Core Maths

AQA Level 3 Certificate in Mathematical Studies

Booklet 2B – Material up to Benchmark 2

Name _____

Critical Path Analysis (Paper 2)

Godalming College Ματησ

Starter

1. Jack wants to do a random sample of people attending a football match. How might he carry one out?

2. Dog House kennels weighed a certain dog breed, and put the results in a frequency table. Find the mean and standard deviation:

Weight (Kg)	0≤W<5	5≤W<10	10≤W<15	15≤W<20	20≤W<25
f	34	53	16	10	5

3. Barker's Lodge kennels also weighed their dogs and found theirs had a mean weight of 9.6kg and a standard deviation of 3.9. Compare the weights of dogs at the two kennels

4. Kim borrows £750, which she will repay in two equal, monthly, instalments. If the APR is 45%, how much will her repayments be?

✎ **Critical Path Analysis** can be used for any multi-task complex project to ensure that the entire project is completed in the minimum time.

Example 1 – Precedence Tables:

The first step in scheduling a complex project is to break it down into activities

For example to build an extension to a home:

A   Prepare the foundations
B   Have foundations passed by inspector
C   Obtain bricks
D   Erect walls
E   Construct roof
F   Install plumbing
G   Install wiring
H   Plaster walls
I   Decorate
J   Landscape garden

We identify which activities depend on which others being completed first and summarise it in a precedence table:

Activity	Depends On
A	
B	
C	
D	
E	
F	
G	
H	
I	
J	

Critical Path Analysis (Paper 2)

Your Turn 1 – Precedence Tables:

A project has been broken down into activities A, B, C, D, E, F, G and H. After a committee meeting the information below was produced. Draw a precedence table that summarises this information.

A and D do not depend on any other activity

- A must be completed before B and C can start

- A, B, C and D must be completed for E to start

- A, B, C, D and E must be completed for F to start

- G is the last activity and can start when A, B, C, D, E and F are completed

- A and B must be completed for H to start

Activity	Depends On
A	
B	
C	
D	
E	
F	
G	
H	

✎ **Constructing an Activity Network**

✎ Procedure:
1. Draw a START vertex and add a "column" of vertices representing those activities which can be started straight away. Join each vertex back to the START vertex.
2. Add a "column" of vertices which can now be started given the activities which are already on the network. Join each new vertex back to each of the activities that it directly depends on.
3. Repeat step 2 until all activities have been placed on the network.
4. Add a FINISH vertex and join it back to any vertices which have no arc out from them.
5. Label each arc out from a vertex with the activity's duration.

Example 2:

Construct an Activity Network for the project in "Your Turn 1"

Critical Path Analysis (Paper 2)

Your Turn 2A:

Draw an activity network based on the information below:

Task	Duration (hours)	Immediate predecessors
A	3	-
B	4	-
C	6	-
D	5	A
E	1	B
F	6	B
G	7	C, D, E

Your Turn 2B:

Draw an activity network based on the information below:

Task	Duration (days)	Immediate predecessors
A	2	-
B	3	-
C	5	-
D	6	A, B
E	8	C
F	2	C
G	4	D, E

# Critical Path Analysis (Paper 2)

Godalming College
Ματησ

Starter

1. Oli wants to do some research on the different sizes of leaves in a local woodland. What would be the best sampling method for him to use and how might he carry it out?

2. Complete the table below, what formula would Sina use to find the result in cell D3?

3. What is the AER on a bank account which pays interest of 0.03% each week?

◢	A	B	C	D
1	Club	Played	Won	Win %
2	Man City	38	32	
3	Liverpool	38	30	
4	Chelsea	38	21	
5	Tottenham	38		60.50%

4. What is the APR on a loan of £500, which is repaid in a single payment of £600 after one month?

5. How much money do you estimate is spent in the Godalming College canteen each week? State your assumptions.

✏ **Earliest and Latest Start Times**

✏ Each activity has an Earliest Start Time and a Latest Start Time, in order that the project can be completed in the minimum time.

✏ We write the earliest and latest start times along with the duration for an activity inside the vertex box.
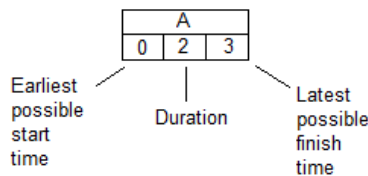
✏ **Finding Earliest Start Times:**

✏ Work through the activity network from the START vertex to the FINISH vertex. (This is called a "forward pass")

✏ Formula:

Earliest Start Time	=	Earliest Start of preceding activity	+	Preceding activity's duration

✏ Where an activity has more than one preceding activity, use the **highest** value obtained.

✏ **Finding Latest Finish Times:**

✏ Work through the activity network from the FINISH vertex to the START vertex. (This is called a "backward pass")

✏ Formula:

Latest Finish Time	=	Latest finish of following activity	-	Following activity's duration

✏ Where an activity has more than one preceding activity, use the **lowest** value obtained.

Critical Path Analysis (Paper 2)

Godalming College

Ματησ

Example 3A:

For previous examples (Your Turn 2A and 2B) find the critical activities and the minimum duration of the project.
The activities networks are shown below:
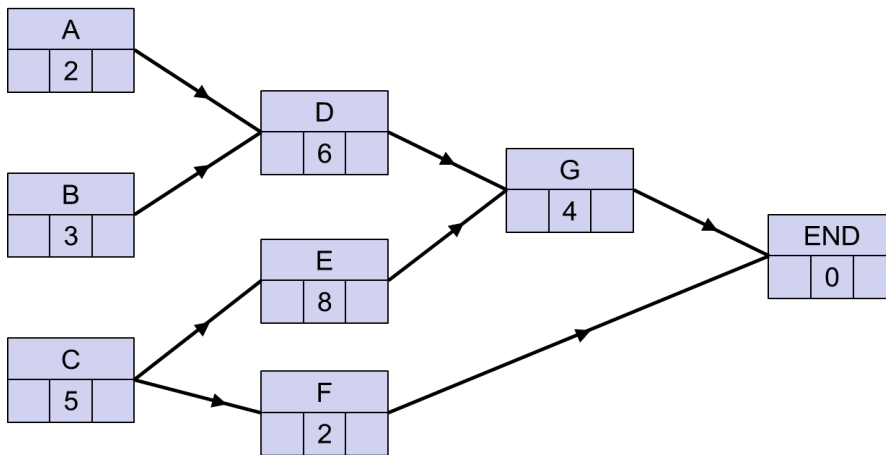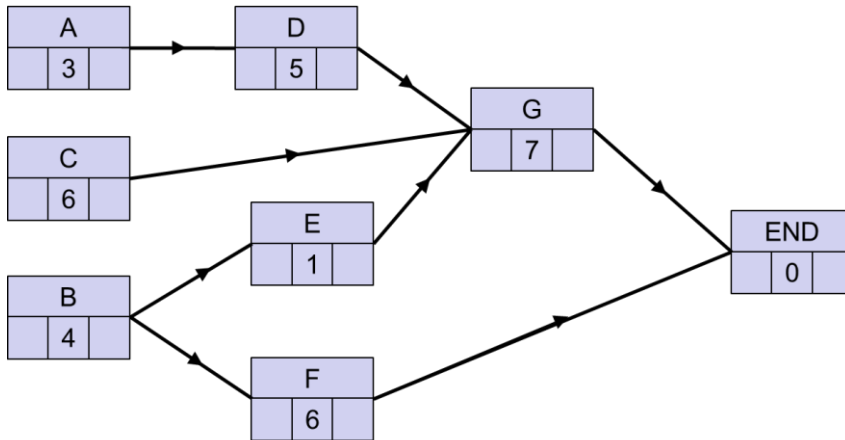
✎ **Float Time and Critical Activities**

✎ Float Time = Latest Finish Time – Duration – Earliest Start Time

✎ The Float time tells us about the 'flexibility' of an activity.

✎ i) If float = 0,   Critical, No flexibility, Has to Start/Finish on time.

✎ ii) If float > 0,   Flexible, Don't need to start on time.

Example 3B:

Calculate the float times for the activities in the projects in Example 3A, and hence write down the critical path(s) for each
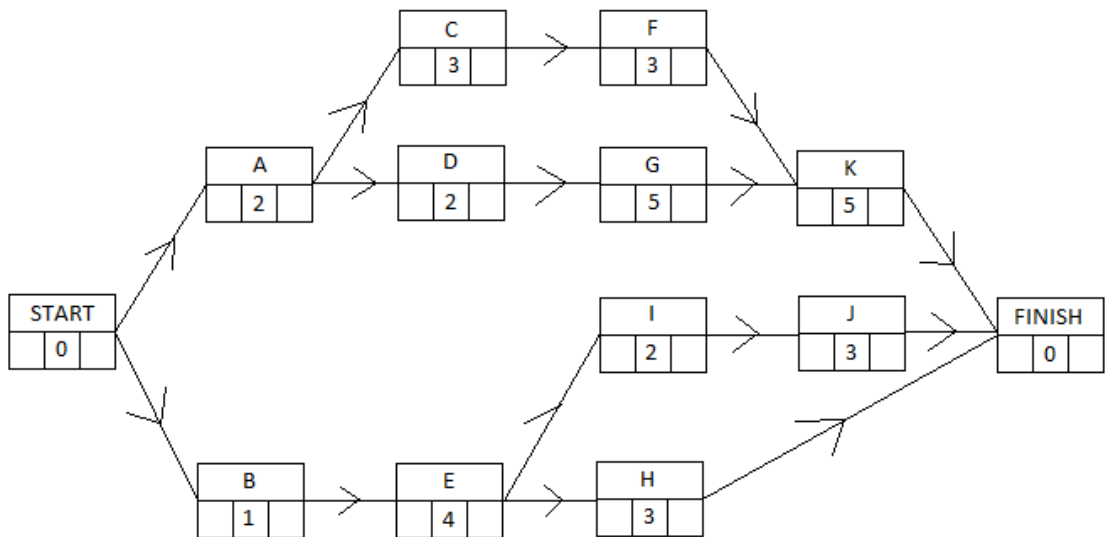
Critical Path Analysis (Paper 2)

Test Yourself 3A:

Given the activity network below find the earliest start times and latest finish times for each activity. Hence find the critical activities
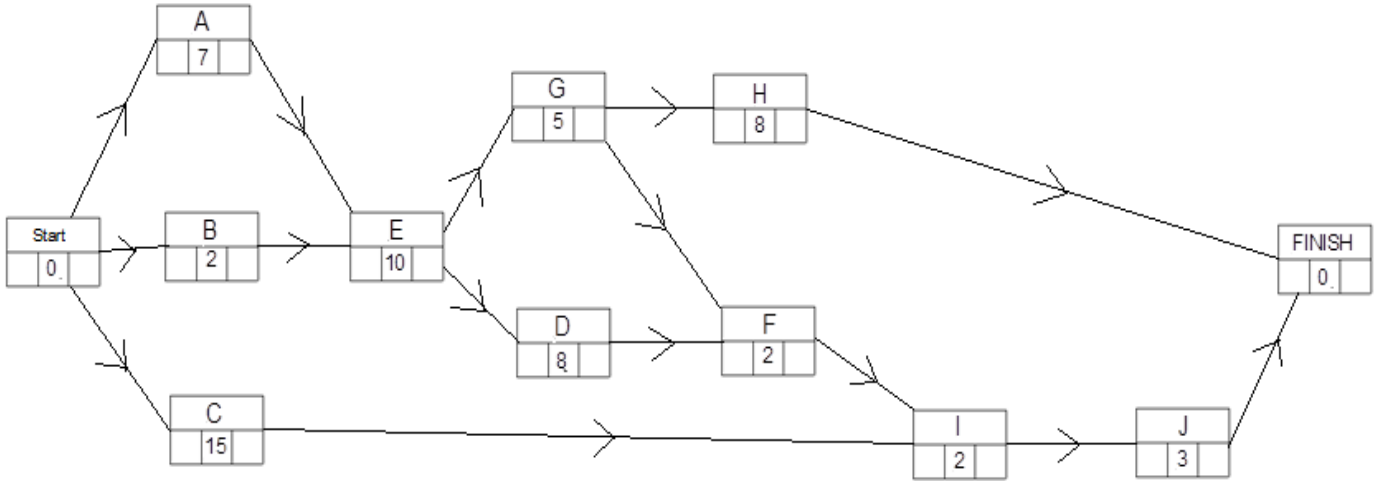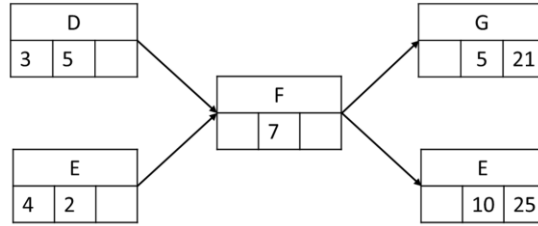

Test Yourself 3B:

Given the activity network below find the earliest start times and latest finish times for each activity. Hence find the critical activities

Critical Path Analysis (Paper 2)

Starter

1. Fill in the blank squares:
   List all the float times and
   identify the critical activities.

2. Calculate the mean and standard deviation of the 25 employees from company A:

   Company B's employees have a mean commuting time of 19m28s and a standard deviation of 20.6.
   Which commute would you rather have?

Daily commuting time (minutes)	Number of employees
0 to less than 10	4
10 to less than 20	9
20 to less than 30	6
30 to less than 40	4
40 to less than 50	2

3. Which bank account has the best AER? One with a nominal rate of 1.6%, compounded weekly, or one which pays out monthly interest of 0.18%?

✎ **Gantt or Cascade Charts**

✎ A Gantt or Cascade Chart is an alternative way of displaying the information gained from completion of the activity network.

1) For each activity, start the bar at its earliest start time.   Bar length = activity duration

2) Float is shown as a shaded extension to the bar (length of extension = float)

Critical Path Analysis (Paper 2)

Example 4A:

Construct a Gantt (cascade) diagram for the project.

A		
0	3	3

D		
3	5	8

G		
8	7	15

C		
0	6	8

E		
4	1	8

END		
15	0	15

B		
0	4	7

F		
4	6	15
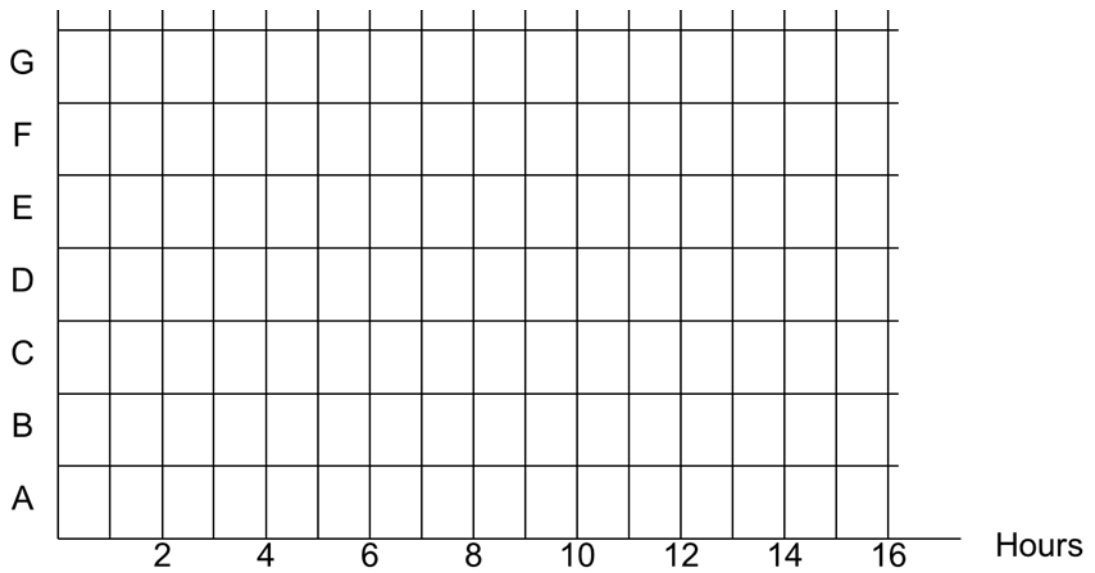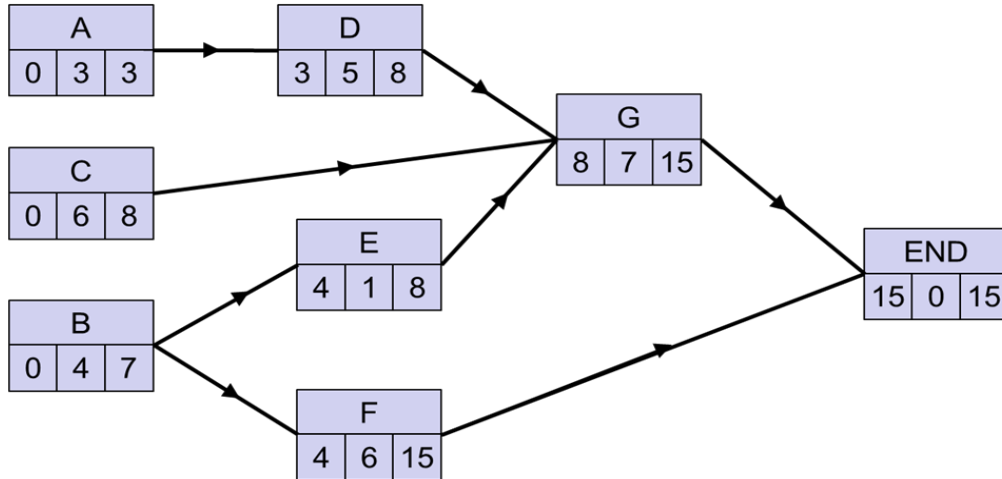
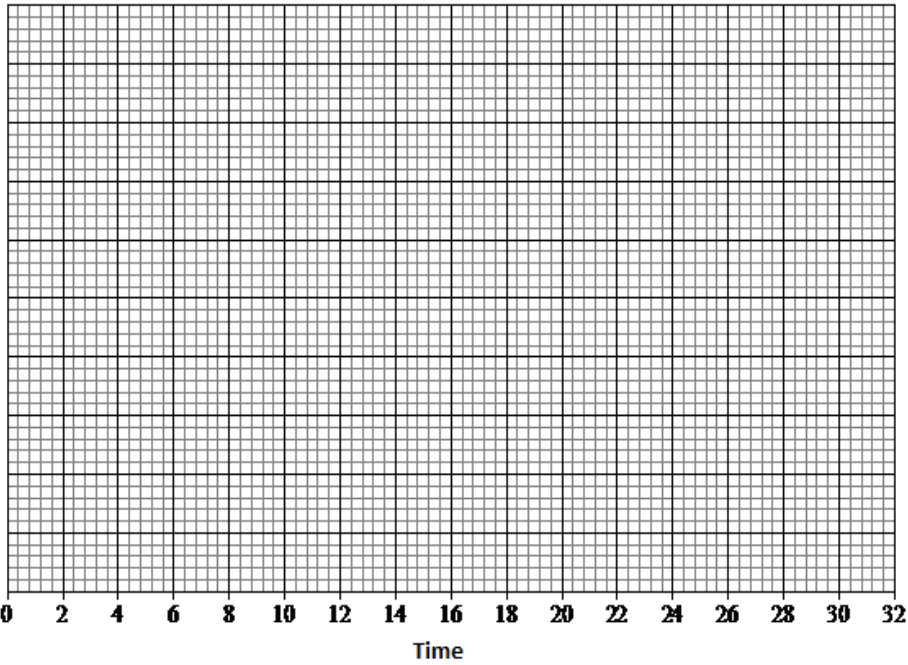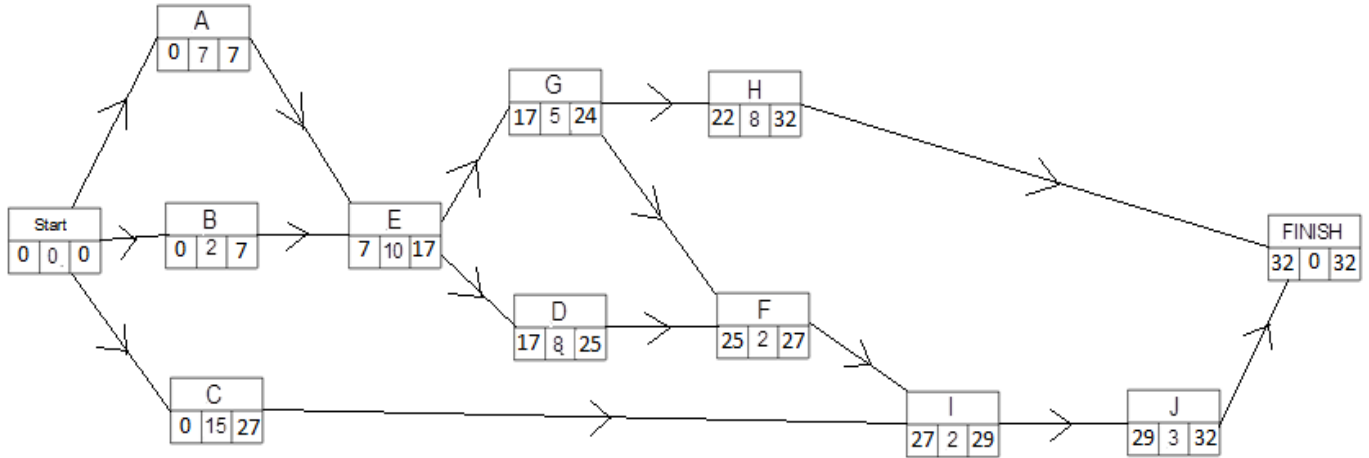# Critical Path Analysis (Paper 2)

Your Turn 4:

Construct a Gantt (cascade) diagram for the project.

Critical Path Analysis (Paper 2)

Your Turn 5:

a) Construct an activity network for the project represented by the table below
b) Find the earliest start times for each activity
c) Find the latest start times for each activity
d) List the critical activities
e) On the grid on the next page construct a Gantt (cascade) diagram for the project

Activity	Immediate predecessor	Planned duration (weeks)
A: Decide on new system	–	1
B: Prepare ICT control room	A	2
C: Buy hardware (including delivery)	A	5
D: Buy software (including delivery)	A	2
E: Train ICT staff	B, C, D	2
F: Install cabling	C	2
G: Install hardware	E, F	1
H: Install software	G	1
I: Prepare pupil/staff data	A	5
J: Install data	H, I	1
K: Train teaching staff	H	2
L: Test system	J, K	1

Critical Path Analysis (Paper 2)

Your Turn 5:
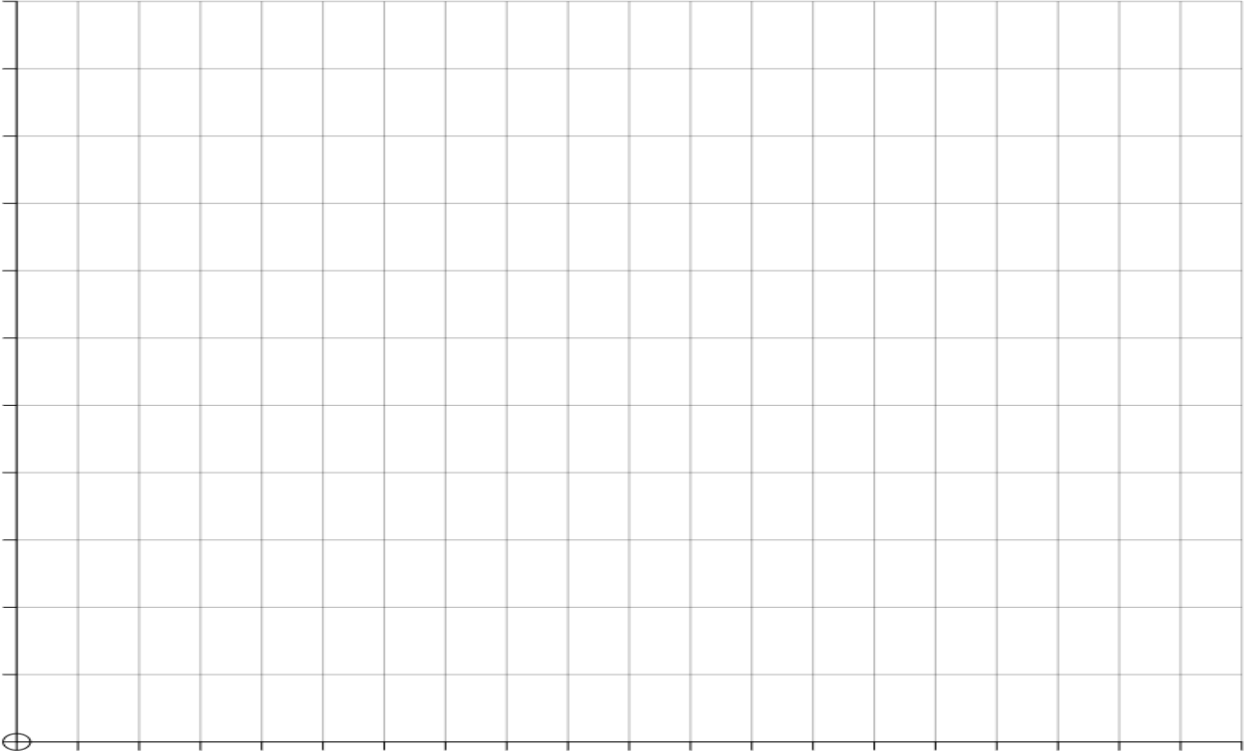
Taxation

Starter

1. Put the data below into a back to back stem and leaf diagram:

    Number of rainy days in a year, Texas cities:
    67, 69, 84, 73, 77, 79, 63, 49, 96, 06, 63, 52, 105, 59, 82, 90, 79, 71
    Number of rainy days in a year, California cities:
    37, 29, 90, 117, 44, 32, 35, 35, 93, 75, 58, 42, 62, 67, 31, 45

2. Comment on the data from question 1.

3. A bank advertises a loan of £10,000 with two yearly repayments of £5,800 and £6,400. It claims the APR is 14.1%. Check whether this claim is accurate.

4. There are approximately 1,980 students at Godalming college, around 400 of which are taking a maths course. 75 students are sampled in a questionnaire. If they are stratified by subject, how many maths students will be sampled?

✏ The UK government collects money through taxation. The three main sources of revenue for the government are **income tax**, **national insurance**, and **value added tax** (**VAT**).

✏ When we buy certain goods and services the UK government adds a tax to the cost. This tax is called **value added tax** (**VAT**).

✏ The price before VAT is called the **net** price, when VAT is added it is called the **gross** price.

✏ The standard rate of VAT is 20%, but for some fuel and health related goods it is 5% and it is 0% for most foot and children's clothes.

Example 6:

a) Samantha is buying a buttonhole to wear at a friend's wedding. The price is £4.00 plus VAT at the standard rate. Calculate the price of the buttonhole

b) Sadiq books a driving test costing £62. The price includes VAT at 20%. Calculate the amount of VAT included in the test price.

Taxation

Your Turn 6:

a) Victoria is buying a pair of jeans. They cost £75 before VAT is added, what is the price after VAT?

b) A game costs £45 after VAT. What is the original cost?

🖉 The UK government collects money through taxation. The three main sources of revenue for the government are **income tax**, **national insurance**, and **value added tax** (**VAT**).

🖉 Income tax is a charge on personal incomes each year the personal tax allowances, income tax rates and tax bands are set by the government.

🖉 Most people have a personal allowance. This is the amount of tax free income you are allowed each year. The personal allowance varies each year. For 2020 to 2021 it was £12,500.

🖉 The amount of income tax you pay depends on how much taxable income you have above your personal allowance. There are different rates of income tax depending on what your income is.

🖉 This table shows the income tax rates on taxable bands for 2020 to 2021:

Rates	Taxable income (after allowance)
Basic: 20%	Up to £37 500
Higher: 40%	Over £37 500
Additional: 45%	Over £150 000

🖉 To calculate your income tax:
  ➢ Find your taxable income by subtracting your personal allowance from your annual gross income.
  ➢ You pay income tax at 20% on the first £37 500 of your taxable income.
  ➢ You pay income tax at 40% on your taxable income over £37 500
  ➢ You pay income tax at 45% on your taxable income over £150 000

Taxation

Godalming College

Ματησ

> ✎ The personal allowance for 2020 to 2021 was £12,500.
>
> ✎ This table shows the income tax rates on taxable bands for 2020 to 2021:
>
> | Rates | Taxable income (after allowance) |
> |---|---|
> | Basic: 20% | Up to £37 500 |
> | Higher: 40% | Over £37 500 |
> | Additional: 45% | Over £150 000 |

Example 7:

Calculate the income tax paid on a gross annual salary of:
a) £10,000
b) £25,000
c) £45,000
d) £200,000

Taxation

✎ The personal allowance for 2020 to 2021 was £12,500.

✎ This table shows the income tax rates on taxable bands for 2020 to 2021:

Rates	Taxable income (after allowance)
Basic: 20%	Up to £37 500
Higher: 40%	Over £37 500
Additional: 45%	Over £150 000

Your turn 7:

Calculate the income tax paid on a gross annual salary of:
a) £12,000
b) £30,000
c) £70,000
d) £185,000

Taxation

✎ The UK government collects money through taxation. The three main sources of revenue for the government are **income tax**, **national insurance**, and **value added tax** (**VAT**).

✎ National Insurance (NI) is another charge on a persons salary. It goes towards thing like the NHS, state pension, statutory sick pay and jobseeker's allowance.

✎ This table shows the NI rates for 2020 to 2021:

Percentage NI due	Minimum monthly income	Maximum monthly income	Minimum yearly income	Maximum yearly income
0%		Up to £792		Up to £9 500
12%	£792.01	£4 167	£9 500.01	£50 000
2%	Above £4 167		Above £50 000	

✎ Examples
  ➢ A person who had a monthly income of £1800 paid 12% on the amount above £792
  ➢ A person who had a yearly income of £62 000 paid 12% on the amount between £9500 and £50 000 plus 2% of the amount above £50 000

Example 8:

Calculate the net pay, after income tax and national insurance has been deducted, for the following gross annual salaries:
a) £10,000
b) £25,000
c) £45,000
d) £200,000

Taxation

> ✎ This table shows the NI rates for 2020 to 2021:

Percentage NI due	Minimum monthly income	Maximum monthly income	Minimum yearly income	Maximum yearly income
0%		Up to £792		Up to £9 500
12%	£792.01	£4 167	£9 500.01	£50 000
2%	Above £4 167		Above £50 000	

Your turn 8:

Calculate the net pay, after income tax and national insurance has been deducted, for the following gross annual salaries:
a) £12,000
b) £30,000
c) £70,000
d) £185,000

Taxation

✏ Another possible deduction is student loan. If you have a student loan you make repayments when your gross income is greater than the threshold.

✏ You pay 9% of the amount of your income which is above the threshold.

✏ The threshold depends upon when you started university:

Started University	Threshold
Before 1/9/2012	£19 390
On or after 1/9/2012	£26 575

Example 8:

Karl started university in 2011, and took out a student loan.
In the year 2020 − 21, Karl earned a gross salary of £3 756 per month. Calculate his monthly net pay.

Taxation

Your Turn 9:

Juno started university in 2017 and took out a student loan. She sees the following job advertised, what will her take-home pay be in her first year?

ALDI	Graduate Area Manager Programme	44k+ £44,000 starting salary Your generous starting salary will rise in increments to £77,870 in year 5.

Exchange Rates

Starter

1. What is the monthly income of someone who earns £68,500 p.a. after tax and national insurance have been deducted?

2. Which savings account would be better to invest your money in? One which pays a nominal interest rate of 3.2%, compounded weekly, or one which pays out a monthly interest rate of 0.35%?

3. Frances spends £258 on items for work. She can claim back the VAT of 20%. How much will she receive back?

4. What is the mean average of visitors to the five most visited US National Parks? Write down the formula that would give the answer on the spreadsheet.

	A	B
1	National Park	Visits per year
2	Great Smoky Mountains	11,421,200
3	Grand Canyon	6,380,495
4	Rocky Mountain	4,590,493
5	Zion	4,320,033
6	Yellowstone	4,115,000
7	Average	

✎ A country's currency has a value that fluctuates relative to other currencies.

✎ Banks buy and sell currencies from each other and these trades set a market price for each currency.

✎ The table shows some currency exchange rates (from John Lewis in Oct 2020) compared to the British pound:

Currency	Sell Rate	Buy Rate
Euro (EUR)	1.0897	1.221
Australian Dollar (AUD)	1.7944	2.0395
Brazilian Real (BRL)	6.9645	8.2947
Canadian Dollar (CAD)	1.6712	1.9053
Chinese Yuan Renminbi (CNY)	8.2191	9.9009
Croatian Kuna (HRK)	8.0149	9.4079
Indonesian Rupiah (IDR)	17651.3749	21815.8735
Kenyan Shilling (KES)	133.1837	160.4538
South African Rand (ZAR)	20.6486	23.8934
US Dollar (USD)	1.2922	1.4457

Exchange Rates

Example 10:

Shahin is going on holiday to Europe. He takes out £500 worth of Euros and returns from holiday with €45.
a) How many Euros does he receive?
b) If he changes his Euros back into pounds, how much will he get?

Ricardo has just moved to the UK from Brazil. He exchanges 5,000 Real into pounds. How many pounds does he receive?

Your turn 10:

Jimmy planned a holiday to Croatia. He took out £600 of Croatian Kuna. A family emergency means he can't go, so he gets the money exchanged back into pounds. How much does he lose?

The iPhone X costs $999 in the US, and £999 in the UK. If Charlie uses cash to buy one in America, how much would she save compared to buying in the UK?

Inflation

Godalming College
Ματησ

> ✎ **Consumer Price Index (CPI).** Each month the UK government Office for National Statistics (ONS) publishes the latest Consumer Price Index (CPI).
>
> ✎ The CPI is calculated using a hypothetical shopping basket of around 700 goods and services on which people typically spend their money. As the prices of individual products vary, so does the total cost of the basket.
>
> ✎ The CPI measures price changes, not price levels. It is therefore expressed in terms of the comparison of prices relative to a base year, when the index was given a value of 100. The base month in the table below is July 2015.

Year	Jan	Feb	Mar	Apr	May	Jun	Jul	Aug	Sep	Oct	Nov	Dec
2020	108.2	108.6	108.6	108.5	108.5	108.6	109.1	108.6				
2019	106.4	106.8	107.0	107.6	107.9	107.9	107.9	108.4	108.5	108.3	108.5	108.5
2018	104.4	104.9	105.0	105.4	105.8	105.8	105.8	106.5	106.6	106.7	107.0	107.1
2017	101.4	102.1	102.5	102.9	103.3	103.3	103.2	103.8	104.1	104.2	104.6	104.9
2016	99.5	99.8	100.2	100.2	100.4	100.6	100.6	100.9	101.1	101.2	101.4	101.9
2015	99.3	99.5	99.7	99.9	100.1	100.2	100.0	100.3	100.2	100.3	100.3	100.3
2014	99.0	99.5	99.7	100.1	100.0	100.2	99.9	100.2	100.3	100.4	100.1	100.1
2013	97.1	97.8	98.1	98.3	98.5	98.3	98.3	98.7	99.1	99.1	99.2	99.6
2012	94.6	95.1	95.4	96.0	95.9	95.5	95.6	96.1	96.5	97.0	97.2	97.6
2011	91.3	92.0	92.2	93.2	93.4	93.3	93.3	93.8	94.4	94.5	94.6	95.1
2010	87.8	88.2	88.7	89.2	89.4	89.5	89.3	89.8	89.8	90.0	90.3	91.2

Example 11:

What was the rate of inflation in February of this year?

If a loaf of bread cost 65p in March 2010, how much might we expect it to cost today?

Your turn 11:

What was the rate of inflation in August of this year?

If four pints of milk costs £1.09 today, how much might we expect it to cost nine years ago?

Inflation

✎ **Retail Price Index (RPI).** The RPI is very similar to the CPI but it is based on a different basket of goods and services, which is weighted according to its own unique system. It is considered less reliable than CPI – but many organisations still use it if the rate is favourable to them!

✎ The series started in 1947, and prices are expressed relative to a base month of January 1987, when the index has a value of 100

Year	Jan	Feb	Mar	Apr	May	Jun	Jul	Aug	Sep	Oct	Nov	Dec
2020	290.6	292.0	292.6	292.6	292.2	292.7	294.2	293.3	294.3			
2019	283.0	285.0	285.1	288.2	289.2	289.6	289.5	291.7	291.0	290.4	291.0	291.9
2018	276.0	278.1	278.3	279.7	280.7	281.5	281.7	284.2	284.1	284.5	284.6	285.6
2017	265.5	268.4	269.3	270.6	271.7	272.3	272.9	274.7	275.1	275.3	275.8	278.1
2016	258.8	260.0	261.1	261.4	262.1	263.1	263.4	264.4	264.9	264.8	265.5	267.1
2015	255.4	256.7	257.1	258.0	258.5	258.9	258.6	259.8	259.6	259.5	259.8	260.6
2014	252.6	254.2	254.8	255.7	255.9	256.3	256.0	257.0	257.6	257.7	257.1	257.5
2013	245.8	247.6	248.7	249.5	250.0	249.7	249.7	251.0	251.9	251.9	252.1	253.4
2012	238.0	239.9	240.8	242.5	242.4	241.8	242.1	243.0	244.2	245.6	245.6	246.8
2011	229.0	231.3	232.5	234.4	235.2	235.2	234.7	236.1	237.9	238.0	238.5	239.4
2010	217.9	219.2	220.7	222.8	223.6	224.1	223.6	224.5	225.3	225.8	226.8	228.4

Example 12:

Holly has agreed to rent a flat for £1250 per month, her contract started in March 2019. In the contract it states that her rent will increase each year by the RPI + 1%, what did the cost of rent change to in March 2020?

Your turn 12:

National Savings and Investments used to run an account which offered interest of RPI +0.5%. If Zoran had £1,500 in his account between Feb 2012 and Feb 2013, how much interest will he accrue?