

Core Maths

AQA Level 3 Certificate in Mathematical Studies

Booklet I – Material up to Benchmark I



Name _____

Estimation and Modelling

Starter

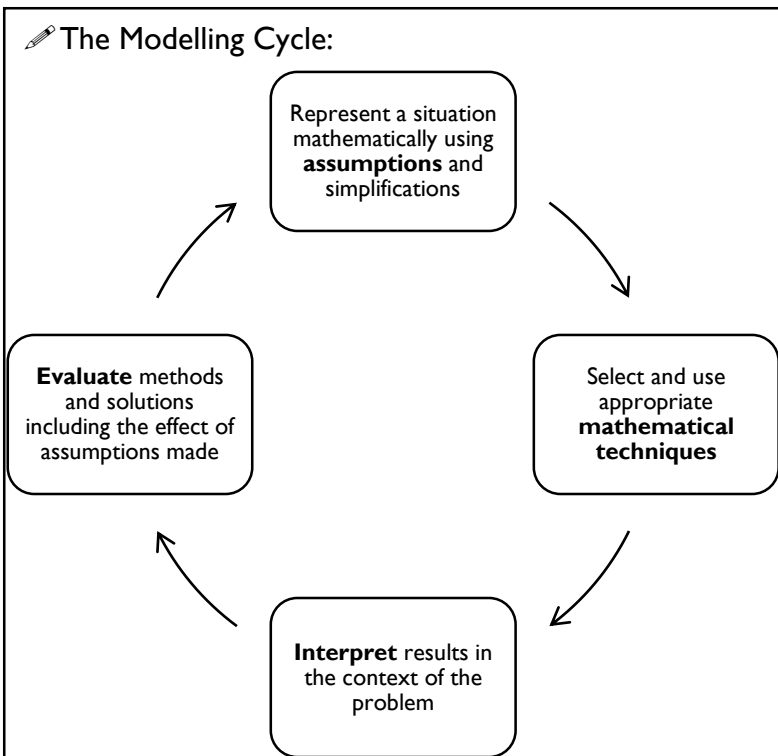
Using Estimates, put these in order from smallest to largest:

- a) The number of passengers a Boeing 747-400 (Jumbo Jet) can accommodate
- b) The age of the world's oldest tortoise, Harriet, when she died in 2006
- c) The length of the River Severn, in miles
- d) The distance, in a straight line, from London to Edinburgh in miles

Example 1:

How many steps would it take someone to walk from Land's End to John O'Groats:

- a) State your assumptions:



b) Working out:

c) Interpretation:

d) Evaluate:

Estimation and Modelling

Example 2:

How many items of luggage pass through Heathrow airport each year?

Test yourself 2:


- a) A ream (500 sheets) of paper is 5.4cm thick. Estimate the number of sheets needed to reach the height of an average man.
- b) There are three rats for every human in London. How many rats are there in London?
- c) How many golf balls would fit in a suitcase?
- d) How many steps would it take for a person to travel around the world once?


Percentages

Starter

Estimate:

- The number of secondary schools in the UK
- How many dentists are there in Bristol?
- How many pizzas are eaten in Godalming each year.

 **Percentages:** These are used throughout the course, so it's important we spend some time recapping what you will have already picked up during GCSE.

 Percentage literally means "out of one hundred"

$$50\% = \frac{50}{100} = 0.5$$

Example 3: Collins p49-50

a) Yolanda pays off 67% of an electricity bill of £84. How much does she pay?

b) Will pays off £93 from a debt of £136. What percentage has he paid?

c) Jack and Beth have sales targets in their shop.

One month, Jack made sales of £4.4k against his £5k target.

Beth made sales of £5.45k against her £6k target.

Who made a greater proportion of sales when compared to their target?

Percentages

Test Yourself 3:

- a) Bryan achieved 87.5% of the marks on a 92 mark test. How many marks did he achieve?
- b) Kiya has paid off £248,345 on her £455,000 mortgage, what percentage has she paid off?
- c) Two items are on sale in a shop. A £34 shirt is on sale for £27 and a £95 pair of jeans is on sale for £79. Which item has been reduced the most?

Example 4: Collins p52

a) Increase £84 by 67%

b) Decrease £84 by 67%

Test Yourself 4:

a) Increase £45 by 83%

b) Decrease £45 by 83%

Percentages

Example 5: Collins p52

a) A price is increased by 15% to £79
What was the original price?

b) A mass is decreased by 8%. The
new mass is 5.3kg.
What was the original mass?

Test Yourself 5:

a) A price is increased by 20% to
£45.50
What was the original price?

b) A shirt is on sale, reduced by 25%.
The new price is £45.
What was the original price?

Test Yourself 4:

No.	Question	Forward or Reverse	Working and Answer	Tick or Cross
1	Increase £10 by 15%			
2	Bill has a wage increase of 10%. He now earns £359.70 a week. What did he earn before?			
3	Kelly buys a jumper in a sale. After a 15% reduction the price is £48.45. What was it before the sale?			
4	The population of an island increased 16% to 2494. What was the population before increase?			

Percentages





5	The mass of a rock is reduced by 22%. The new mass is 193.44g. What was the original mass of the rock?			
6	Decrease 127 by 11%			
7	VAT is charged at 20%. Gina buys a new car with VAT added and pays £21960. She can claim her VAT back. How much can she claim back?			
8	John has a wage increase of 20%. He originally earned £320 a week. How much does he earn now			
9	In a restaurant service is charged at 8.5%. Gemma buys a salad for £6.51. How much would have she paid if the service charge had not been added?			
10	Samuel bought a fridge in a '30% off sale'. Given that he paid £384.30, find its original price before the sale.			
11	A toaster has 18% off in a sale. Given that the toaster was £75 before the sale, find its new price.			
12	John was told to increase his daily exercise by $\frac{1}{4}$. Given that he now does 56 minutes a day, show that he originally did less than $\frac{3}{4}$ of an hour a day.			
13	Sue was told to decrease her daily calorie intake by 15%. Given that she is now eating less than 1600 calories a day, find the maximum number she was originally eating. Give your answer to the nearest calorie.			

Data: Types of data

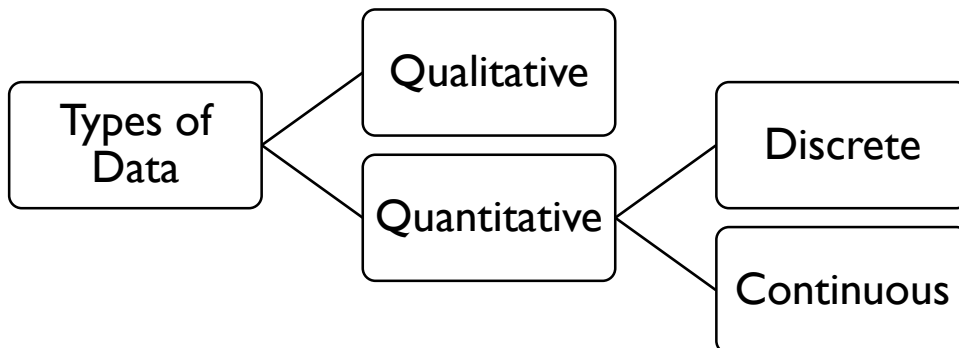
Starter

1. Subhash paid £350 for a computer tablet. He later finds he could have bought the same one for £295. What percentage has the price gone down by?
2. Last year, Simon had 150 followers on Twitter. His number of followers has increased by 120%. How many followers does he have now?
3. A computer game costs £54.60 including VAT at 20%. What was the price before VAT?
4. A car has depreciated in value by 27%. It was £15,650 when new. What is the value now?
5. Estimate how long it would take to type out a novel.

You need to be able to define the following terms:

-  **Qualitative data:** non-numerical data (describes a quality)
-  **Quantitative data:** numerical data
-  **Discrete data:** numerical data which can take only exact values
-  **Continuous data:** numerical data which can take any value

Example 6: Hodder p 320







Place the following into the correct category:

Name, address, date of birth, place of birth, passport number, national insurance number, gender, medical history, blood group, height, weight, waist measurement

Data: Types of data

You need to be able to define the following terms:

-  **Qualitative data:** non-numerical data (describes a quality)
-  **Quantitative data:** numerical data
-  **Discrete data:** numerical data which can take only exact values
-  **Continuous data:** numerical data which can take any value



Test Yourself 6 Hodder p 321

a) Place the following into the correct category:

Age, number of siblings, road name, distance from home to college, bank account number, GCSE maths grade, Core maths grade, salary.

b) Write down three examples of Qualitative, Quantitative discrete, Quantitative continuous data.

You need to be able to define the following terms:

-  **Primary data:** Information that is collected first hand by researchers
-  **Secondary data:** numerical data


Example 7:

Ways of collecting primary data:

Ways of collecting secondary data:

Data: Types of data

You need to be able to define the following terms:

 **Primary data:** Information that is collected first hand by researchers

 **Secondary data:** numerical data

Test Yourself 7:

Which should I use for (justify your answers):

a) *Celebrities' favourite breakfasts*

b) *Average family size*


c) *Favourite clothes shop for 16-19 year olds*

d) *Most common pet*

e) *World's most popular car*


Data: Sampling

You need to be able to define and describe the method for each type of sampling:

 **Random sampling:**

 **Systematic sampling:**

 **Cluster sampling:**

 **Quota sampling:**

 **Stratified Sampling:**

Example 8:

Type	Can use if...	Advantages	Disadvantages
Random			
Cluster			
Stratified			
Quota			

Data: Sampling

Use simple random sampling to pick 10 countries

Geography	Population mid-2018 (millions)	Births per 1,000 Population	Deaths per 1,000 Population
AFRICA	1,284	35	9
NORTHERN AFRICA	235	26	6
Algeria	42.7	26	4
Egypt	97	27	6
Libya	6.5	20	5
Morocco	35.2	18	5
Sudan	41.7	34	8
Tunisia	11.6	19	6
Western Sahara	0.6	20	5
WESTERN AFRICA	382	38	11
Benin	11.5	37	9
Burkina Faso	20.3	40	9
Cape Verde	0.6	19	5
Cote d'Ivoire	24.9	35	12
Gambia	2.2	39	8
Ghana	29.5	30	8
Guinea	11.9	39	11
Guinea-Bissau	1.9	36	11
Liberia	4.9	33	8
Mali	19.4	45	10
Mauritania	4.5	34	8
Niger	22.2	48	10
Nigeria	195.9	39	12
Senegal	16.3	33	6
Sierra Leone	7.7	33	13
Togo	8	34	9
EASTERN AFRICA	432	36	8
Burundi	11.8	41	11
Comoros	0.8	33	7
Djibouti	1	23	8
Eritrea	6	30	7
Ethiopia	107.5	33	7
Kenya	51	31	6
Madagascar	26.3	32	6
Malawi	19.1	34	7
Mauritius	1.3	11	8
Mozambique	30.5	39	10
Rwanda	12.6	32	6
Seychelles	0.1	17	8
Somalia	15.2	43	11
South Sudan	13	37	11
Tanzania	59.1	39	7
Uganda	44.1	41	9
Zambia	17.7	39	8
Zimbabwe	14	34	10
MIDDLE AFRICA	169	42	10
Angola	30.4	45	10
Cameroon	25.6	36	10
Central African Republic	4.7	36	14
Chad	15.4	45	13
Congo	5.4	33	10
Congo, Dem. Rep.	84.3	43	10
Equatorial Guinea	1.3	34	10
Gabon	2.1	29	8
Sao Tome and Principe	0.2	33	7
SOUTHERN AFRICA	66	22	9
Botswana	2.2	22	10
eSwatini	1.4	29	10
Lesotho	2.3	29	13
Namibia	2.5	28	8
South Africa	57.7	21	9

Data: Sampling

Use systematic sampling to pick 10 countries

NORTHERN AMERICA	365	12	9
Canada	37.2	11	8
United States	328	12	9
LATIN AMERICA AND THE CARIBBEAN	649	17	6
CENTRAL AMERICA	179	19	5
Belize	0.4	23	5
Costa Rica	5	14	5
El Salvador	6.5	20	7
Guatemala	17.2	24	5
Honduras	9	22	5
Mexico	130.8	19	6
Nicaragua	6.3	20	5
Panama	4.2	19	5
CARIBBEAN	43	17	8
Antigua and Barbuda	0.1	15	6
Bahamas	0.4	13	6
Barbados	0.3	11	9
Cuba	11.1	10	9
Curacao	0.2	10	9
Dominica	0.07	12	8
Dominican Republic	10.8	20	6
Grenada	0.1	16	9
Haiti	10.8	25	8
Jamaica	2.9	17	7
Puerto Rico	3.3	7	9
St. Kitts-Nevis	0.05	14	9
Saint Lucia	0.2	12	6
St. Vincent and the Grenadines	0.1	16	9
Trinidad and Tobago	1.4	13	8
SOUTH AMERICA	427	16	6
Argentina	44.5	17	8
Bolivia	11.3	23	7
Brazil	209.4	14	6
Chile	18.6	14	6
Colombia	49.8	15	6
Ecuador	17	20	5
Guyana	0.8	21	8
Paraguay	6.9	21	6
Peru	32.2	19	6
Suriname	0.6	18	7
Uruguay	3.5	14	9
Venezuela	31.8	19	5

Data: Sampling

Use cluster sampling to pick 10 countries

EUROPE	746	10	11
NORTHERN EUROPE	105	11	9
Denmark	5.8	11	9
Estonia	1.3	10	12
Finland	5.5	9	10
Iceland	0.4	12	7
Ireland	4.9	13	6
Latvia	1.9	11	15
Lithuania	2.8	10	14
Norway	5.3	11	8
Sweden	10.2	11	9
United Kingdom	66.4	12	9
WESTERN EUROPE	195	10	10
Austria	8.8	10	9
Belgium	11.4	10	10
France	65.1	11	9
Germany	82.8	9	11
Liechtenstein	0.04	10	7
Luxembourg	0.6	10	7
Monaco	0.04	7	6
Netherlands	17.2	10	9
Switzerland	8.5	10	8
EASTERN EUROPE	293	11	12
Belarus	9.5	11	13
Bulgaria	7	9	16
Czechia	10.6	11	11
Hungary	9.8	9	14
Moldova	3.5	11	11
Poland	38.4	11	11
Romania	19.5	10	13
Russia	147.3	12	12
Slovakia	5.4	11	10
Ukraine	42.3	9	14
SOUTHERN EUROPE	153	8	10
Albania	2.9	11	8
Andorra	0.08	8	4
Bosnia-Herzegovina	3.5	9	10
Croatia	4.1	9	12
Greece	10.6	9	11
Italy	60.6	8	11
Kosovo	1.8	13	5
Macedonia	2.1	11	10
Malta	0.5	10	7
Montenegro	0.6	12	11
Portugal	10.3	8	11
San Marino	0.03	7	8
Serbia	7	9	14
Slovenia	2.1	10	10
Spain	46.7	8	9

Data: Sampling

ASIA	4,536	17	7
WESTERN ASIA	272	21	5
Armenia	3	13	9
Azerbaijan	9.9	15	6
Bahrain	1.5	14	2
Cyprus	1.2	11	6
Georgia	3.9	14	13
Iraq	40.2	31	4
Israel	8.5	21	5
Jordan	10.2	24	3
Kuwait	4.2	14	2
Lebanon	6.1	14	5
Oman	4.7	20	2
Qatar	2.7	10	1
Saudi Arabia	33.4	17	3
Syria	18.3	22	6
Turkey	81.3	16	5
United Arab Emirates	9.5	10	2
Yemen	28.9	32	6
CENTRAL ASIA	72	24	6
Kazakhstan	18.4	23	7
Kyrgyzstan	6.1	26	6
Tajikistan	9.1	33	5
Turkmenistan	5.9	26	7
Uzbekistan	32.9	22	5
SOUTH ASIA	1,909	21	6
Afghanistan	36.5	35	7
Bangladesh	166.4	19	5
Bhutan	0.8	18	6
India	1,371.30	20	6
Iran	81.6	19	5
Maldives	0.4	20	3
Nepal	29.7	20	6
Pakistan	200.6	26	7
Sri Lanka	21.7	16	6
SOUTHEAST ASIA	650	18	7
Brunei	0.4	16	4
Cambodia	16	23	6
Indonesia	265.2	19	7
Laos	7	24	7
Malaysia	32.5	16	5
Myanmar	53.9	18	8
Philippines	107	21	6
Singapore	5.8	9	5
Thailand	66.2	11	8
Timor-Leste	1.2	30	7
Vietnam	94.7	16	7
EAST ASIA	1,633	12	7
China	1,393.80	12	7
Japan	126.5	8	11
Korea, North	25.6	14	9
Korea, South	51.8	7	6
Mongolia	3.2	24	6
Taiwan	23.6	9	7

Use stratified sampling to pick 10 countries

Data: Sampling

OCEANIA	41	17	7
Australia	24.1	13	7
Federated States of Micronesia	0.1	22	6
Fiji	0.9	18	7
French Polynesia	0.3	14	5
Guam	0.2	21	6
Kiribati	0.1	29	7
Marshall Islands	0.06	27	4
Nauru	0.01	30	8
New Caledonia	0.3	15	6
New Zealand	4.9	12	7
Palau	0.02	12	10
Papua New Guinea	8.5	29	7
Samoa	0.2	25	4
Solomon Islands	0.7	28	5
Tonga	0.1	24	6
Tuvalu	0.01	25	9
Vanuatu	0.3	27	5


Use quota sampling to pick 10 countries

How could we improve the reliability of the samples?

Data: Averages

Starter

- a) Describe how one would carry out a random sample.
- b) Give 3 examples of discrete data.
- c) An item of clothing has been reduced by 30% and Harry buys it for £70. How much did it cost before it was reduced?
- d) Estimate how many sheets of paper are used by Godalming College students every day.

 You will have met the different measures of average at GCSE:

 **Mode**

 **Mean**

 **Median**

A driving test centre records the gender and age of learners who pass their practical driving test. The list gives the results for one day:

F19 M20 F26 F17 M21 M26 M18 F49 M20 M18 F25 F21 M17
F22 F19 F17 M19 F18 M23 M17 M22 M18 F17 M22 M18

Example 9: Oxford p14

For the **female** drivers:

a) Find the Median

b) Find the Mode

c) Find the Mean

Test yourself 9:

For the **male** drivers:


a) Find the Median





b) Find the Mode

c) Find the Mean

Data: Averages

Average	Advantages	Disadvantages
Mode		
Mean		
Median		

 We can represent data using a stem and leaf diagram; to draw a stem and leaf diagram:

-  List the data in order
-  Choose a stem (usually all but the last digit of the data values)
-  List the leaves (usually the final digit of the data values)
-  Include a title and key

Example 10: Oxford p 15

The list gives the marks achieved by students in a test out of 60:

39 37 56 44 32 40 26 58 35
31 42 37 51 29 38 28 42 49

a) Draw a stem and leaf diagram

b) Find the mode and the median

Test Yourself 10: Oxford p 15






The list gives the ages of a group of holiday makers:

32 18 23 19 43 27 21 35
26 19 23 31 24 37 29 23

a) Draw a stem and leaf diagram

b) Find the mode and the median

Data: Averages

-  We can represent data using a stem and leaf diagram; to draw a stem and leaf diagram:
-  List the data in order
 -  Choose a stem (usually all but the last digit of the data values)
 -  List the leaves (usually the final digit of the data values)
 -  Include a title and key

Example 11: Oxford p 15

Represent the data from the driving test centre in example 1 in a back-to-back stem and leaf diagram

F19 M20 F26 F17 M21 M26 M18 F49 M20 M18 F25 F21 M17
 F22 F19 F17 M19 F18 M23 M17 M22 M18 F17 M22 M18


Test Yourself 11: Oxford p 15

An office worker records the number of emails he receives and sends each day:


Received	36	29	19	44	26	18	32	24	25	42	20	26	31	29	38
Sent	15	21	32	27	19	17	39	28	14	35	9	27	19	35	27

- a) Draw a back to back stem and leaf diagram
- b) Compare the modes, medians, and spread of the Received and Sent datasets

Data: Measures of Spread

 You will have met two measures of spread at GCSE:

 Range:

 Interquartile Range

Example 12: Oxford p 16

The marks out of 100 achieved by two classes in a maths test are listed below

Class A 8 54 55 56 58 59 59 59 62 62 63 63 63 64 65

Class B 25 28 33 37 41 47 59 59 63 69 70 73 76 80

Find the median and interquartile range, and compare the two classes.




Test Yourself 12: Oxford p 16

Compare the times taken by two groups of apprentices to complete a task:


Group P (mins) 32 18 23 19 43 27 21 35 26 19 23

Group Q (mins) 31 24 37 29 20 35 69 21 25 33

Data: Measures of Spread

-  **Standard Deviation** is another way of measuring spread. It is used far more often in statistical analysis than either the range or IQR.
-  It essentially measures the average distance of every value in the dataset away from the mean – the higher the standard deviation the more spread out the data.
-  The formula for standard deviation is as follows:

$$\sigma_{n-1} = \sqrt{\frac{(\sum x - \bar{x})^2}{n - 1}}$$

-  For this course you don't need to use the formula, you can use your calculator – write down the instructions here:

Example 13: Oxford p 16

Use the data from Class A and Class B in example 4

Find the mean and standard deviation, and compare the two classes.

Test Yourself 13: Oxford p 16

Use the data from Group P and Group Q in Test Yourself 4


Find the mean and standard deviation, and compare the two groups.


Measure of Spread	Advantages	Disadvantages
Range		
IQR		
Standard Deviation		

Finance: Spreadsheets


Starter

- a) Describe how one would carry out a stratified sample.
- b) Give 3 examples of continuous data.
- c) Paulo bought some clothes for £135, including 20% VAT. How much did they cost before tax?
- d) On average Kyle scored 65% on his maths homework and Edith scored 70%. The standard deviation of Kyle's scores was 4.32% and Edith's was 15.2%.
- Compare their scores.
 - If it takes around 70% in the final exam to be get a B, should either of them be predicted a B grade? Why or why not?

 You will need to be familiar with some simple spreadsheet functions:

 Spreadsheet formulae always start with _____

 Multiplication is represented by: _____

 If the number you want to calculate is already on the spreadsheet you should use _____

	A	B	C	D
1	Name	Monthly income	Monthly housing cost	Percentage
2	Alan	£1,325	£435	33
3	Bev	£1,993	£605	30
4	Charlie	£934	£345	37
5	Dot	£2,012	£820	41

Example 14: Hodder p305

What formula could have been used to calculate the percentage that Alan spends on housing?

Test yourself 14:

What formula could have been used to calculate the percentage that Bev spends on housing?

Finance: Spreadsheets


	A	B	C
1	Week	Hours worked	Earnings
2	1	45	£342.00
3	2	35	£266.00
4	3	39	£296.40
5		39.67	£301.47


Example 15: Hodder p305

Row 5 shows the mean - what formula could have been used to calculate the average hours worked?

Test yourself 15:

What formula could have been used to calculate the average earnings?

 We are going to use a spreadsheet to practice calculating simple and compound interest.

 Simple interest is where the interest does not change depending on the amount of money in the account. Compound interest is re-calculated at regular intervals (could be yearly, monthly, weekly or another time period depending on the bank)

Example 16:

The interest on an account is 3.5% per annum calculate the interest earned for:

a) Simple interest

Year	Start of year	Interest	End of year
1	£450.00		
2			
3			
4			
5			

b) Compound interest

Year	Start of year	Interest	End of year
1	£450.00		
2			
3			
4			
5			

Finance: AER




- ✎ AER is banking term – it stands for “Annual Equivalent Rate”, every bank needs to display it to help customers compare accounts.
- ✎ It is the rate of interest a bank will pay a customer over one year.
- ✎ The formula for AER (r) is:

$$r = \left(1 + \frac{i}{n}\right)^n - 1$$

where i is the nominal annual interest rate and n is the number of times during the year the interest is compounded.




Example 17:

Compare the following bank accounts:

- a)  4.2% per annum with interest calculated annually
- b) **HSBC**  4.0% per annum with interest calculated monthly
- c)  3.9% per annum with interest calculated weekly

Test Yourself 17:

Compare the following bank accounts:

- a)  2.4% per annum with interest calculated annually
- b) **HSBC**  2.41% per annum with interest calculated monthly
- c)  2.38% per annum with interest calculated weekly

Finance: AER

- ✎ AER is banking term – it stands for “Annual Equivalent Rate”, every bank needs to display it to help customers compare accounts.
- ✎ It is the rate of interest a bank will pay a customer over one year.
- ✎ The formula for AER (r) is:

$$r = \left(1 + \frac{i}{n}\right)^n - 1$$

where i is the nominal annual interest rate and n is the number of times during the year the interest is compounded.

Example 18:

Watch out for the difference between **nominal annual** interest and interest which has already been quoted over a different period:

- a) Find the AER for an account with an annual rate of 3.2%, compounded weekly.
- b) Find the AER for an account with interest of 0.03% which is paid each week
- c) Find the AER for an account with interest of 1.24% which is paid every 6 months
- d) Find the AER for an account with a nominal rate of 2.4%, compounded 3 times per year

Test Yourself 18:

Compare the following bank accounts:

- a) Find the AER for an account with an annual rate of 2.68%, compounded monthly.
- b) Find the AER for an account with interest of 0.2% which is paid each month

Finance: Mortgages


MATHS


Starter

1. A bank account pays interest of 0.25% each week. What is the AER?
2. Find the mean and standard deviation from this table:
3. If there are 750 people in the audience of a cinema, and 58 of them are men between 25 and 34, how many would be sampled using a stratified sample of 100?
4. It has been suggested that the speed limit on UK motorways should be increased from 70mph to 80mph. What is the percentage increase?
5. Estimate how many people could fit on the Isle of Wight

Time in seconds	Frequency
$85 \leq t < 90$	1
$90 \leq t < 95$	5
$95 \leq t < 100$	28
$100 \leq t < 105$	19
$105 \leq t < 110$	7



 Mortgages are used when someone borrows large sums of money to buy a house. To pay off the mortgage, payments are made each month, and the lender adds interest to the sum owed.

 The amount owed can be represented by a formula known as a recurrence relation, this recurrence relation represents someone who has a mortgage with a monthly interest rate of 0.23% and monthly payments of £950:

$$A_n = 1.0023A_{n-1} - 950$$

Where A_n represents the amount owed in month n and A_{n-1} represents the amount owed the month before. To use this formula, type in the amount owed into your calculator and press "=", then type in the formula, and instead of " A_{n-1} ", use the "ANS" button.

Example 19:

- a) Using the recurrence relation above, complete the table to show how much someone who takes out a mortgage worth £350,000 owes after 4 months
- b) How much has been paid to the lender in interest?
- c) What could the spreadsheet formula be to calculate the amount in cell B3?

	A	B
1	Month	Amount outstanding
2	0	£350,000
3	1	£349,855
4	2	
5	3	
6	4	

Finance: Mortgages

Test Yourself 19:

- a) Amil takes out a mortgage worth £412,000, he pays £1,200 each month and the lender adds 0.19% interest each month. This can be represented by the following recurrence relation:

$$A_n = 1.0019A_{n-1} - 1200$$

Using this information, fill out the table.

- b) How much has been paid to the lender in interest?

- c) What could the spreadsheet formula be to calculate the amount in cell B3?

▲	A	B
1	Month	Amount outstanding
2	0	£412,000
3	1	£411,582.80
4	2	
5	3	
6	4	

Example 20:

Write down a recurrence relation for the following mortgages:

- a) Monthly interest rate: 0.3%
Monthly payments: £356

- b) Yearly interest rate: 2.19%
Yearly payments: £11,400

- c) Monthly interest rate: 0.425%
Monthly payments: £823

Test Yourself 20:

Write down a recurrence relation for the following mortgages:

- a) Monthly interest rate: 0.2%
Monthly payments: £720

- b) Yearly interest rate: 4.24%
Yearly payments: £15,240

- c) Monthly interest rate: 0.399%
Monthly payments: £1,020

Finance: APR

✎ APR is banking term – it stands for “Annual Percentage Rate”, it differs from AER in that AER is what a customer can earn in interest from a bank, whilst APR is what a customer will pay to a lender for borrowing money.

✎ It is the rate of interest a customer will pay on their borrowing over one year..

✎ The formula for APR (given in the formula book) is:

$$C = \sum_{k=1}^m \left(\frac{A_k}{(1+i)^{t_k}} \right)$$

where C is the amount borrowed, m is the number of repayments, i is the APR, A_k is the amount of the k^{th} repayment and t_k is the amount of time, in years, between the start of the loan and the k^{th} repayment

✎ This is a complicated looking formula, but we can make it simpler. The symbol sigma (Σ), means “sum of”. They use this in formula book because the number of terms in our sum will vary depending on the number of repayments. We can (and should) write the formula without sigma:

$$C = \frac{A_1}{(1+i)^{t_1}} + \frac{A_2}{(1+i)^{t_2}} + \frac{A_3}{(1+i)^{t_3}} + \dots + \frac{A_m}{(1+i)^{t_m}}$$

✎ Write down the formula used for 1 repayment:

✎ 2 repayments:

✎ 3 repayments

Example 21:

A loan of £2000 is repaid by a payment of £2500 made 3 years later Calculate the APR.

Test Yourself 21:

£1000 repaid by single repayment of £1200 made 2 years later. Calculate the APR

Finance: APR

Example 22: Oxford p62

A loan was taken out at APR 30%. £1,000 was repaid after 1 year, a further £1,000 was repaid after 2 years and a final payment of £1,404 was made after 3 years. How much was the loan for?

Test Yourself 22:

A loan was taken out at APR 18.9%. £1,500 was repaid after 1 year, a further £1,600 was repaid after 2 years and a final payment of £2,300 was made after 3 years. How much was the loan for?

Example 23: Oxford p62

Rachel borrows £4,000. A lender offers APR of 16.26%, with two equal annual repayments. Calculate the value of the repayments

Test Yourself 23: Oxford p62

Rachel borrows £4,000. A different lender offers APR of 15%, with two equal annual repayments. Calculate the value of the repayments

Finance: APR

 The formula for APR (given in the formula book) is:

$$C = \sum_{k=1}^m \left(\frac{A_k}{(1+i)^{t_k}} \right)$$

where C is the amount borrowed, m is the number of repayments, i is the APR, A_k is the amount of the k^{th} repayment and t_k is the amount of time, in years, between the start of the loan and the k^{th} repayment

Example 24: Collins p66

You borrow £400 from a money lender to be paid back in five equal monthly instalments at an APR of 1509%. How much are the monthly instalments?

Test Yourself 24:

You borrow £120 from a money lender to be paid back in four equal monthly instalments at an APR of 1783%. How much are the monthly instalments?

Finance: APR

 The formula for APR (given in the formula book) is:

$$C = \sum_{k=1}^m \left(\frac{A_k}{(1+i)^{t_k}} \right)$$

where C is the amount borrowed, m is the number of repayments, i is the APR, A_k is the amount of the k^{th} repayment and t_k is the amount of time, in years, between the start of the loan and the k^{th} repayment

Example 25:

A loan of £7000 is repaid in annual instalments of £1000, £2000, £3000 and £4000.

- Show that the APR lies between 12.6% and 13%
- Find the APR correct to 1 decimal place.

Test Yourself 25:

A borrower repays a debt of £4000 in 2 annual instalments of £2300.

- Show that the APR lies between 8% and 12%
- Find the APR to the nearest %.