alternative to battery power.

(a) A flywheel for an energy storage system has a moment of inertia of 0.60 kg m^2 and a maximum safe angular speed of 22 000 rev min^{-1}.

Show that the energy stored in the flywheel when rotating at its maximum safe speed is 1.6 MJ.

....

....

....

(2)

(b) In a test the flywheel was taken up to maximum safe speed and then allowed to run freely until it came to rest. The average power dissipated in overcoming friction was 8.7 W.

Calculate

(i) the time taken for the flywheel to come to rest from its maximum speed,

....

....

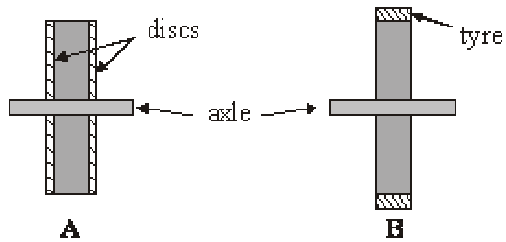
(ii) the average frictional torque acting on the flywheel.

....

....

(2)

(c) The energy storage capacity of the flywheel can be improved by adding solid discs to the flywheel as shown in cross-section in **A** in the figure below, or by adding a hoop or tyre to the rim of the flywheel as shown in **B** in the same diagram. The same mass of material is added in each case. State, with reasons, which arrangement stores the more energy when rotating at a given angular speed.



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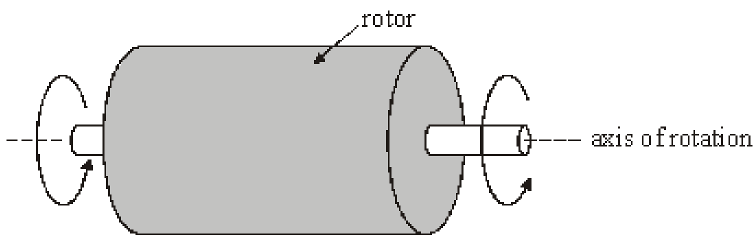
(2)
(Total 6 marks)

Q12. 'Low inertia' motors are used in applications requiring rapid changes of speed and direction of rotation. These motors are designed so that the rotor has a very low moment of inertia about its axis of rotation.



(a) (i) Explain why a low moment of inertia is desirable when the speed and direction of rotation must be changed quickly.

....

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....

(ii) State, giving a reason in each case, **two** features of rotor design which would lead to a low moment of inertia about the axis of rotation.

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(4)

(b) In one application, a rotor of moment of inertia 4.4×10^{-5} kg m^2 about its axis of rotation is required to reverse direction from an angular speed of 120 rad s^{-1} to the same speed in the opposite direction in a time of 50 ms. Assuming that the torque acting is constant throughout the change, calculate

(i) the angular acceleration of the rotor,

....

....

(ii) the torque needed to achieve this acceleration,

....

....

(iii) the angular impulse given to the rotor during the time the torque is acting,

....

....

(iv) the angle turned through by the rotor in coming to rest momentarily before reversing direction.

....

....

(4)
(Total 8 marks)

Q13. An early form of four-stroke gas engine stores kinetic energy in a large flywheel driven by the crankshaft. The engine is started from rest with its load disconnected and produces a torque which accelerates the flywheel to its off-load running speed of 110 rev min^{-1}.

(a) The flywheel has a moment of inertia of 150 kg m^2 and takes 15 s to accelerate from rest to an angular speed of 110 rev min^{-1}.

(i) Show that the rotational kinetic energy stored in the flywheel at this speed is approximately 10 kJ.

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....

(ii) Calculate the average useful power output of the engine during the acceleration.

....

....

(iii) Use your answer to part (ii) to calculate the average net torque acting on the flywheel during the acceleration.

....

....

(5)

(b) When the engine is running at 110 rev min^{-1} off-load, the gas supply to the engine is suddenly cut off and the flywheel continues to rotate for a further 35 complete turns before coming to rest. Calculate the average retarding torque acting on the flywheel.

....

....

(2)
(Total 7 marks)

Q14. **Figure 1** shows a 'firewheel' used at a firework display. Thrust produced by the captive rockets creates a torque which rotates the beam about a horizontal pivot at its centre. The shower of brilliant sparks in the exhaust gases of the rapidly orbiting rockets creates the illusion of a solid wheel.

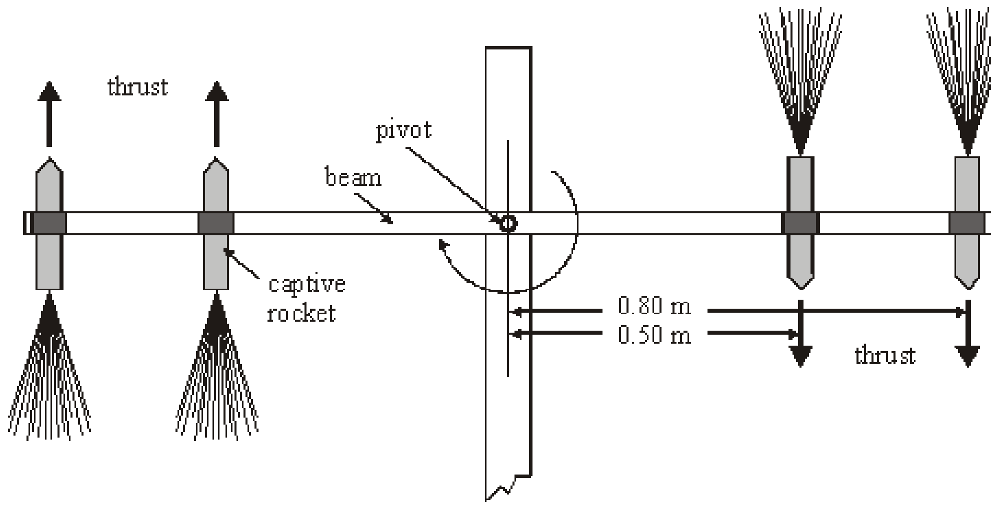

Figure 1

(a) The rockets are fixed symmetrically about the pivot at distances of 0.50 m and 0.80 m from the pivot. The initial mass of each rocket is 0.54 kg and the moment of inertia of the beam about the pivot is 0.14 kg m^2.

Show that the initial moment of inertia of the firewheel about the pivot is 1.10 kg m^2.

....

....

....

(2)

(b) The rockets are ignited simultaneously and each produces a constant thrust of 3.5 N. The frictional torque at the pivot is negligible. Calculate

(i) the total torque about the pivot when all the rockets are producing thrust,

....

....

(ii) the initial angular acceleration of the firewheel,

....

....

(iii) the time taken for the firewheel to make its first complete turn, starting from rest.

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(4)

(c) The total thrust exerted by the rockets remains constant as the firewheel accelerates. Explain why, after a short time, the firewheel is rotating at a constant angular speed which is maintained until the rocket fuel is exhausted.

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(2)
(Total 8 marks)

Q15. In electrical resistance welding, two steel components are pressed together and a pulse of current passed through the junction between them. Local heating in the junction softens the metal and the components fuse together. One heavy-duty welding rig uses a rotating flywheel as the energy source for the welding operation. The flywheel drives a generator which sets up a current in the junction until the flywheel comes to rest.

(a) The flywheel is driven from rest up to its working angular speed by a motor which produces an output power of 15 kW for 3.0 minutes. The moment of inertia of the flywheel is 9.5 kg m^2.
Assuming that frictional losses are negligible, show that the working angular speed of the flywheel is about 750 rad s^{-1}.

....

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(2)

(b) When the flywheel reaches an angular speed of 750 rad s^{-1}, it is disconnected from the motor and connected to the generator. The energy stored in the flywheel is dissipated as heat in the junction between the steel components and the flywheel comes to rest in 4.5 s. Assuming that friction can be neglected, calculate

(i) the angular impulse acting on the flywheel during the welding operation,

....

....

....

(ii) the average torque acting on the flywheel during the time it takes to come to rest.

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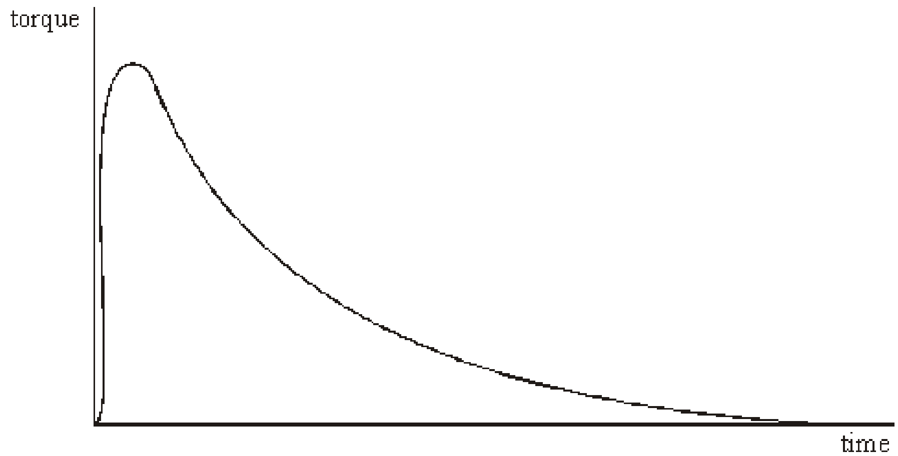
(2)

(c) The torque is not constant during the retardation but is a maximum just after the current is established in the junction. The graph below shows the way that the torque varies with time during any welding operation.



Explain how you could use the graph, if the axes were fully calibrated, to estimate the average torque acting on the system during a welding operation.

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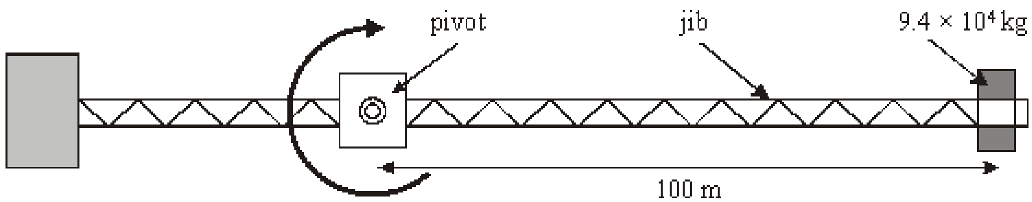
(3)
(Total 7 marks)

Q16. The diagram shows an overhead view of the jib of a tower crane carrying its maximum permitted load of 9.4×10^4 kg at a distance of 100 m from the pivot of the jib.
The manufacturer states that, under these conditions, at least 2.5 minutes must be allowed for one complete turn of the jib at constant speed.



(a) The moment of inertia of the unloaded jib about the pivot is 5.3×10^8 kg m^2.

(i) Calculate the moment of inertia of the jib when carrying its maximum load, as shown in the diagram.

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(ii) Calculate the maximum permitted angular speed of the jib, in rad s^{-1}.

....

....

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(iii) Show that the rotational kinetic energy of the jib when rotating at this angular speed is 1.3×10^6 J.

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(3)

(b) When the jib is at maximum loading, the motor used to rotate it accelerates the jib from rest to its maximum permitted angular speed in 25 s. Assuming that frictional forces can be neglected, calculate the output power of the motor.

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....

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(1)

(c) When frictional forces in the bearing are taken into account and the jib is rotating at its maximum permitted speed, the motor must produce an output power of 3.0 kW to do work against the frictional torque. Calculate

(i) the frictional torque in the bearings,

....

....

....

(ii) the total work done against friction in making one turn.

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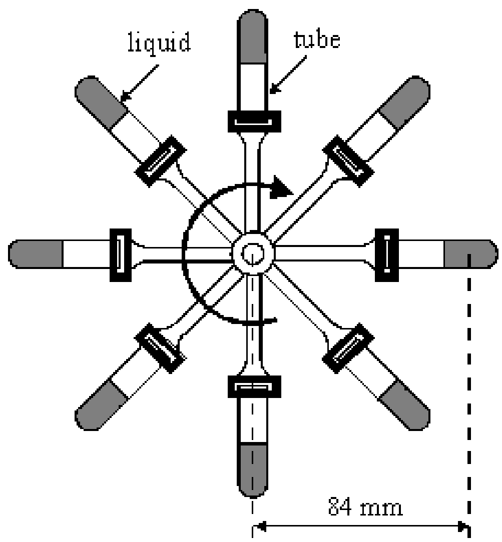
(2)
(Total 6 marks)

Q17. The diagram shows an overhead view of the load carrier of a spinning centrifuge, used to separate solid particles from the liquid in which they are suspended.



(a) When the centrifuge is operated with empty tubes, it reaches its working angular speed of 1100 rad s^{-1} in a time of 4.2 s, starting from rest. The moment of inertia of this system is 7.6×10^{-4} kg m^2. Calculate

(i) the angular acceleration of the system,

....

....

(ii) the torque required to produce this angular acceleration.

....

(2)

(b) In normal operation, each of the eight tubes contains 3.0×10^{-3} kg of liquid, whose centre of mass, when spinning, is 84 mm from the axis of rotation. The torque produced by the motor is the same as when the tubes are empty.

Show that this system takes approximately 5 s to reach its working speed of 1100 rad s^{-1}, starting from rest.

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(3)

(c) The normal operating cycle of the centrifuge takes a total time of 1 min. The centrifuge accelerates uniformly during the first 5.0 s to a speed of 1100 rad s^{-1}, after which the speed remains constant until the final 6.0 s of the cycle, during which it is brought to rest uniformly.
Calculate the angle turned by a tube during one complete operating cycle.

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(3)
(Total 8 marks)

Q18. An electric motor drives a machine which stamps out washers from a metal sheet. The motor is coupled to a flywheel of moment of inertia 38 kg m^2, which is accelerated until it is rotating at 480 rev min^{-1}. The motor drive to the flywheel is then disconnected and some of the kinetic energy of the flywheel is used to do work in the stamping operation.

(a) (i) Calculate the angular speed of the flywheel, in rad s^{-1}, when it is rotating at 480 rev min^{-1}.

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(ii) Show that the rotational kinetic energy of the flywheel when it is rotating at this speed is 48 kJ.

....

....

(2)

(b) During the stamping operation, the punch is in contact with the metal sheet for 150 ms. The kinetic energy of the flywheel is reduced by 12 kJ during this time. Calculate

(i) the angular speed of the flywheel immediately after the stamping,

....

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....

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(ii) the angular impulse acting on the flywheel during the stamping,

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....

(iii) the mean retarding torque acting on the flywheel during the stamping.

....

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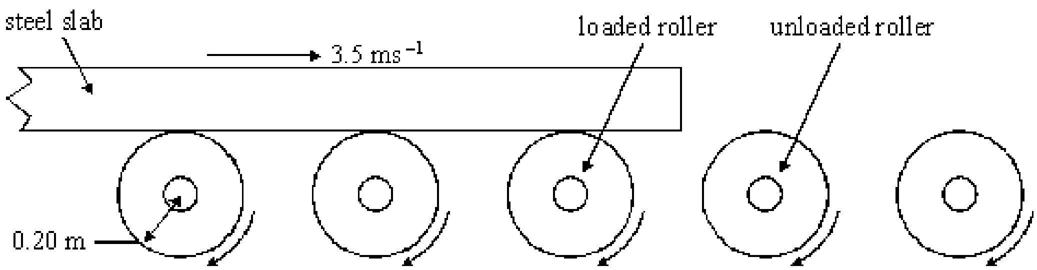
(3)
(Total 5 marks)

Q19. The diagram below shows a method used in a steel mill to transport steel slabs during the manufacture of steel beams. The slab rests on rollers of radius 0.20 m, each of which is driven by its own electric motor. In one operation, a slab moving at 3.5 m s^{-1} along the rollers must be brought to rest and its direction of motion reversed.



(a) Assuming that no sliding occurs between the surfaces of a loaded roller and the slab, calculate the angular speed of the roller when the slab is moving at a steady speed of 3.5 m s^{-1}.

....

....

(1)

(b) While the slab is moving at 3.5 m s^{-1}, the motor driving each roller exerts a uniform torque for 4.6 s on its roller such that the direction of motion of the slab is reversed. At the end of this time, the slab is moving at 3.5 m s^{-1} in the opposite direction.

Calculate

(i) the angular acceleration of a roller during this reversal,

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(ii) the uniform torque that its motor must exert to produce this angular acceleration in an unloaded roller. The moment of inertia of each unloaded roller system is 40 kg m^2,

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(iii) the angular impulse imparted when this torque acts on the system for 4.6 s,

....

....

(iv) the number of complete turns made by a loaded roller in bringing the slab momentarily to rest from a speed of 3.5 m s^{-1} before reversing its direction of motion.

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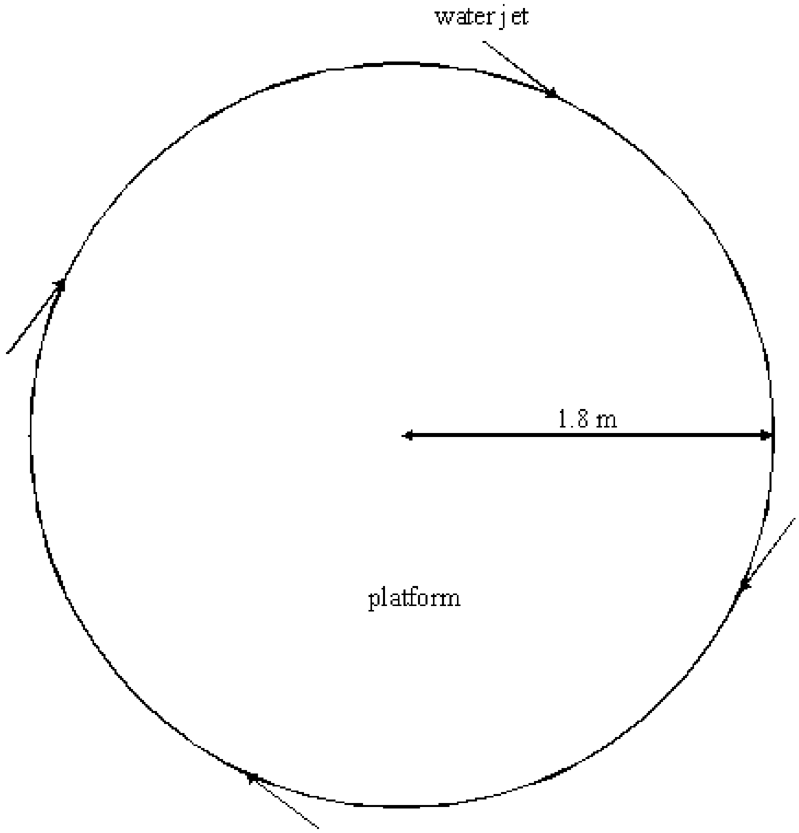
(6)
(Total 7 marks)

Q20. A rotating flower bed forms a novelty feature in the annual display of a horticultural society. The circular platform supporting the plants floats in a water tank and is caused to rotate by means of four water jets directed at the rim of the platform.



Each of the four jets exerts a tangential force of 0.60 N on the platform at a distance of 1.8 m from the axis of the rotation. The platform rotates at a steady angular speed, making one complete revolution in 110 s.

(a) Calculate

(i) the total torque exerted on the platform by the four jets,

....

....

(ii) the power dissipated by the frictional couple acting on the rotating platform, showing your reasoning.

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....

(4)

(b) When the water jets are switched off, all the kinetic energy of the loaded platform is dissipated as heat by the frictional couple and the platform comes to rest from its normal steady speed in 12 s.

(i) The kinetic energy of the loaded platform when rotating at its normal steady speed is 1.5 J.
Show that this value is consistent with your answer to part (a)(ii).

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(ii) Calculate the moment of inertia of the loaded platform.

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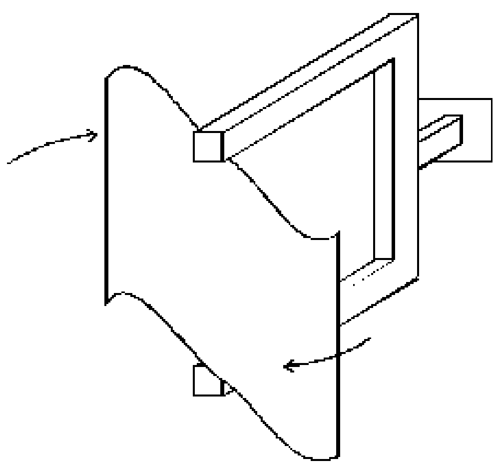
(3)
(Total 7 marks)

Q21. The diagram shows a street sign designed to rotate under windy conditions.



(a) On a still day, a gust of wind from a passing vehicle imparts an angular impulse of 1.2 kg m^2 rad s^{-1} to the sign, which accelerates from rest during a time of 2.8 s. The moment of inertia of the sign about its axis of rotation is 4.8×10^{-2} kg m^2. Assuming that the frictional couple acting on the sign is negligible, calculate

(i) the angular momentum acquired by the sign as a result of the angular impulse, showing your reasoning clearly,

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(ii) the angular speed of the sign immediately after the impulse has been imparted,

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(iii) the average torque acting on the sign during the time the impulse was imparted.

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(4)

(b) A second sign, of the same type and with the same moment of inertia, continues to rotate for 14 s after an impulse has been imparted, before friction brings it to rest from an angular speed of 30 rad s^{-1}. Calculate the number of complete turns made by the sign during this time.

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(2)
(Total 6 marks)

Q22. **Figure 1** shows a human centrifuge used in pilot training to simulate the large 'g' forces experienced by pilots during aerial manoeuvres. The trainee sits in the capsule at the end of the rotating centrifuge arm, which is driven by an electric motor.

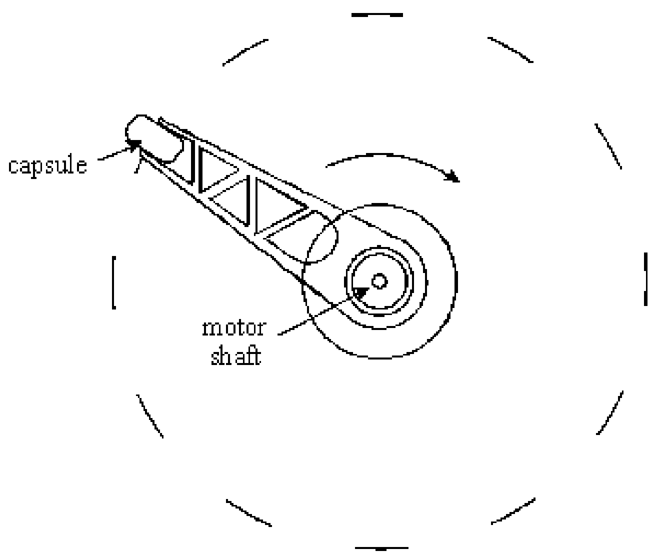

Figure 1

(a) When working at maximum power, the motor is capable of increasing the angular speed of the arm from its minimum working speed of 1.6 rad s^{-1} to its maximum speed of 7.4 rad s^{-1} in 4.4 s. The net power needed to achieve this acceleration is 150 kW.

(i) Assuming that this power remains constant during the acceleration, calculate the energy supplied to the centrifuge by the motor.

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(ii) Hence estimate the moment of inertia of the rotating system.

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(4)

(b) Bending stresses on the central shaft could have been reduced at the design stage by extending the arm beyond the shaft and fixing a counterweight, as shown in **Figure 2.** Its designers rejected this because savings in the manufacturing and maintenance costs of the system would be far less than the increased costs associated with a higher power motor.

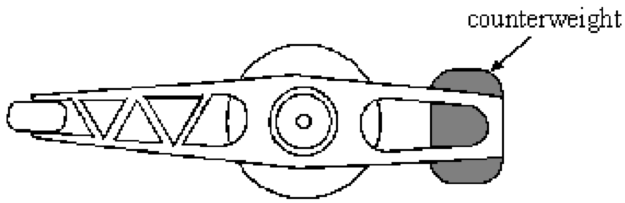

Figure 2

State and explain why, apart from increased friction associated with a heavier arm, the use of a counterweight would require greatly increased motor power. You may be awarded marks for the quality of written communication provided in your answer.

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(2)
(Total 6 marks)