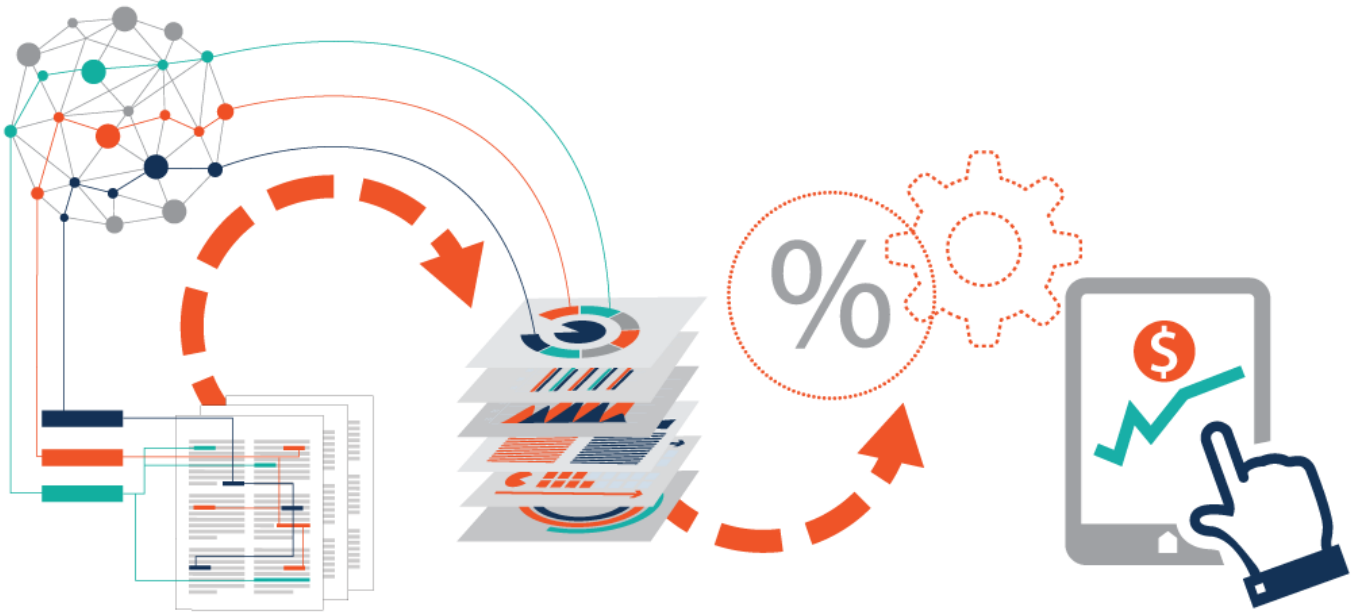
Godalming College

Ματησ

AQA Level 3 Certificate in Mathematical Studies (Core Maths)

Paper 1 Content



Name _____

Estimation and Modelling

Starter

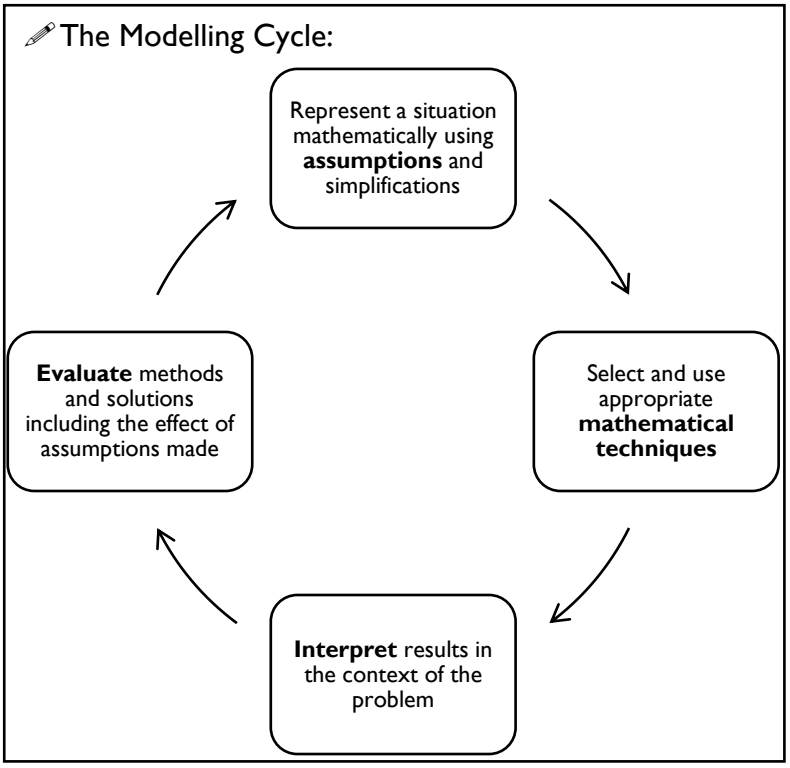
Using Estimates, put these in order from smallest to largest:

a) The number of passengers a Boeing 747-400 (Jumbo Jet) can accommodate
b) The age of the world's oldest tortoise, Harriet, when she died in 2006
c) The length of the River Severn, in miles
d) The distance, in a straight line, from London to Edinburgh in miles

Example 1:

How many steps would it take someone to walk from Land's End to John O'Groats:

a) State your assumptions:



b) Working out:

c) Interpretation:

d) Evaluate:

Estimation and Modelling

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Example 2:

How many items of luggage pass through Heathrow airport each year?

Test yourself 2:

a) A ream (500 sheets) of paper is 5.4cm thick. Estimate the number of sheets needed to reach the height of an average man.

b) There are three rats for every human in London. How many rats are there in London?

c) How many golf balls would fit in a suitcase?

d) How many steps would it take for a person to travel around the world once?

Percentages

Starter

Estimate:

a) The number of secondary schools in the UK
b) How many dentists are there in Bristol?
c) How many pizzas are eaten in Godalming each year.

- **Percentages**: These are used throughout the course, so it's important we spend some time recapping what you will have already picked up during GCSE.
- Percentage literally means "out of one hundred"

$$50\% = \frac{50}{100} = 0.5$$

Example 3: Collins p49-50

a) Yolanda pays off 67% of an electricity bill of £84. How much does she pay?

b) Will pays off £93 from a debt of £136. What percentage has he paid?

c) Jack and Beth have sales targets in their shop.

One month, Jack made sales of £4.4k against his £5k target.

Beth made sales of £5.45k against her £6k target.

Who made a greater proportion of sales when compared to their target?

Percentages

Test Yourself 3:

a) Bryan achieved 87.5% of the marks on a 96 mark test. How many marks did he achieve?

b) Kiya has paid off £248,345 on her £455,000 mortage, what percentage has she paid off?

c) Two items are on sale in a shop. A £34 shirt is on sale for £27 and a £95 pair of jeans is on sale for £79. Which item has been reduced the most?

Example 4: Collins p52

a) Increase £84 by 67%

b) Decrease £84 by 67%

Test Yourself 4:

a) Increase £45 by 83%

b) Decrease £45 by 83%

Percentages

Example 5: Collins p52

a) A price is increased by 15% to £79
What was the original price?

b) A mass is decreased by 8%. The new mass is 5.3kg.
What was the original mass?

Test Yourself 5:

a) A price is increased by 20% to £45.50
What was the original price?

b) A shirt is on sale, reduced by 25%. The new price is £45.
What was the original price?

Test Yourself 4:

No.	Question	Forward or Reverse	Working and Answer	Tick or Cross
1	Increase £10 by 15%			
2	Bill has a wage increase of 10%. He now earns £359.70 a week. What did he earn before?			
3	Kelly buys a jumper in a sale. After a 15% reduction the price is £48.45. What was it before the sale?			
4	The population of an island increased 16% to 2494. What was the population before increase?			

Percentages

5	The mass of a rock is reduced by 22%. The new mass is 193.44g. What was the original mass of the rock?			
6	Decrease 127 by 11%			
7	VAT is charged at 20%. Gina buys a new car with VAT added and pays £21960. She can claim her VAT back. How much can she claim back?			
8	John has a wage increase of 20%. He originally earned £320 a week. How much does he earn now			
9	In a restaurant service is charged at 8.5%. Gemma buys a salad for £6.51. How much would have she paid if the service charge had not been added?			
10	Samuel bought a fridge in a '30% off sale'. Given that he paid £384.30, find its original price before the sale.			
11	A toaster has 18% off in a sale. Given that the toaster was £75 before the sale, find its new price.			
12	John was told to increase his daily exercise by ¼. Given that he now does 56 minutes a day, show that he originally did less than ¾ of an hour a day.			
13	Sue was told to decrease her daily calorie intake by 15%. Given that she is now eating less than 1600 calories a day, find the maximum number she was originally eating. Give your answer to the nearest calorie.			

Data: Types of data

Starter

1. Subhash paid £350 for a computer tablet. He later finds he could have bought the same one for £295. What percentage has the price gone down by?

2. Last year, Simon had 150 followers on Twitter. His number of followers has increased by 120%. How many followers does he have now?

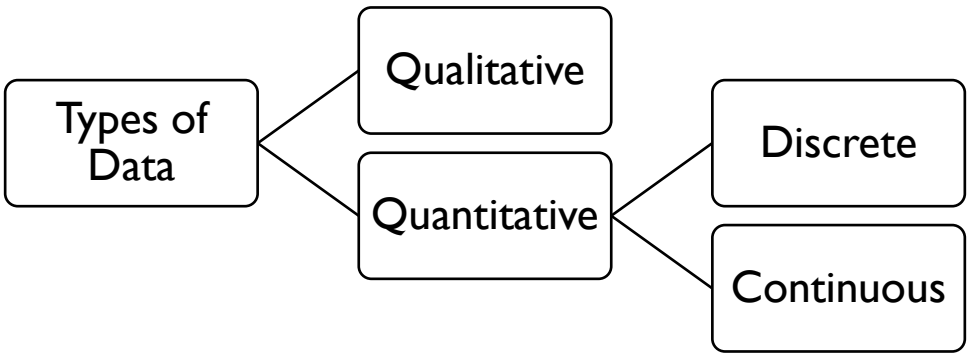
3. A computer game costs £54.60 including VAT at 20%. What was the price before VAT?

4. A car has depreciated in value by 27%. It was £15,650 when new. What is the value now?

5. Estimate how long it would take to type out a novel.

You need to be able to define the following terms:
- **Qualitative data**: non-numerical data (describes a quality)
- **Quantitative data**: numerical data
- **Discrete data**: numerical data which can take only exact values
- **Continuous data**: numerical data which can take any value

Example 6: Hodder p 320



Place the following into the correct category:
Name, address, date of birth, place of birth, passport number, national insurance number, gender, medical history, blood group, height, weight, waist measurement

Data: Types of data

You need to be able to define the following terms:
- **Qualitative data**: non-numerical data (describes a quality)
- **Quantitative data**: numerical data
- **Discrete data**: numerical data which can take only exact values
- **Continuous data**: numerical data which can take any value

Test Yourself 6 Hodder p 321

a) Place the following into the correct category:
Age, number of siblings, road name, distance from home to college, bank account number, GCSE maths grade, Core maths grade, salary.

b) Write down three examples of Qualitative, Quantitative discrete, Quantitative continuous data.

You need to be able to define the following terms:
- **Primary data**: Information that is collected first hand by researchers
- **Secondary data**: numerical data

Example 7:

Ways of collecting primary data:

Ways of collecting secondary data:

Data: Types of data

You need to be able to define the following terms:
- **Primary data**: Information that is collected first hand by researchers
- **Secondary data**: numerical data

Test Yourself 7:

Which should I use for (justify your answers):

a) Celebrities' favourite breakfasts

b) Average family size

c) Favourite clothes shop for 16-19 year olds

d) Most common pet

e) World's most popular car

Data: Sampling

You need to be able to define and describe the method for each type of sampling:

- **Random sampling**:

- **Systematic sampling:**

- **Cluster sampling**:

- **Quota sampling**:

- **Stratified Sampling:**

Example 8:

Type	Can use if…	Advantages	Disadvantages
Random			
Cluster			
Stratified			
Quota			

Data: Sampling

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Use simple random sampling to pick 10 countries

	Geography	Population mid-2018 (millions)	Births per 1,000 Population	Deaths per 1,000 Population
	AFRICA	1,284	35	9
	NORTHERN AFRICA	235	26	6
	Algeria	42.7	26	4
	Egypt	97	27	6
	Libya	6.5	20	5
	Morocco	35.2	18	5
	Sudan	41.7	34	8
	Tunisia	11.6	19	6
	Western Sahara	0.6	20	5
	WESTERN AFRICA	382	38	11
	Benin	11.5	37	9
	Burkina Faso	20.3	40	9
	Cape Verde	0.6	19	5
	Cote d'Ivoire	24.9	35	12
	Gambia	2.2	39	8
	Ghana	29.5	30	8
	Guinea	11.9	39	11
	Guinea-Bissau	1.9	36	11
	Liberia	4.9	33	8
	Mali	19.4	45	10
	Mauritania	4.5	34	8
	Niger	22.2	48	10
	Nigeria	195.9	39	12
	Senegal	16.3	33	6
	Sierra Leone	7.7	33	13
	Togo	8	34	9
	EASTERN AFRICA	432	36	8
	Burundi	11.8	41	11
	Comoros	0.8	33	7
	Djibouti	1	23	8
	Eritrea	6	30	7
	Ethiopia	107.5	33	7
	Kenya	51	31	6
	Madagascar	26.3	32	6
	Malawi	19.1	34	7
	Mauritius	1.3	11	8
	Mozambique	30.5	39	10
	Rwanda	12.6	32	6
	Seychelles	0.1	17	8
	Somalia	15.2	43	11
	South Sudan	13	37	11
	Tanzania	59.1	39	7
	Uganda	44.1	41	9
	Zambia	17.7	39	8
	Zimbabwe	14	34	10
	MIDDLE AFRICA	169	42	10
	Angola	30.4	45	10
	Cameroon	25.6	36	10
	Central African Republic	4.7	36	14
	Chad	15.4	45	13
	Congo	5.4	33	10
	Congo, Dem. Rep.	84.3	43	10
	Equatorial Guinea	1.3	34	10
	Gabon	2.1	29	8
	Sao Tome and Principe	0.2	33	7
	SOUTHERN AFRICA	66	22	9
	Botswana	2.2	22	10
	eSwatini	1.4	29	10
	Lesotho	2.3	29	13
	Namibia	2.5	28	8
	South Africa	57.7	21	9

Data: Sampling

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Use systematic sampling to pick 10 countries

	NORTHERN AMERICA	365	12	9
	Canada	37.2	11	8
	United States	328	12	9
	LATIN AMERICA AND THE CARIBBEAN	649	17	6
	CENTRAL AMERICA	179	19	5
	Belize	0.4	23	5
	Costa Rica	5	14	5
	El Salvador	6.5	20	7
	Guatemala	17.2	24	5
	Honduras	9	22	5
	Mexico	130.8	19	6
	Nicaragua	6.3	20	5
	Panama	4.2	19	5
	CARIBBEAN	43	17	8
	Antigua and Barbuda	0.1	15	6
	Bahamas	0.4	13	6
	Barbados	0.3	11	9
	Cuba	11.1	10	9
	Curacao	0.2	10	9
	Dominica	0.07	12	8
	Dominican Republic	10.8	20	6
	Grenada	0.1	16	9
	Haiti	10.8	25	8
	Jamaica	2.9	17	7
	Puerto Rico	3.3	7	9
	St. Kitts-Nevis	0.05	14	9
	Saint Lucia	0.2	12	6
	St. Vincent and the Grenadines	0.1	16	9
	Trinidad and Tobago	1.4	13	8
	SOUTH AMERICA	427	16	6
	Argentina	44.5	17	8
	Bolivia	11.3	23	7
	Brazil	209.4	14	6
	Chile	18.6	14	6
	Colombia	49.8	15	6
	Ecuador	17	20	5
	Guyana	0.8	21	8
	Paraguay	6.9	21	6
	Peru	32.2	19	6
	Suriname	0.6	18	7
	Uruguay	3.5	14	9
	Venezuela	31.8	19	5

Data: Sampling

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	EUROPE	746	10	11
	NORTHERN EUROPE	105	11	9
	Denmark	5.8	11	9
	Estonia	1.3	10	12
	Finland	5.5	9	10
	Iceland	0.4	12	7
	Ireland	4.9	13	6
	Latvia	1.9	11	15
	Lithuania	2.8	10	14
	Norway	5.3	11	8
	Sweden	10.2	11	9
	United Kingdom	66.4	12	9
	WESTERN EUROPE	195	10	10
	Austria	8.8	10	9
	Belgium	11.4	10	10
	France	65.1	11	9
	Germany	82.8	9	11
	Liechtenstein	0.04	10	7
	Luxembourg	0.6	10	7
	Monaco	0.04	7	6
	Netherlands	17.2	10	9
	Switzerland	8.5	10	8
	EASTERN EUROPE	293	11	12
	Belarus	9.5	11	13
	Bulgaria	7	9	16
	Czechia	10.6	11	11
	Hungary	9.8	9	14
	Moldova	3.5	11	11
	Poland	38.4	11	11
	Romania	19.5	10	13
	Russia	147.3	12	12
	Slovakia	5.4	11	10
	Ukraine	42.3	9	14
	SOUTHERN EUROPE	153	8	10
	Albania	2.9	11	8
	Andorra	0.08	8	4
	Bosnia-Herzegovina	3.5	9	10
	Croatia	4.1	9	12
	Greece	10.6	9	11
	Italy	60.6	8	11
	Kosovo	1.8	13	5
	Macedonia	2.1	11	10
	Malta	0.5	10	7
	Montenegro	0.6	12	11
	Portugal	10.3	8	11
	San Marino	0.03	7	8
	Serbia	7	9	14
	Slovenia	2.1	10	10
	Spain	46.7	8	9

Use cluster sampling to pick 10 countries

Data: Sampling

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	ASIA	4,536	17	7
	WESTERN ASIA	272	21	5
	Armenia	3	13	9
	Azerbaijan	9.9	15	6
	Bahrain	1.5	14	2
	Cyprus	1.2	11	6
	Georgia	3.9	14	13
	Iraq	40.2	31	4
	Israel	8.5	21	5
	Jordan	10.2	24	3
	Kuwait	4.2	14	2
	Lebanon	6.1	14	5
	Oman	4.7	20	2
	Qatar	2.7	10	1
	Saudi Arabia	33.4	17	3
	Syria	18.3	22	6
	Turkey	81.3	16	5
	United Arab Emirates	9.5	10	2
	Yemen	28.9	32	6
	CENTRAL ASIA	72	24	6
	Kazakhstan	18.4	23	7
	Kyrgyzstan	6.1	26	6
	Tajikistan	9.1	33	5
	Turkmenistan	5.9	26	7
	Uzbekistan	32.9	22	5
	SOUTH ASIA	1,909	21	6
	Afghanistan	36.5	35	7
	Bangladesh	166.4	19	5
	Bhutan	0.8	18	6
	India	1,371.30	20	6
	Iran	81.6	19	5
	Maldives	0.4	20	3
	Nepal	29.7	20	6
	Pakistan	200.6	26	7
	Sri Lanka	21.7	16	6
	SOUTHEAST ASIA	650	18	7
	Brunei	0.4	16	4
	Cambodia	16	23	6
	Indonesia	265.2	19	7
	Laos	7	24	7
	Malaysia	32.5	16	5
	Myanmar	53.9	18	8
	Philippines	107	21	6
	Singapore	5.8	9	5
	Thailand	66.2	11	8
	Timor-Leste	1.2	30	7
	Vietnam	94.7	16	7
	EAST ASIA	1,633	12	7
	China	1,393.80	12	7
	Japan	126.5	8	11
	Korea, North	25.6	14	9
	Korea, South	51.8	7	6
	Mongolia	3.2	24	6
	Taiwan	23.6	9	7

Use stratified sampling to pick 10 countries

Data: Sampling

	OCEANIA	41	17	7
	Australia	24.1	13	7
	Federated States of Micronesia	0.1	22	6
	Fiji	0.9	18	7
	French Polynesia	0.3	14	5
	Guam	0.2	21	6
	Kiribati	0.1	29	7
	Marshall Islands	0.06	27	4
	Nauru	0.01	30	8
	New Caledonia	0.3	15	6
	New Zealand	4.9	12	7
	Palau	0.02	12	10
	Papua New Guinea	8.5	29	7
	Samoa	0.2	25	4
	Solomon Islands	0.7	28	5
	Tonga	0.1	24	6
	Tuvalu	0.01	25	9
	Vanuatu	0.3	27	5

Use quota sampling to pick 10 countries

How could we improve the reliability of the samples?

Data: Averages

Starter

a) Describe how one would carry out a random sample.

b) Give 3 examples of discrete data.

c) An item of clothing has been reduced by 30% and Harry buys it for £70. How much did it cost before it was reduced?

d) Estimate how many sheets of paper are used by Godalming College students every day.

- You will have met the different measures of average at GCSE:
  - **Mode**
  - **Mean**
  - **Median**

A driving test centre records the gender and age of learners who pass their practical driving test. The list gives the results for one day:

F19 M20 F26 F17 M21 M26 M18 F49 M20 M18 F25 F21 M17
F22 F19 F17 M19 F18 M23 M17 M22 M18 F17 M22 M18

Example 9: Oxford p14

For the **female** drivers:

a) Find the Median

b) Find the Mode

c) Find the Mean

Test yourself 9:

For the **male** drivers:

a) Find the Median

b) Find the Mode

c) Find the Mean

Data: Averages

Average	Advantages	Disadvantages
Mode		
Mean		
Median		

- We can represent data using a stem and leaf diagram; to draw a stem and leaf diagram:
  - List the data in order
  - Choose a stem (usually all but the last digit of the data values)
  - List the leaves (usually the final digit of the data values)
  - Include a title and key

Example 10: Oxford p 15

The list gives the marks achieved by students in a test out of 60:

39 37 56 44 32 40 26 58 35
31 42 37 51 29 38 28 42 49

a) Draw a stem and leaf diagram

b) Find the mode and the median

Test Yourself 10: Oxford p 15

The list gives the ages of a group of holiday makers:

32 18 23 19 43 27 21 35
26 19 23 31 24 37 29 23

a) Draw a stem and leaf diagram

b) Find the mode and the median

Data: Averages

- We can represent data using a stem and leaf diagram; to draw a stem and leaf diagram:
  - List the data in order
  - Choose a stem (usually all but the last digit of the data values)
  - List the leaves (usually the final digit of the data values)
  - Include a title and key

Example 11: Oxford p 15

Represent the data from the driving test centre in example 1 in a back-to-back stem and leaf diagram

F19 M20 F26 F17 M21 M26 M18 F49 M20 M18 F25 F21 M17
F22 F19 F17 M19 F18 M23 M17 M22 M18 F17 M22 M18

Test Yourself 11: Oxford p 15

An office worker records the number of emails he receives and sends each day:

Received	36	29	19	44	26	18	32	24	25	42	20	26	31	29	38
Sent	15	21	32	27	19	17	39	28	14	35	9	27	19	35	27

a) Draw a back to back stem and leaf diagram
b) Compare the modes, medians, and spread of the Received and Sent datasets

Data: Measures of Spread

- You will have met two measures of spread at GCSE:
  - Range:
  - Interquartile Range

Example 12: Oxford p 16

The marks out of 100 achieved by two classes in a maths test are listed below

Class A 8 54 55 56 58 59 59 59 62 62 63 63 63 64 65
Class B 25 28 33 37 41 47 59 59 63 69 70 73 76 80

Find the median and interquartile range, and compare the two classes.

Test Yourself 12: Oxford p 16

Compare the times taken by two groups of apprentices to complete a task:

Group P (mins) 32 18 23 19 43 27 21 35 26 19 23
Group Q (mins) 31 24 37 29 20 35 69 21 25 33

Data: Measures of Spread

- **Standard Deviation** is another way of measuring spread. It is used far more often in statistical analysis than either the range or IQR.
- It essentially measures the average distance of every value in the dataset away from the mean – the higher the standard deviation the more spread out the data.
- The formula for standard deviation is as follows:

$$\sigma_{n-1} = \sqrt{\frac{(\Sigma x - \bar{x})^2}{n-1}}$$

- For this course you don't need to use the formula, you can use your calculator – write down the instructions here:

Example 13: Oxford p 16

Use the data from Class A and Class B in example 4

Find the mean and standard deviation, and compare the two classes.

Test Yourself 13: Oxford p 16

Use the data from Group P and Group Q in Test Yourself 4

Find the mean and standard deviation, and compare the two groups.

Measure of Spread	Advantages	Disadvantages
Range		
IQR		
Standard Deviation		

Finance: Spreadsheets

Starter

a) Describe how one would carry out a stratified sample.

b) Give 3 examples of continuous data.

c) Paulo bought some clothes for £135, including 20% VAT. How much did they cost before tax?

d) On average Kyle scored 65% on his maths homework and Edith scored 70%. The standard deviation of Kyle's scores was 4.32% and Edith's was 15.2%.

i. Compare their scores.

ii. If it takes around 70% in the final exam to be get a B, should either of them be predicted a B grade? Why or why not?

✎ You will need to be familiar with some simple spreadsheet functions:

✎ Spreadsheet formulae always start with _____

✎ Multiplication is represented by: _____

✎ If the number you want to calculate is already on the spreadsheet you should use _____

	A	B	C	D
1	Name	Monthly income	Monthly housing cost	Percentage
2	Alan	£1,325	£435	33
3	Bev	£1,993	£605	30
4	Charlie	£934	£345	37
5	Dot	£2,012	£820	41

Example 14: Hodder p305

What formula could have been used to calculate the percentage that Alan spends on housing?

Test yourself 14:

What formula could have been used to calculate the percentage that Bev spends on housing?

Finance: Spreadsheets

	A	B	C
1	Week	Hours worked	Earnings
2	1	45	£342.00
3	2	35	£266.00
4	3	39	£296.40
5		39.67	£301.47

Example 15: Hodder p305

Row 5 shows the mean - what formula could have been used to calculate the average hours worked?

Test yourself 15:

What formula could have been used to calculate the average earnings?

- We are going to use a spreadsheet to practice calculating simple and compound interest.
- Simple interest is where the interest does not change depending on the amount of money in the account. Compound interest is re-calculated at regular intervals (could be yearly, monthly, weekly or another time period depending on the bank)

Example 16:

The interest on an account is 3.5% per annum calculate the interest earned for:

a) Simple interest

Year	Start of year	Interest	End of year
1	£450.00		
2			
3			
4			
5			

b) Compound interest

Year	Start of year	Interest	End of year
1	£450.00		
2			
3			
4			
5			

Finance: AER

- AER is banking term – it stands for "Annual Equivalent Rate", every bank needs to display it to help customers compare accounts.
- It is the rate of interest a bank will pay a customer over one year.
- The formula for AER (r) is:

$$r = \left(1 + \frac{i}{n}\right)^n - 1$$

where i is the nominal annual interest rate and n is the number of times during the year the interest is compounded.

Example 17:

Compare the following bank accounts:

a) BARCLAYS 4.2% per annum with interest calculated annually

b) HSBC 4.0% per annum with interest calculated monthly

c) HALIFAX 3.9% per annum with interest calculated weekly

Test Yourself 17:

Compare the following bank accounts:

a) BARCLAYS 2.4% per annum with interest calculated annually

b) HSBC 2.41% per annum with interest calculated monthly

c) HALIFAX 2.38% per annum with interest calculated weekly

Finance: AER

- AER is banking term – it stands for "Annual Equivalent Rate", every bank needs to display it to help customers compare accounts.
- It is the rate of interest a bank will pay a customer over one year.
- The formula for AER (r) is:

$$r = \left(1 + \frac{i}{n}\right)^n - 1$$

where i is the nominal annual interest rate and n is the number of times during the year the interest is compounded.

Example 18:

Watch out for the difference between **nominal annual** interest and interest which has already been quoted over a different period:

a) Find the AER for an account with an annual rate of 3.2%, compounded weekly.
b) Find the AER for an account with interest of 0.03% which is paid each week
c) Find the AER for an account with interest of 1.24% which is paid every 6 months
d) Find the AER for an account with a nominal rate of 2.4%, compounded 3 times per year

Test Yourself 18:

Compare the following bank accounts:

a) Find the AER for an account with an annual rate of 2.68%, compounded monthly.
b) Find the AER for an account with interest of 0.2% which is paid each month

Godalming College Maths

Finance: Mortgages

Starter

1. A bank account pays interest of 0.25% each week. What is the AER?

2. Find the mean and standard deviation from this table:

Time in seconds	Frequency
$85 \leq t < 90$	1
$90 \leq t < 95$	5
$95 \leq t < 100$	28
$100 \leq t < 105$	19
$105 \leq t < 110$	7

3. If there are 750 people in the audience of a cinema, and 58 of them are men between 25 and 34, how many would be sampled using a stratified sample of 100?

4. It has been suggested that the speed limit on UK motorways should be increased from 70mph to 80mph. What is the percentage increase?

5. Estimate how many people could fit on the Isle of Wight

Mortgages are used when someone borrows large sums of money to buy a house. To pay off the mortgage, payments are made each month, and the lender adds interest to the sum owed.

The amount owed can be represented by a formula known as a recurrence relation, this recurrence relation represents someone who has a mortgage with a monthly interest rate of 0.23% and monthly payments of £950:

$$A_n = 1.0023A_{n-1} - 950$$

Where A_n represents the amount owed in month n and A_{n-1} represents the amount owed the month before. To use this formula, type in the amount owed into your calculator and press "=", then type in the formula, and instead of "A_{n-1}", use the "ANS" button.

Example 19:

a) Using the recurrence relation above, complete the table to show how much someone who takes out a mortgage worth £350,000 owes after 4 months

b) How much has been paid to the lender in interest?

c) What could the spreadsheet formula be to calculate the amount in cell B3?

	A	B
1	**Month**	**Amount outstanding**
2	0	£350,000
3	1	£349,855
4	2	
5	3	
6	4	

Finance: Mortgages

Test Yourself 19:

a) Amil takes out a mortgage worth £412,000, he pays £1,200 each month and the lender adds 0.19% interest each month. This can be represented by the following recurrence relation:

$$A_n = 1.0019A_{n-1} - 1200$$

Using this information, fill out the table.

◢	A	B
1	Month	Amount outstanding
2	0	£412,000
3	1	£411,582.80
4	2	
5	3	
6	4	

b) How much has been paid to the lender in interest?

c) What could the spreadsheet formula be to calculate the amount in cell B3?

Example 20:

Write down a recurrence relation for the following mortgages:

a) Monthly interest rate: 0.3%
Monthly payments: £356

b) Yearly interest rate: 2.19%
Yearly payments: £11,400

c) Monthly interest rate: 0.425%
Monthly payments: £823

Test Yourself 20:

Write down a recurrence relation for the following mortgages:

a) Monthly interest rate: 0.2%
Monthly payments: £720

b) Yearly interest rate: 4.24%
Yearly payments: £15,240

c) Monthly interest rate: 0.399%
Monthly payments: £1,020

Finance: APR

- APR is banking term – it stands for "Annual Percentage Rate", it differs from AER in that AER is what a customer can earn in interest from a bank, whilst APR is what a customer will pay to a lender for borrowing money.

- It is the rate of interest a customer will pay on their borrowing over one year..

- The formula for APR (given in the formula book) is:

$$C = \sum_{k=1}^{m} \left(\frac{A_k}{(1+i)^{t_k}} \right)$$

where C is the amount borrowed, m is the number of repayments, i is the APR, A_k is the amount of the k^{th} repayment and t_k is the amount of time, in years, between the start of the loan and the k^{th} repayment

- This is a complicated looking formula, but we can make it simpler. The symbol sigma (Σ), means "sum of". They use this in formula book because the number of terms in our sum will vary depending on the number of repayments. We can (and should) write the formula without sigma:

$$C = \frac{A_1}{(1+i)^{t_1}} + \frac{A_2}{(1+i)^{t_2}} + \frac{A_3}{(1+i)^{t_3}} + \cdots + \frac{A_m}{(1+i)^{t_m}}$$

- Write down the formula used for 1 repayment:

- 2 repayments:

- 3 repayments

Example 21:

A loan of £2000 is repaid by a payment of £2500 made 3 years later Calculate the APR.

Test Yourself 21:

£1000 repaid by single repayment of £1200 made 2 years later. Calculate the APR

Finance: APR

Example 22: Oxford p62

A loan was taken out at APR 30%. £1,000 was repaid after 1 year, a further £1,000 was repaid after 2 years and a final payment of £1,404 was made after 3 years. How much was the loan for?

Test Yourself 22:

A loan was taken out at APR 18.9%. £1,500 was repaid after 1 year, a further £1,600 was repaid after 2 years and a final payment of £2,300 was made after 3 years. How much was the loan for?

Example 23: Oxford p62

Rachel borrows £4,000. A lender offers APR of 16.26%, with two equal annual repayments. Calculate the value of the repayments

Test Yourself 23: Oxford p62

Rachel borrows £4,000. A different lender offers APR of 15%, with two equal annual repayments. Calculate the value of the repayments

Finance: APR

The formula for APR (given in the formula book) is:

$$C = \sum_{k=1}^{m} \left(\frac{A_k}{(1+i)^{t_k}} \right)$$

where C is the amount borrowed, m is the number of repayments, i is the APR, A_k is the amount of the k^{th} repayment and t_k is the amount of time, in years, between the start of the loan and the k^{th} repayment

Example 24: Collins p66

You borrow £400 from a money lender to be paid back in five equal monthly instalments at an APR of 1509%. How much are the monthly instalments?

Test Yourself 24:

You borrow £120 from a money lender to be paid back in four equal monthly instalments at an APR of 1783%. How much are the monthly instalments?

Finance: APR

The formula for APR (given in the formula book) is:

$$C = \sum_{k=1}^{m} \left(\frac{A_k}{(1+i)^{t_k}} \right)$$

where C is the amount borrowed, m is the number of repayments, i is the APR, A_k is the amount of the k^{th} repayment and t_k is the amount of time, in years, between the start of the loan and the k^{th} repayment

Example 25:

A loan of £7000 is repaid in annual instalments of £1000, £2000, £3000 and £4000.
a) Show that the APR lies between 12.6% and 13%
b) Find the APR correct to 1 decimal place.

Test Yourself 25:

A borrower repays a debt of £4000 in 2 annual instalments of £2300.
a) Show that the APR lies between 8% and 12%
b) Find the APR to the nearest %.

Taxation

Starter

1. Put the data below into a back to back stem and leaf diagram:

Number of rainy days in a year, Texas cities:
67, 69, 84, 73, 77, 79, 63, 49, 96, 06, 63, 52, 105, 59, 82, 90, 79, 71
Number of rainy days in a year, California cities:
37, 29, 90, 117, 44, 32, 35, 35, 93, 75, 58, 42, 62, 67, 31, 45

2. Comment on the data from question 1.

3. A bank advertises a loan of £10,000 with two yearly repayments of £5,800 and £6,400. It claims the APR is 14.1%. Check whether this claim is accurate.

4. There are approximately 1,980 students at Godalming college, around 400 of which are taking a maths course. 75 students are sampled in a questionnaire. If they are stratified by subject, how many maths students will be sampled?

- The UK government collects money through taxation. The three main sources of revenue for the government are **income tax**, **national insurance**, and **value added tax** (**VAT**).
- When we buy certain goods and services the UK government adds a tax to the cost. This tax is called **value added tax** (**VAT**).
- The price before VAT is called the **net** price, when VAT is added it is called the **gross** price.
- The standard rate of VAT is 20%, but for some fuel and health related goods it is 5% and it is 0% for most foot and children's clothes.

Example 6:

a) Samantha is buying a buttonhole to wear at a friend's wedding. The price is £4.00 plus VAT at the standard rate. Calculate the price of the buttonhole

b) Sadiq books a driving test costing £62. The price includes VAT at 20%. Calculate the amount of VAT included in the test price.

Taxation

Your Turn 6:

a) Victoria is buying a pair of jeans. They cost £75 before VAT is added, what is the price after VAT?

b) A game costs £45 after VAT. What is the original cost?

- The UK government collects money through taxation. The three main sources of revenue for the government are **income tax**, **national insurance**, and **value added tax** (**VAT**).
- Income tax is a charge on personal incomes each year the personal tax allowances, income tax rates and tax bands are set by the government.
- Most people have a personal allowance. This is the amount of tax free income you are allowed each year. The personal allowance varies each year. For 2020 to 2021 it was £12,500.
- The amount of income tax you pay depends on how much taxable income you have above your personal allowance. There are different rates of income tax depending on what your income is.
- This table shows the income tax rates on taxable bands for 2020 to 2021:

Rates	Taxable income (after allowance)
Basic: 20%	Up to £37 500
Higher: 40%	Over £37 500
Additional: 45%	Over £150 000

- To calculate your income tax:
  - Find your taxable income by subtracting your personal allowance from your annual gross income.
  - You pay income tax at 20% on the first £37 500 of your taxable income.
  - You pay income tax at 40% on your taxable income over £37 500
  - You pay income tax at 45% on your taxable income over £150 000

Taxation

- The personal allowance for 2020 to 2021 was £12,500.
- This table shows the income tax rates on taxable bands for 2020 to 2021:

Rates	Taxable income (after allowance)
Basic: 20%	Up to £37 500
Higher: 40%	Over £37 500
Additional: 45%	Over £150 000

Example 7:

Calculate the income tax paid on a gross annual salary of:

a) £10,000
b) £25,000
c) £45,000
d) £200,000

Taxation

- The personal allowance for 2020 to 2021 was £12,500.
- This table shows the income tax rates on taxable bands for 2020 to 2021:

Rates	Taxable income (after allowance)
Basic: 20%	Up to £37 500
Higher: 40%	Over £37 500
Additional: 45%	Over £150 000

Your turn 7:

Calculate the income tax paid on a gross annual salary of:

a) £12,000
b) £30,000
c) £70,000
d) £185,000

Taxation

- The UK government collects money through taxation. The three main sources of revenue for the government are **income tax**, **national insurance**, and **value added tax** (**VAT**).

- National Insurance (NI) is another charge on a persons salary. It goes towards thing like the NHS, state pension, statutory sick pay and jobseeker's allowance.

- This table shows the NI rates for 2020 to 2021:

Percentage NI due	Minimum monthly income	Maximum monthly income	Minimum yearly income	Maximum yearly income
0%		Up to £792		Up to £9 500
12%	£792.01	£4 167	£9 500.01	£50 000
2%	Above £4 167		Above £50 000	

- Examples
  - A person who had a monthly income of £1800 paid 12% on the amount above £792
  - A person who had a yearly income of £62 000 paid 12% on the amount between £9500 and £50 000 plus 2% of the amount above £50 000

Example 8:

Calculate the net pay, after income tax and national insurance has been deducted, for the following gross annual salaries:

a) £10,000
b) £25,000
c) £45,000
d) £200,000

Taxation

This table shows the NI rates for 2020 to 2021:

Percentage NI due	Minimum monthly income	Maximum monthly income	Minimum yearly income	Maximum yearly income
0%		Up to £792		Up to £9 500
12%	£792.01	£4 167	£9 500.01	£50 000
2%	Above £4 167		Above £50 000	

Your turn 8:

Calculate the net pay, after income tax and national insurance has been deducted, for the following gross annual salaries:

a) £12,000
b) £30,000
c) £70,000
d) £185,000

Taxation

- Another possible deduction is student loan. If you have a student loan you make repayments when your gross income is greater than the threshold.
- You pay 9% of the amount of your income which is above the threshold.
- The threshold depends upon when you started university:

Started University	Threshold
Before 1/9/2012	£19 390
On or after 1/9/2012	£26 575

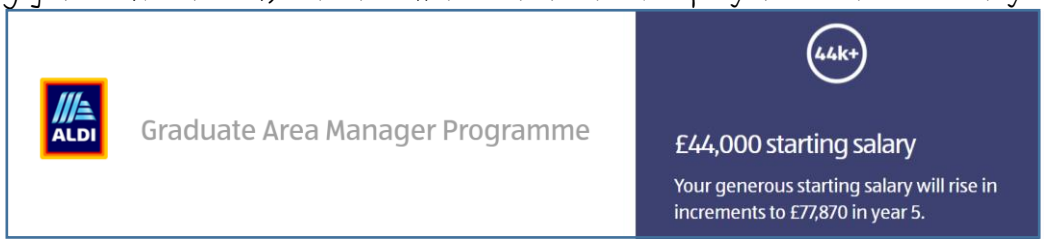
Example 8:

Karl started university in 2011, and took out a student loan.
In the year 2020 – 21, Karl earned a gross salary of £3 756 per month. Calculate his monthly net pay.

Taxation

Your Turn 9:

Juno started university in 2017 and took out a student loan. She sees the following job advertised, what will her take-home pay be in her first year?



Exchange Rates

Starter

1. What is the monthly income of someone who earns £68,500 p.a. after tax and national insurance have been deducted?

2. Which savings account would be better to invest your money in? One which pays a nominal interest rate of 3.2%, compounded weekly, or one which pays out a monthly interest rate of 0.35%?

3. Frances spends £258 on items for work. She can claim back the VAT of 20%. How much will she receive back?

4. What is the mean average of visitors to the five most visited US National Parks? Write down the formula that would give the answer on the spreadsheet.

	A	B
1	**National Park**	**Visits per year**
2	Great Smoky Mountains	11,421,200
3	Grand Canyon	6,380,495
4	Rocky Mountain	4,590,493
5	Zion	4,320,033
6	Yellowstone	4,115,000
7	Average	

- A country's currency has a value that fluctuates relative to other currencies.
- Banks buy and sell currencies from each other and these trades set a market price for each currency.
- The table shows some currency exchange rates (from John Lewis in Oct 2020) compared to the British pound:

Currency	Sell Rate	Buy Rate
Euro (EUR)	1.0897	1.221
Australian Dollar (AUD)	1.7944	2.0395
Brazilian Real (BRL)	6.9645	8.2947
Canadian Dollar (CAD)	1.6712	1.9053
Chinese Yuan Renminbi (CNY)	8.2191	9.9009
Croatian Kuna (HRK)	8.0149	9.4079
Indonesian Rupiah (IDR)	17651.3749	21815.8735
Kenyan Shilling (KES)	133.1837	160.4538
South African Rand (ZAR)	20.6486	23.8934
US Dollar (USD)	1.2922	1.4457

Exchange Rates

Example 10:

Shahin is going on holiday to Europe. He takes out £500 worth of Euros and returns from holiday with €45.
a) How many Euros does he receive?
b) If he changes his Euros back into pounds, how much will he get?

Your turn 10:

Jimmy planned a holiday to Croatia. He took out £600 of Croatian Kuna. A family emergency means he can't go, so he gets the money exchanged back into pounds. How much does he lose?

Ricardo has just moved to the UK from Brazil. He exchanges 5,000 Real into pounds. How many pounds does he receive?

The iPhone X costs $999 in the US, and £999 in the UK. If Charlie uses cash to buy one in America, how much would she save compared to buying in the UK?

Inflation

Godalming College Maths

- **Consumer Price Index (CPI).** Each month the UK government Office for National Statistics (ONS) publishes the latest Consumer Price Index (CPI).

- The CPI is calculated using a hypothetical shopping basket of around 700 goods and services on which people typically spend their money. As the prices of individual products vary, so does the total cost of the basket.

- The CPI measures price changes, not price levels. It is therefore expressed in terms of the comparison of prices relative to a base year, when the index was given a value of 100. The base month in the table below is July 2015.

Year	Jan	Feb	Mar	Apr	May	Jun	Jul	Aug	Sep	Oct	Nov	Dec
2020	108.2	108.6	108.6	108.5	108.5	108.6	109.1	108.6				
2019	106.4	106.8	107.0	107.6	107.9	107.9	107.9	108.4	108.5	108.3	108.5	108.5
2018	104.4	104.9	105.0	105.4	105.8	105.8	105.8	106.5	106.6	106.7	107.0	107.1
2017	101.4	102.1	102.5	102.9	103.3	103.3	103.2	103.8	104.1	104.2	104.6	104.9
2016	99.5	99.8	100.2	100.2	100.4	100.6	100.6	100.9	101.1	101.2	101.4	101.9
2015	99.3	99.5	99.7	99.9	100.1	100.2	100.0	100.3	100.2	100.3	100.3	100.3
2014	99.0	99.5	99.7	100.1	100.0	100.2	99.9	100.2	100.3	100.4	100.1	100.1
2013	97.1	97.8	98.1	98.3	98.5	98.3	98.3	98.7	99.1	99.1	99.2	99.6
2012	94.6	95.1	95.4	96.0	95.9	95.5	95.6	96.1	96.5	97.0	97.2	97.6
2011	91.3	92.0	92.2	93.2	93.4	93.3	93.3	93.8	94.4	94.5	94.6	95.1
2010	87.8	88.2	88.7	89.2	89.4	89.5	89.3	89.8	89.8	90.0	90.3	91.2

Example 11:

What was the rate of inflation in February of this year?

If a loaf of bread cost 65p in March 2010, how much might we expect it to cost today?

Your turn 11:

What was the rate of inflation in August of this year?

If four pints of milk costs £1.09 today, how much might we expect it to cost nine years ago?

Inflation

Godalming College Maths

- **Retail Price Index (RPI).** The RPI is very similar to the CPI but it is based on a different basket of goods and services, which is weighted according to its own unique system. It is considered less reliable than CPI – but many organisations still use it if the rate is favourable to them!

- The series started in 1947, and prices are expressed relative to a base month of January 1987, when the index has a value of 100

Year	Jan	Feb	Mar	Apr	May	Jun	Jul	Aug	Sep	Oct	Nov	Dec
2020	290.6	292.0	292.6	292.6	292.2	292.7	294.2	293.3	294.3			
2019	283.0	285.0	285.1	288.2	289.2	289.6	289.5	291.7	291.0	290.4	291.0	291.9
2018	276.0	278.1	278.3	279.7	280.7	281.5	281.7	284.2	284.1	284.5	284.6	285.6
2017	265.5	268.4	269.3	270.6	271.7	272.3	272.9	274.7	275.1	275.3	275.8	278.1
2016	258.8	260.0	261.1	261.4	262.1	263.1	263.4	264.4	264.9	264.8	265.5	267.1
2015	255.4	256.7	257.1	258.0	258.5	258.9	258.6	259.8	259.6	259.5	259.8	260.6
2014	252.6	254.2	254.8	255.7	255.9	256.3	256.0	257.0	257.6	257.7	257.1	257.5
2013	245.8	247.6	248.7	249.5	250.0	249.7	249.7	251.0	251.9	251.9	252.1	253.4
2012	238.0	239.9	240.8	242.5	242.4	241.8	242.1	243.0	244.2	245.6	245.6	246.8
2011	229.0	231.3	232.5	234.4	235.2	235.2	234.7	236.1	237.9	238.0	238.5	239.4
2010	217.9	219.2	220.7	222.8	223.6	224.1	223.6	224.5	225.3	225.8	226.8	228.4

Example 12:

Holly has agreed to rent a flat for £1250 per month, her contract started in March 2019. In the contract it states that her rent will increase each year by the RPI + 1%, what did the cost of rent change to in March 2020?

Your turn 12:

National Savings and Investments used to run an account which offered interest of RPI +0.5%. If Zoran had £1,500 in his account between Feb 2012 and Feb 2013, how much interest will he accure?

Box Plots

Starter:

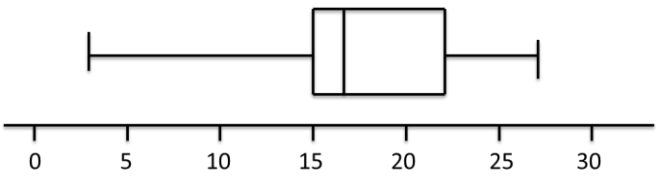
1. Find the median and interquartile ranges for the two classes' test scores:
   Class A 8 54 55 56 58 59 59 59 62 62 63 63 63 64 65
   Class B 25 28 33 37 41 47 59 59 63 69 70 73 76 80

2. Two machines produce piston rings, use the mean and standard deviation to compare them:
   Machine A 20.4 mm, 20.4 mm, 20.3 mm, 20.2 mm, 20.4 mm, 20.1 mm, 20.5 mm
   Machine B 20.2 mm, 20.3 mm, 20.3 mm, 20.4 mm, 20.3 mm, 20.3 mm, 20.0 mm

3. What is the monthly take home pay for someone with a salary of £64,568 per year? They have a student loan, and begun university in Sept 2011.

4. Calculate the APR on a loan of £550, paid back in a single repayment of £770 after two months.

Box plots allow us to visually represent the distribution of data:



Example 9A Oxford 20:

Using the values from Q1 in the starter, draw box and whisker plots, and compare the classes' test results.

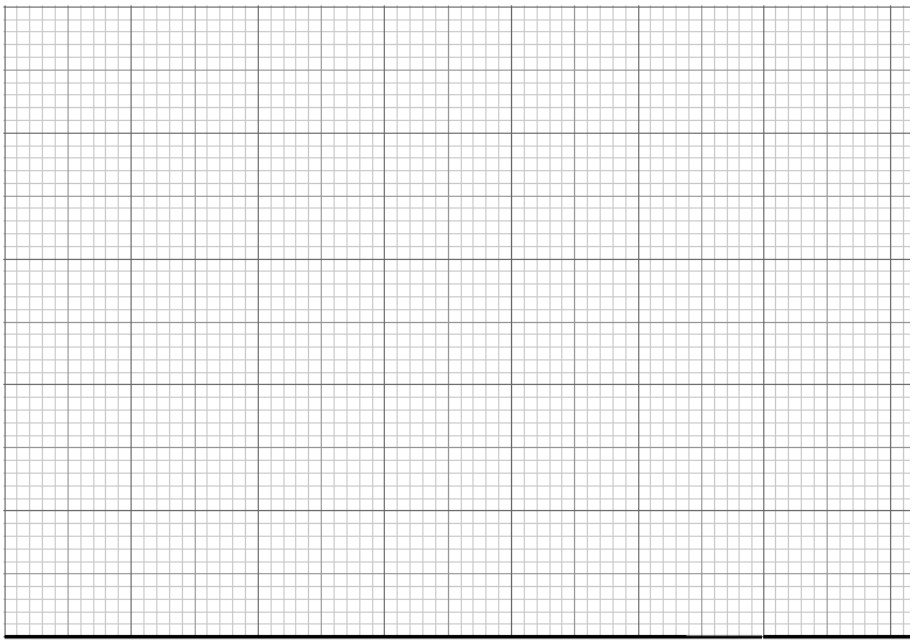
Box Plots

Example 9B Oxford 20:

The frequency table shows the number of goals scored by Manchester United and Manchester City in matches in the 2013-14 season

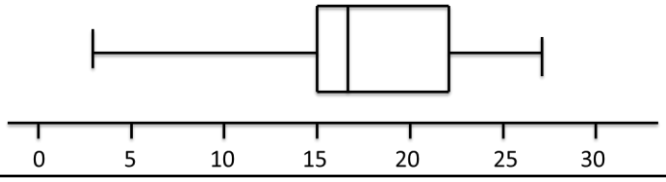
No. of goals	0	1	2	3	4	5	6	7
No. of matches (Man U)	9	9	9	7	4	0	0	0
No. of matches (Man C)	4	5	10	8	6	2	2	1

Using the data, draw box and whisker plots, and compare the two teams.



Box Plots

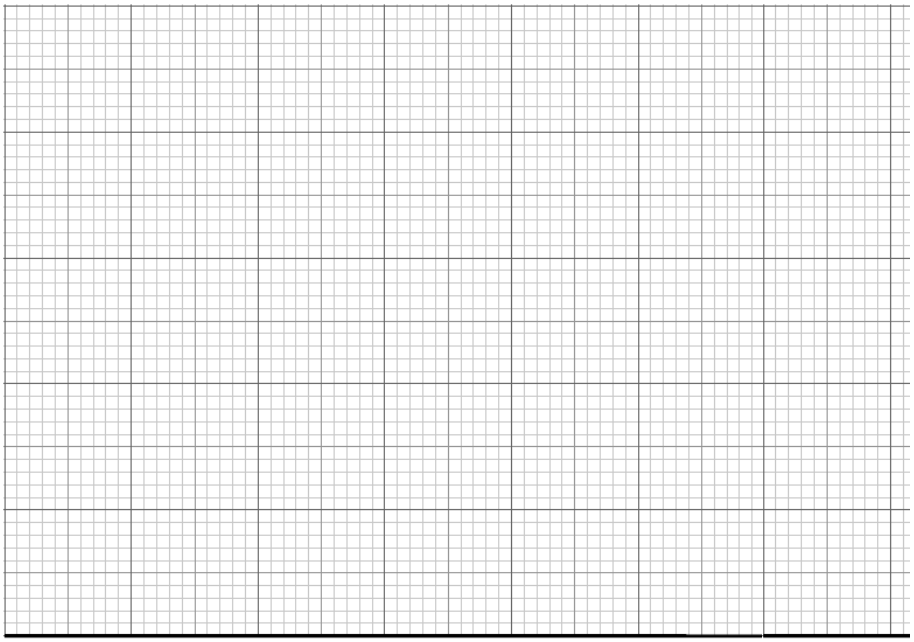
Box plots allow us to visually represent the distribution of data:



Your Turn 9 Oxford 20:

Using a box and whisker diagram, compare the times taken by two groups of apprentices to complete a task:

Group P (mins) 32 18 23 19 43 27 21 35 26 19 23
Group Q (mins) 31 24 37 29 20 35 69 21 25 33



Cumulative Frequency diagrams

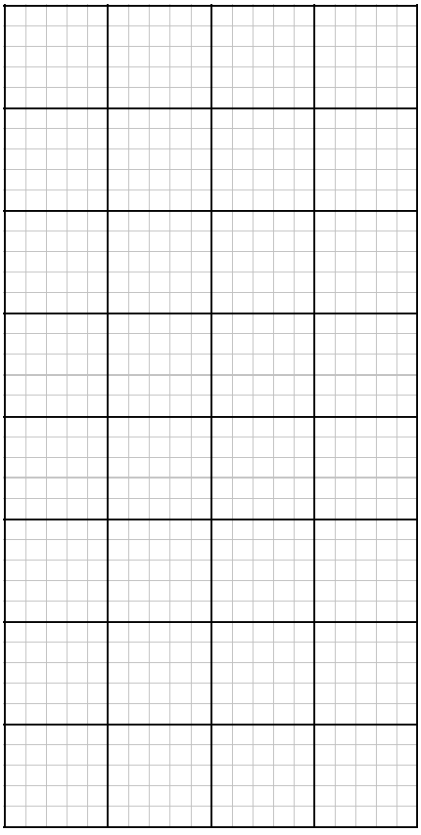
Godalming College Μαθσ

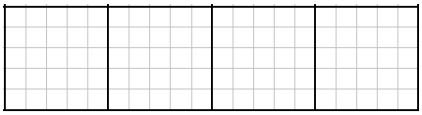
These graphs are intended to show the running total of people/things up to a particular value, and are particularly useful in estimating the median and quartiles.

Example 10:

The data in the table shows the heights in metres, of 80 giraffes.

a) Draw a cumulative frequency diagram for the data
b) Using the cumulative frequency diagram, estimate the median and quartiles
c) Draw a box plot to represent the diagram





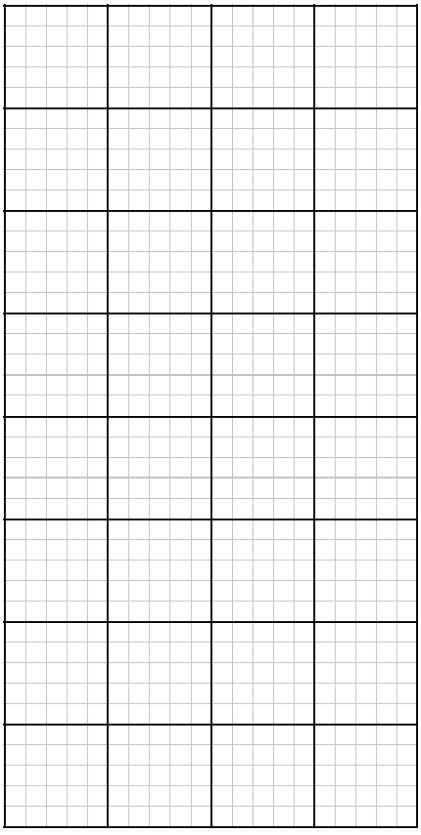
Height, h (m)	Frequency	
$4.6 \leq h < 4.8$	4	
$4.8 \leq h < 5.0$	7	
$5.0 \leq h < 5.2$	15	
$5.2 \leq h < 5.4$	33	
$5.4 \leq h < 5.6$	17	
$5.6 \leq h \leq 5.8$	4	

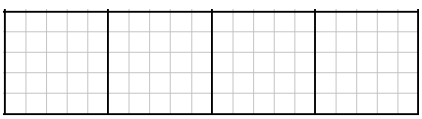
Cumulative Frequency diagrams

- These graphs are intended to show the running total of people/things up to a particular value, and are particularly useful in estimating the median and quartiles.
- You plot the **upper** value of the group against the cumulative frequency.
- You can find the quartiles and produce a box plot:

Example 10:

The data in the table shows the heights in metres, of 80 giraffes.
a) Draw a cumulative frequency diagram for the data
b) Using the cumulative frequency diagram, estimate the median and quartiles
c) Estimate how many giraffes are taller than 5.5 m
d) Draw a box plot to represent the diagram





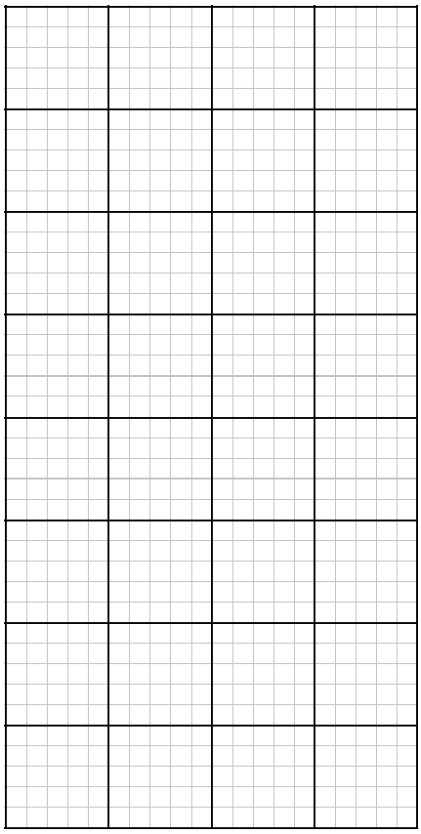
Height, h (m)	Frequency	
$4.6 \leq h < 4.8$	4	
$4.8 \leq h < 5.0$	7	
$5.0 \leq h < 5.2$	15	
$5.2 \leq h < 5.4$	33	
$5.4 \leq h < 5.6$	17	
$5.6 \leq h \leq 5.8$	4	

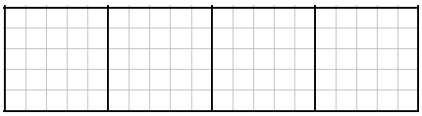
Cumulative Frequency diagrams

Your Turn 10:

The data in the table shows times some professional athletes took to run 100m.

a) Draw a cumulative frequency diagram for the data
b) Using the cumulative frequency diagram, estimate the median and quartiles
c) Estimate how many people took more than 10 seconds to run 100m
d) The top 10 runners qualified for the next round, estimate the qualification time?
e) Draw a box plot to represent the diagram





Time (s)	Frequency	C. F.
$9.6 < t \leq 9.7$	1	
$9.7 < t \leq 9.9$	4	
$9.9 < t \leq 10.05$	10	
$10.05 < t \leq 10.2$	17	

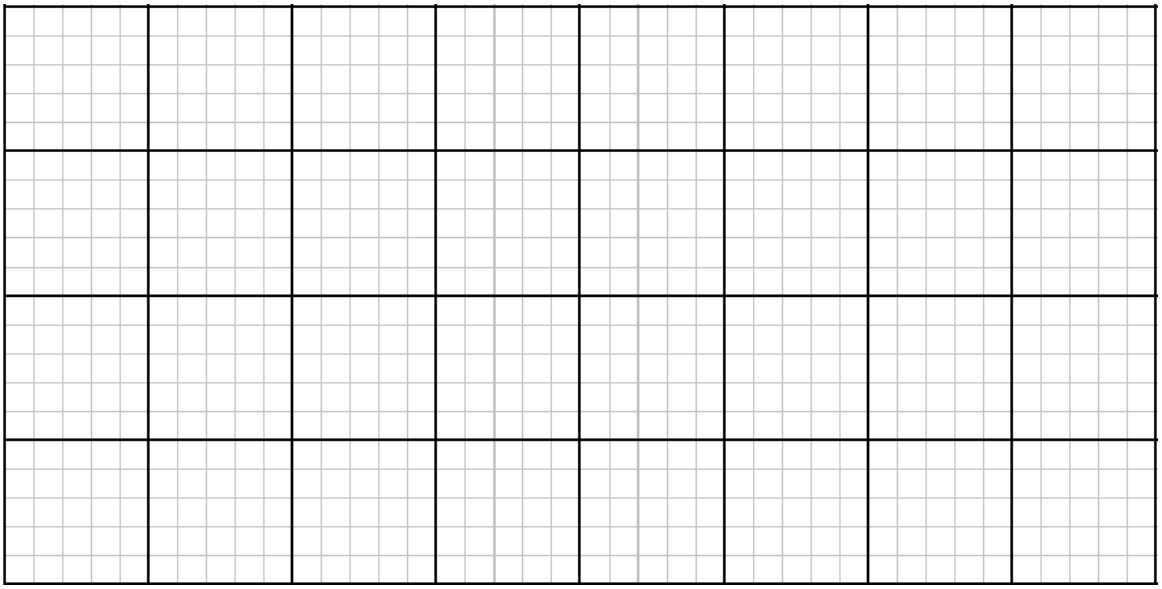
Histograms

- Used for **continuous** data.
- Data divided into (potentially uneven) intervals.
- Height of bars is **frequency density.** Frequency given by area of bars.
- No gaps between bars.

Example 11:

Complete the table below and draw a histogram representing the data:

Time taken (t seconds)	Frequency	Frequency Density
$10 < t \leq 30$	5	
$30 < t \leq 35$	4	
$35 < t \leq 40$	8	
$40 < t \leq 50$	27	
$50 < t \leq 70$	24	



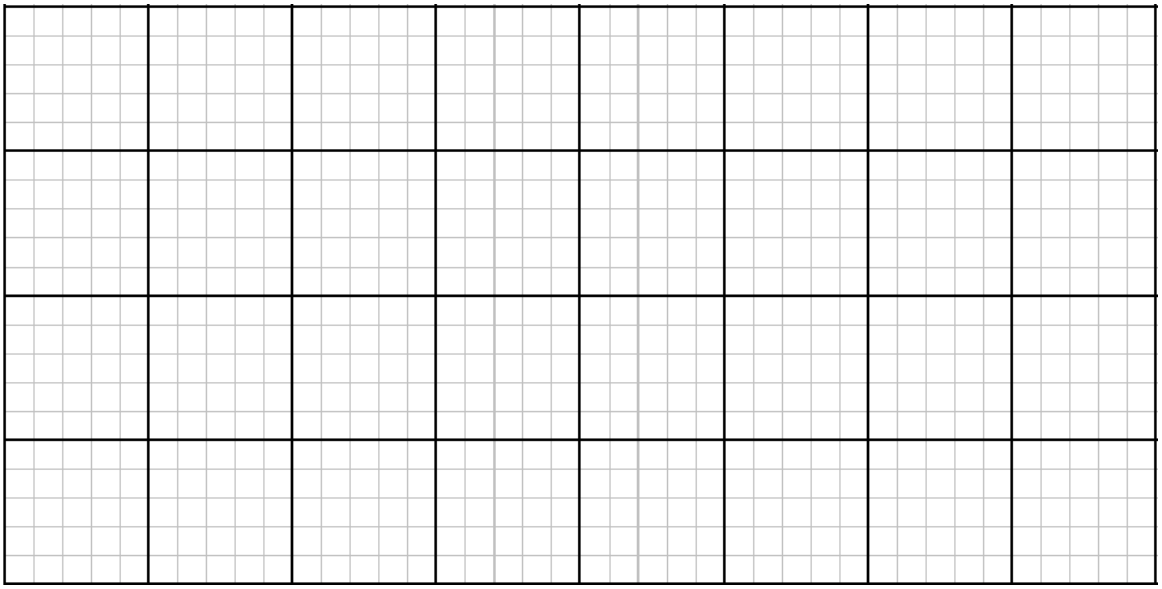
Histograms

Remember Frequency Density = Frequency ÷ Class Width

Your Turn 11:

Complete the table below and draw a histogram representing the data:

Time taken (t seconds)	Frequency	Frequency Density
$0 < t \leq 20$	30	
$20 < t \leq 30$	40	
$30 < t \leq 55$	25	

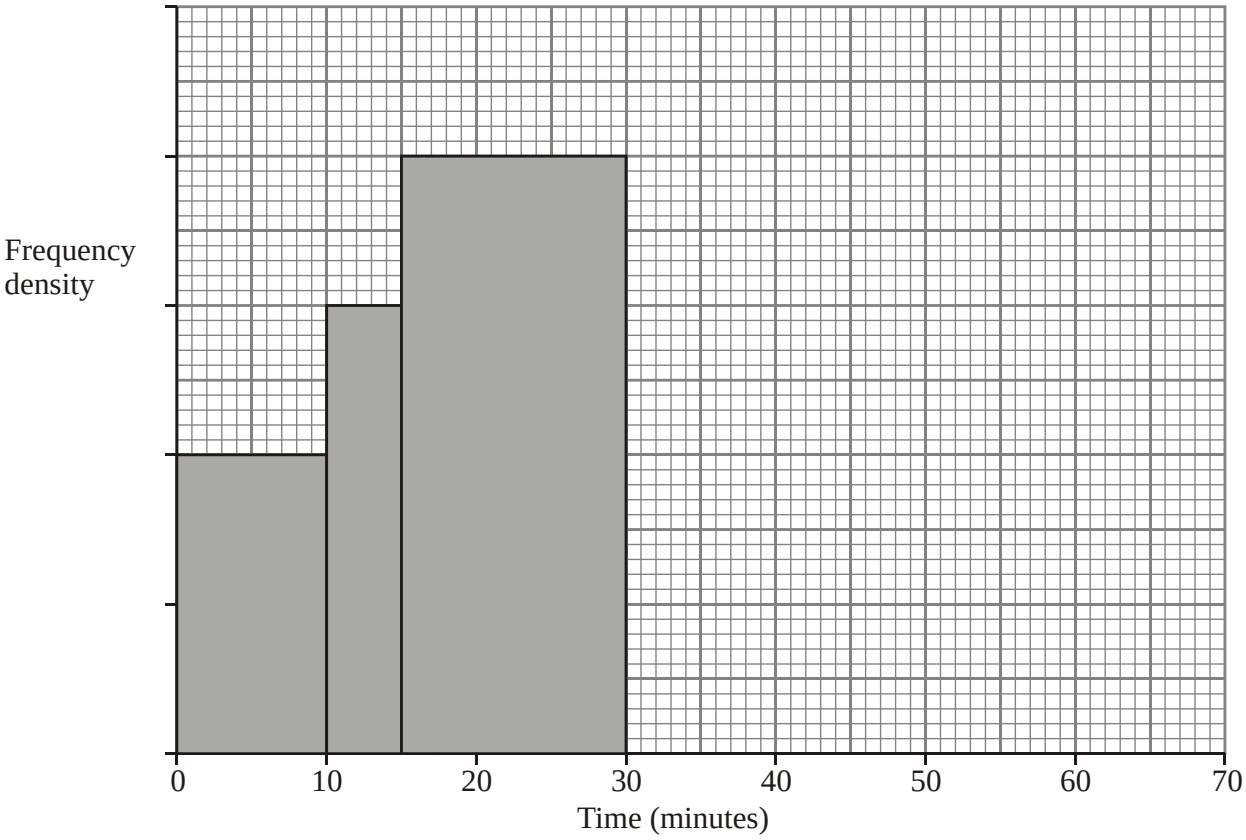


Histograms

Example 12:

Complete the table and the histogram below:

Time (m)	Frequency	
$0 < t \leq 10$	20	
$10 < t \leq 15$		
$15 < t \leq 30$		
$30 < t \leq 50$	62	
$50 < t \leq 60$	23	

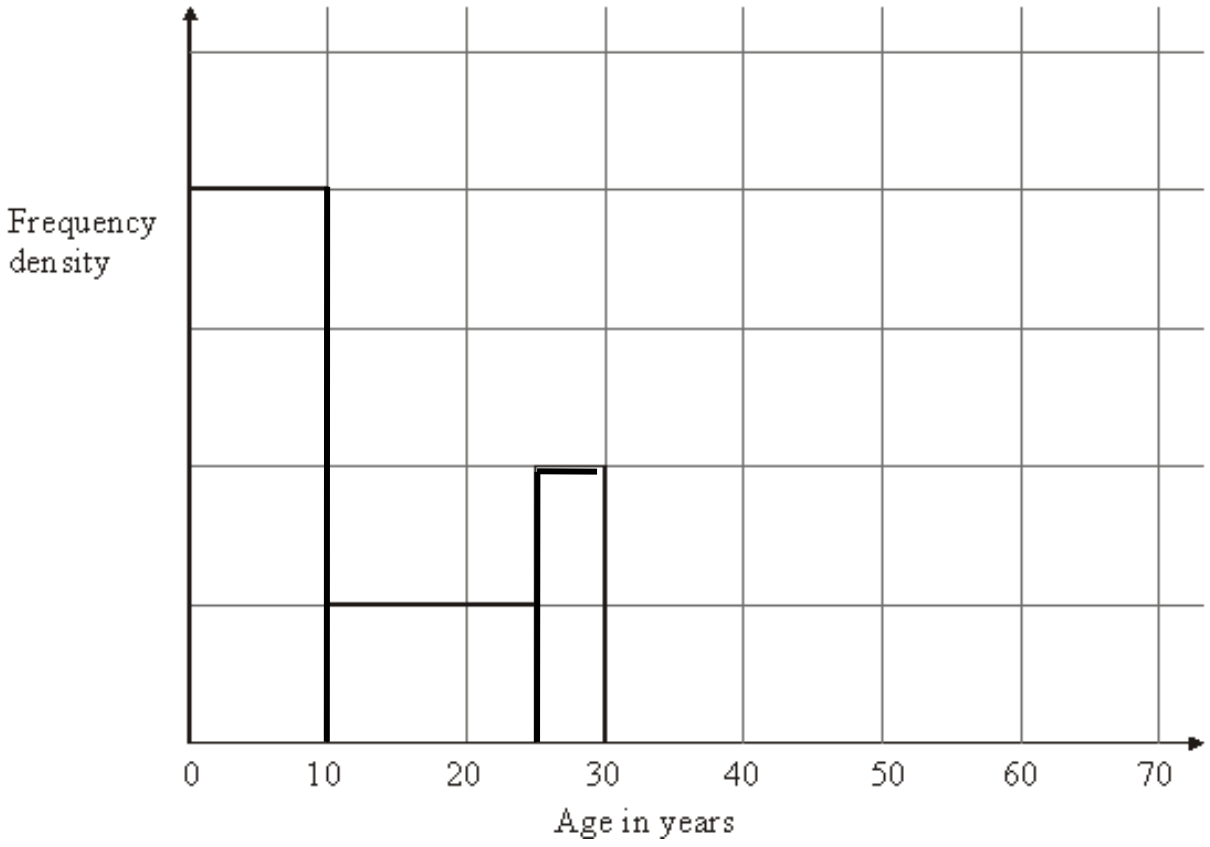


Histograms

Your Turn 12:

Complete the table and the histogram below:

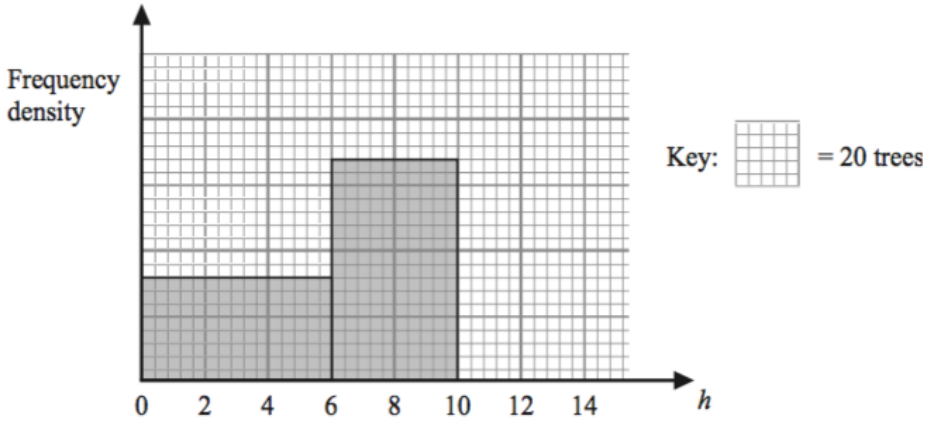
Age (x) in years	Frequency
$0 < x \leq 10$	160
$10 < x \leq 25$	
$25 < x \leq 30$	
$30 < x \leq 40$	100
$40 < x \leq 70$	120



Histograms

Example 13:

The unfinished histogram shows information about the heights, h metres, of some trees:



Calculate an estimate for the number of trees with heights in the interval $4.5 < h \leq 10$

Histograms

Your Turn 13

The incomplete table shows information about the times, in minutes, that runners took to complete a race.

Runners who achieved a time between 37 and 48 minutes to complete the race were each awarded a silver medal.

a) Complete the table and histogram
b) calculate an estimate of the number of runners awarded silver medals.

Time (t minutes)	$30 \leqslant t < 35$	$35 \leqslant t < 40$	$40 \leqslant t < 50$	$50 \leqslant t < 60$	$60 \leqslant t < 80$
Number of runners	12	20		12	16