Godalming College

Μαθσ

AQA Level 3 Certificate in Mathematical Studies (Core Maths)

Paper 2B Content

Name _____

Critical Path Analysis

Starter

1. Jack wants to do a random sample of people attending a football match. How might he carry one out?

2. Dog House kennels weighed a certain dog breed, and put the results in a frequency table. Find the mean and standard deviation:

Weight (Kg)	0≤W<5	5≤W<10	10≤W<15	15≤W<20	20≤W<25
f	34	53	16	10	5

3. Barker's Lodge kennels also weighed their dogs and found theirs had a mean weight of 9.6kg and a standard deviation of 3.9. Compare the weights of dogs at the two kennels

4. Kim borrows £750, which she will repay in two equal, monthly, instalments. If the APR is 45%, how much will her repayments be?

Critical Path Analysis can be used for any multi-task complex project to ensure that the entire project is completed in the minimum time.

Example 1 – Precedence Tables:

The first step in scheduling a complex project is to break it down into activities

For example to build an extension to a home:

A Prepare the foundations
B Have foundations passed by inspector
C Obtain bricks
D Erect walls
E Construct roof
F Install plumbing
G Install wiring
H Plaster walls
I Decorate
J Landscape garden

We identify which activities depend on which others being completed first and summarise it in a precedence table:

Activity	Depends On
A	
B	
C	
D	
E	
F	
G	
H	
I	
J	

Critical Path Analysis

Your Turn 1 – Precedence Tables:

A project has been broken down into activities A, B, C, D, E, F, G and H. After a committee meeting the information below was produced. Draw a precedence table that summarises this information.

A and D do not depend on any other activity

- A must be completed before B and C can start
- A, B, C and D must be completed for E to start
- A, B, C, D and E must be completed for F to start
- G is the last activity and can start when A, B, C, D, E and F are completed
- A and B must be completed for H to start

Activity	Depends On
A	
B	
C	
D	
E	
F	
G	
H	

Constructing an Activity Network

Procedure:

1. Draw a START vertex and add a "column" of vertices representing those activities which can be started straight away. Join each vertex back to the START vertex.
2. Add a "column" of vertices which can now be started given the activities which are already on the network. Join each new vertex back to each of the activities that it directly depends on.
3. Repeat step 2 until all activities have been placed on the network.
4. Add a FINISH vertex and join it back to any vertices which have no arc out from them.
5. Label each arc out from a vertex with the activity's duration.

Example 2:

Construct an Activity Network for the project in "Your Turn 1"

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Critical Path Analysis

Your Turn 2A:

Draw an activity network based on the information below:

Task	Duration (hours)	Immediate predecessors
A	3	-
B	4	-
C	6	-
D	5	A
E	1	B
F	6	B
G	7	C, D, E

Your Turn 2B:

Draw an activity network based on the information below:

Task	Duration (days)	Immediate predecessors
A	2	-
B	3	-
C	5	-
D	6	A, B
E	8	C
F	2	C
G	4	D, E

Critical Path Analysis

Starter

1. Oli wants to do some research on the different sizes of leaves in a local woodland. What would be the best sampling method for him to use and how might he carry it out?

2. Complete the table below, what formula would Sina use to find the result in cell D3?

3. What is the AER on a bank account which pays interest of 0.03% each week?

	A	B	C	D
1	Club	Played	Won	Win %
2	Man City	38	32	
3	Liverpool	38	30	
4	Chelsea	38	21	
5	Tottenham	38		60.50%

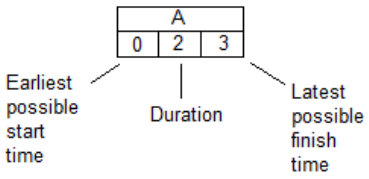
4. What is the APR on a loan of £500, which is repaid in a single payment of £600 after one month?

5. How much money do you estimate is spent in the Godalming College canteen each week? State your assumptions.

Earliest and Latest Start Times

Each activity has an Earliest Start Time and a Latest Start Time, in order that the project can be completed in the minimum time.

We write the earliest and latest start times along with the duration for an activity inside the vertex box.



Finding Earliest Start Times:

Work through the activity network from the START vertex to the FINISH vertex. (This is called a "forward pass")

Formula:

Earliest Start Time = Earliest Start of preceding activity + Preceding activity's duration

Where an activity has more than one preceding activity, use the **highest** value obtained.

Finding Latest Finish Times:

Work through the activity network from the FINISH vertex to the START vertex. (This is called a "backward pass")

Formula:

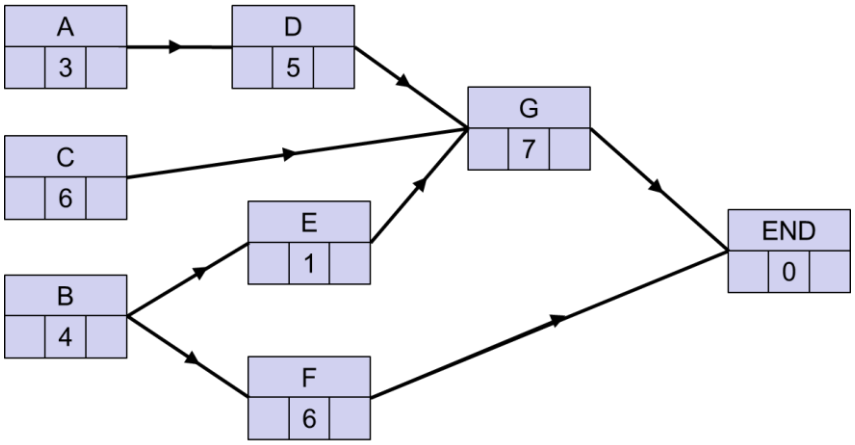
Latest Finish Time = Latest finish of following activity - Following activity's duration

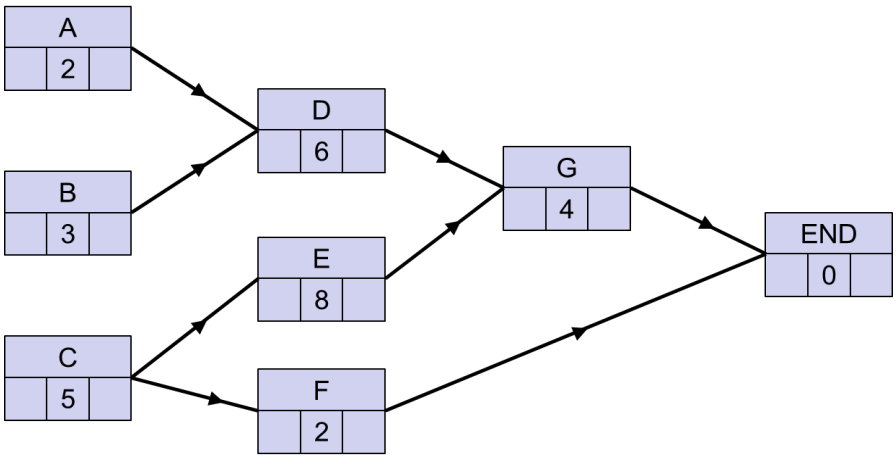
Where an activity has more than one preceding activity, use the **lowest** value obtained.

Critical Path Analysis

Example 3A:

For previous examples (Your Turn 2A and 2B) find the critical activities and the minimum duration of the project.
The activities networks are shown below:





Float Time and Critical Activities

Float Time = Latest Finish Time – Duration – Earliest Start Time

The Float time tells us about the 'flexibility' of an activity.

i) If float = 0, Critical, No flexibility, Has to Start/Finish on time.

ii) If float > 0, Flexible, Don't need to start on time.

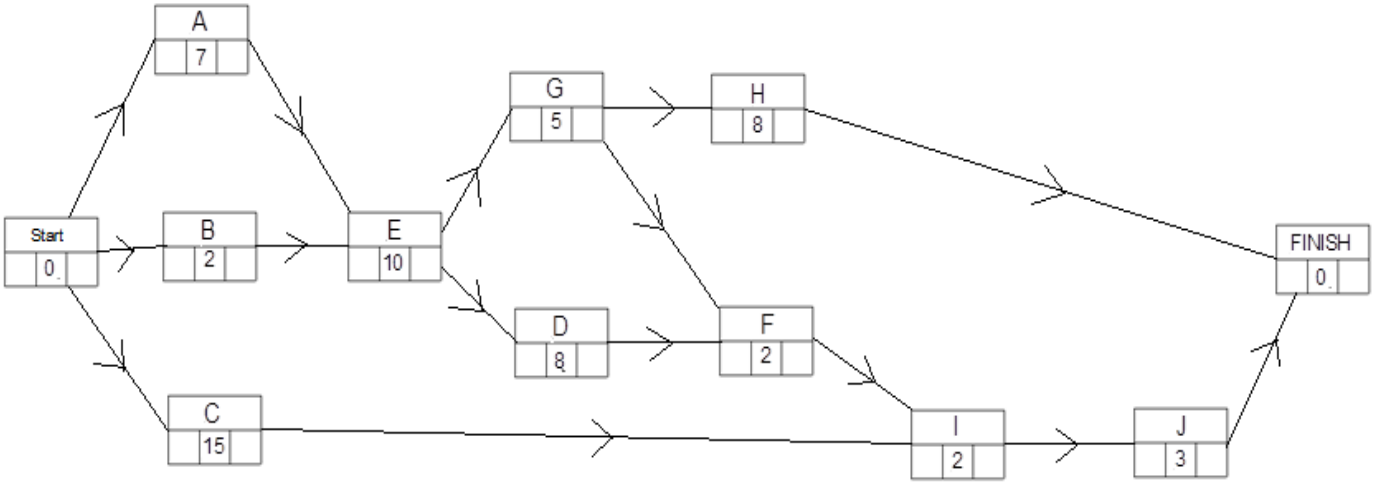
Example 3B:

Calculate the float times for the activities in the projects in Example 3A, and hence write down the critical path(s) for each

Critical Path Analysis

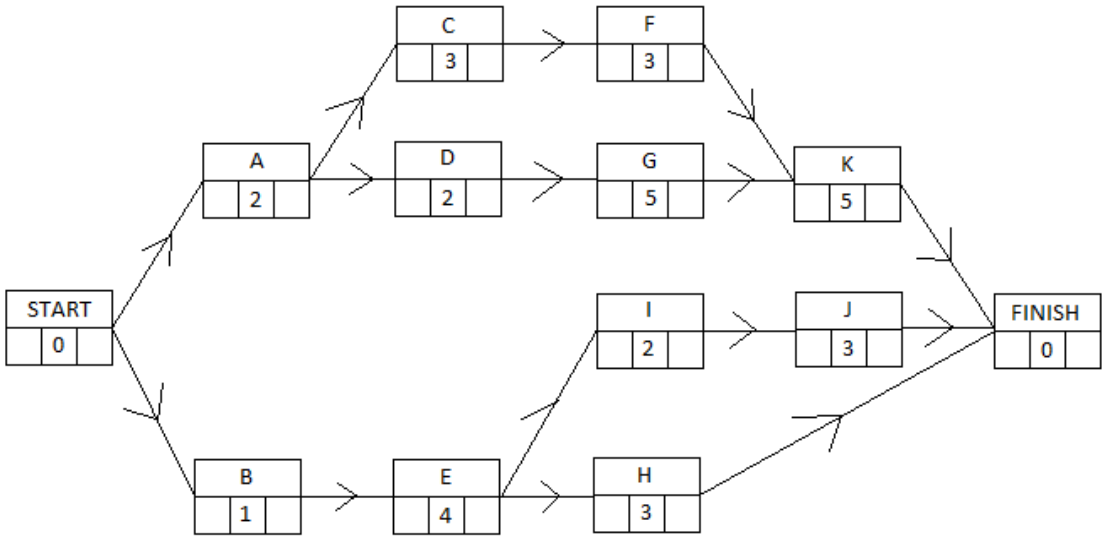
Test Yourself 3A:

Given the activity network below find the earliest start times and latest finish times for each activity. Hence find the critical activities



Test Yourself 3B:

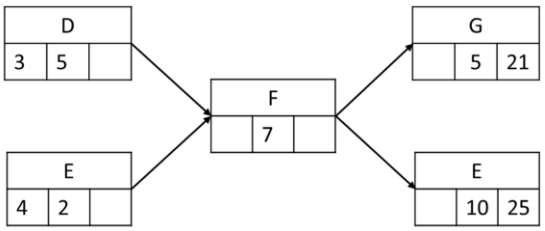
Given the activity network below find the earliest start times and latest finish times for each activity. Hence find the critical activities



Critical Path Analysis

Starter

1. Fill in the blank squares:
   List all the float times and identify the critical activities.



2. Calculate the mean and standard deviation of the 25 employees from company A:
   Company B's employees have a mean commuting time of 19m28s and a standard deviation of 20.6.
   Which commute would you rather have?

Daily commuting time (minutes)	Number of employees
0 to less than 10	4
10 to less than 20	9
20 to less than 30	6
30 to less than 40	4
40 to less than 50	2

3. Which bank account has the best AER? One with a nominal rate of 1.6%, compounded weekly, or one which pays out monthly interest of 0.18%?

Gantt or Cascade Charts

A Gantt or Cascade Chart is an alternative way of displaying the information gained from completion of the activity network.

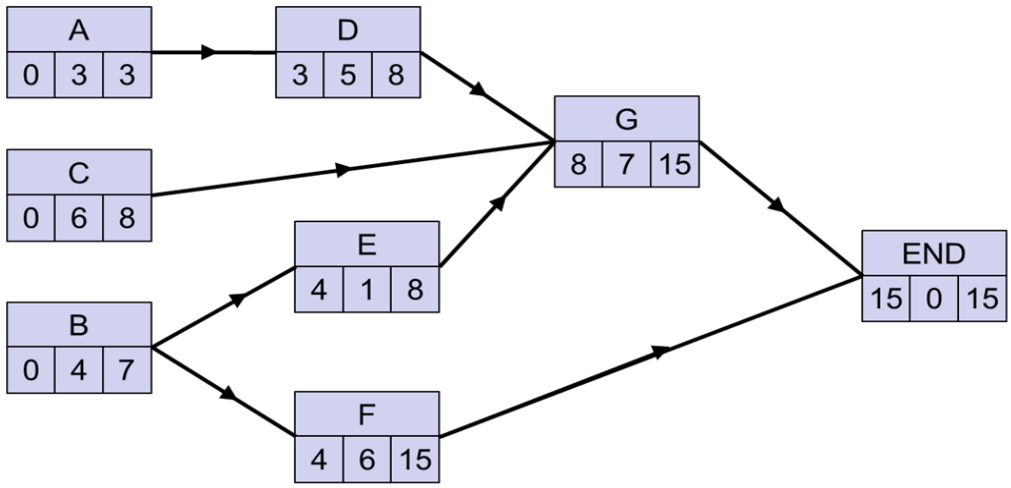
1) For each activity, start the bar at its earliest start time. Bar length = activity duration

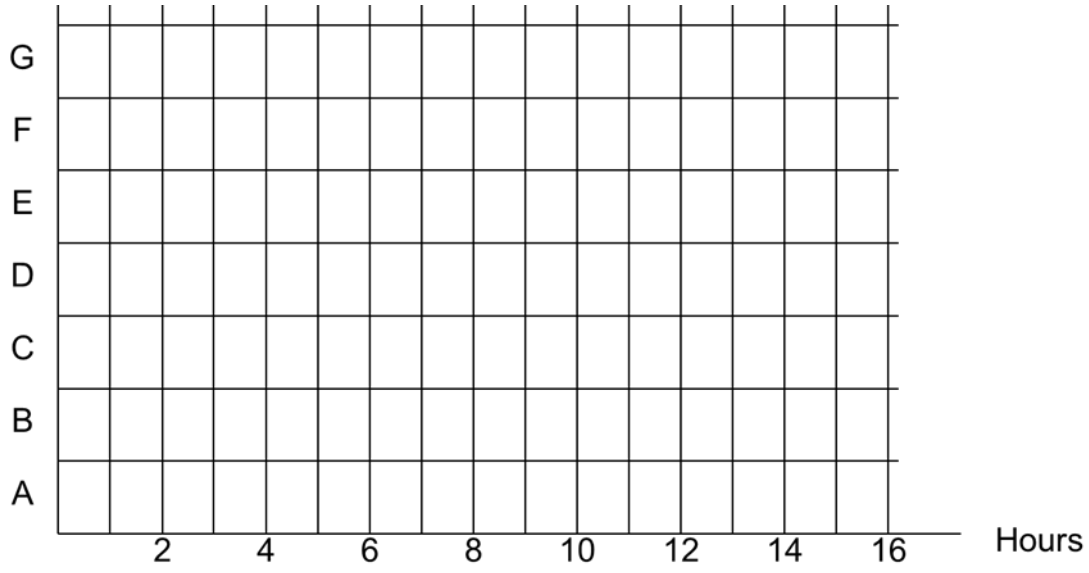
2) Float is shown as a shaded extension to the bar (length of extension = float)

Critical Path Analysis

Example 4A:

Construct a Gantt (cascade) diagram for the project.

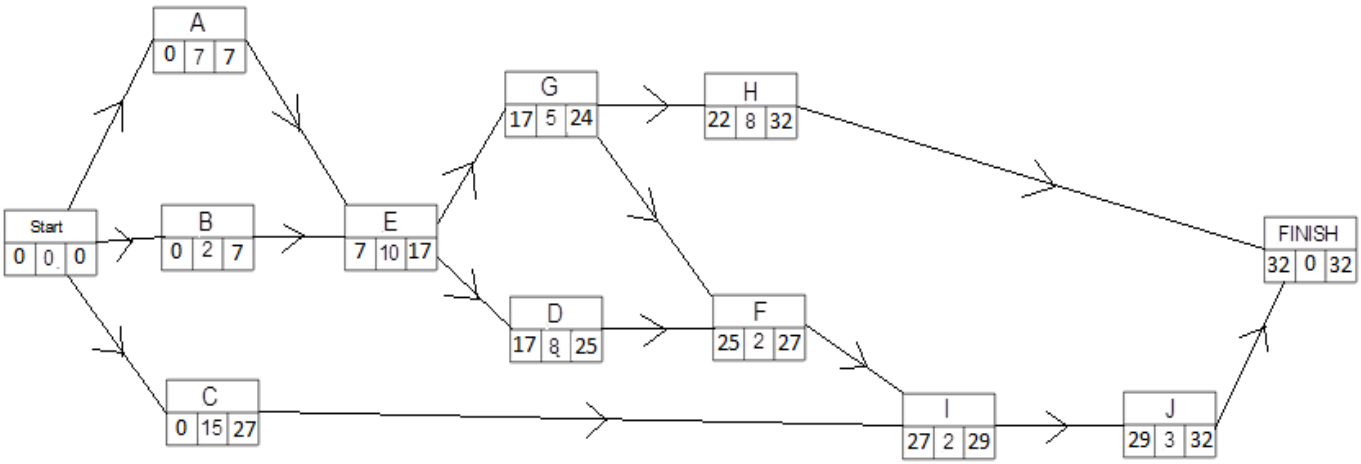


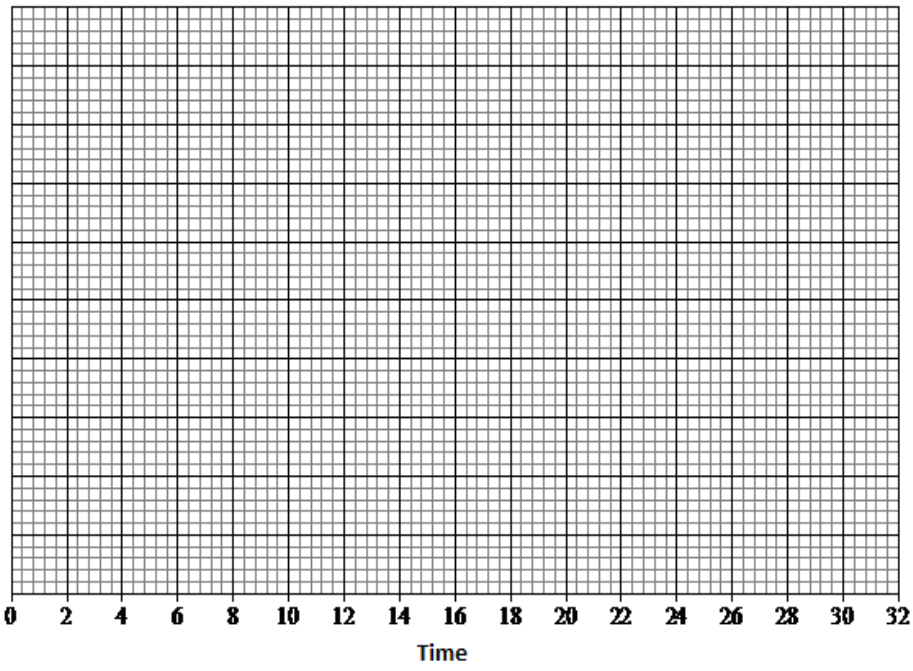


Critical Path Analysis

Your Turn 4:

Construct a Gantt (cascade) diagram for the project.





Critical Path Analysis

Your Turn 5:

a) Construct an activity network for the project represented by the table below
b) Find the earliest start times for each activity
c) Find the latest start times for each activity
d) List the critical activities
e) On the grid on the next page construct a Gantt (cascade) diagram for the project

Activity	Immediate predecessor	Planned duration (weeks)
A: Decide on new system	–	1
B: Prepare ICT control room	A	2
C: Buy hardware (including delivery)	A	5
D: Buy software (including delivery)	A	2
E: Train ICT staff	B, C, D	2
F: Install cabling	C	2
G: Install hardware	E, F	1
H: Install software	G	1
I: Prepare pupil/staff data	A	5
J: Install data	H, I	1
K: Train teaching staff	H	2
L: Test system	J, K	1

Critical Path Analysis

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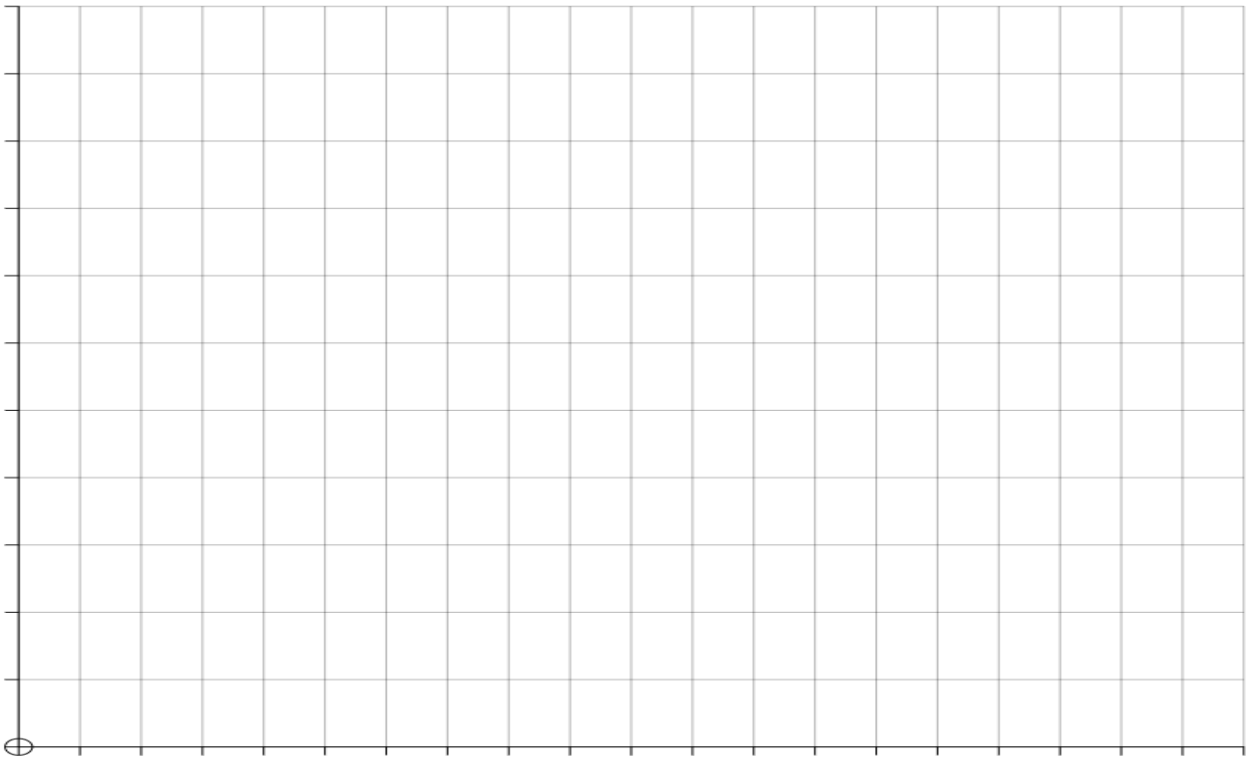
Your Turn 5:



Probability, Venn diagrams

Starter

1. What is the monthly income of someone who earns £211,500 p.a. after tax and national insurance have been deducted?

2. Calculate the mean and standard deviation of the cricket balls in the table.

Diameter (d mm)	Number of cricket balls
$65 < d \leqslant 66$	5
$66 < d \leqslant 67$	9
$67 < d \leqslant 68$	12
$68 < d \leqslant 69$	15
$69 < d \leqslant 70$	10
$70 < d \leqslant 71$	7
$71 < d \leqslant 72$	2
Total	**60**

3. Kevin works in a battery factory. Describe how he might conduct a quota sample to test the lifetime of 5 different types of battery.

4. Calculate the AER on a bank account which has a monthly interest rate of 0.45%. Is it better than an account which has nominal 4% interest, compounded monthly?

5. Estimate the total number of GCSE papers that will be taken by year 11 students this summer, in the UK.

✎ **This section focuses on probability.** Much of it will be familiar from GCSE

Example 1A: Hodder p 218

A raffle is held at a local fundraising event. There are 100 tickets numbered from 1 to 100.

The prizes are:

Number ends in a 7	free drink at a local café
number is a multiple of 25	2 tickets for the local cinema

Jim buys one ticket. What is the probability that is not a winning ticket?

Probability, Venn diagrams

Example 1B: Hodder p 219

An examiner is designing a multiple-choice examination paper and wants to make sure that a candidate who takes random guesses ends up with zero marks. The paper has 20 questions and each question has four possible options. A question answered correctly is to be given 6 marks. Work out how many marks must be deducted for a wrong answer so is 0 marks will be awarded for a candidate who answers the questions randomly

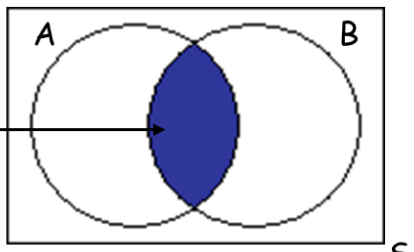
- We can represent probabilities using **Venn diagrams**.
- In a Venn diagram all possible outcomes are listed – so probabilities will add up to 1
- The following notation (used at GCSE) is important:
  - $P(A)$
  - $P(A')$
  - $P(A \cap B)$
  - $P(A \cup B)$

Using Venn Diagrams

The Area in the middle represents the Probability of A and B happening together.

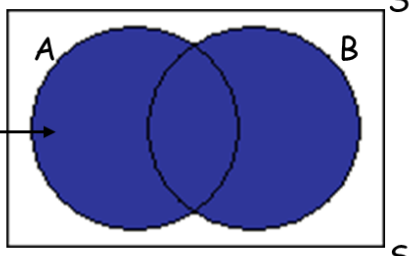
$P(A \cap B)$

'n' → 'and'



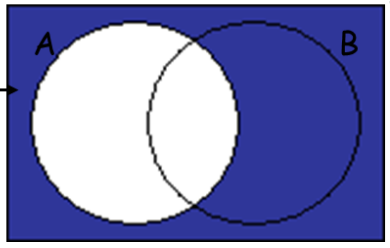
The whole area represents the Probability of A or B happening (or them together).

$P(A \cup B)$



The area outside of A represents the Probability of A not happening.

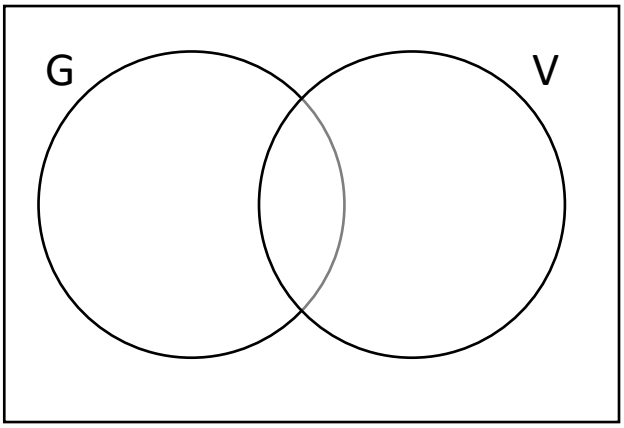
$P(A') = 1 - P(A)$



Probability, Venn diagrams

Example 2: Collins p 213

60 students are discussing music festivals. Of these students 11 have tickets for Glastonbury, 19 have tickets for the V festival, and 6 students have tickets for both festivals. Put this information in the Venn diagram below.

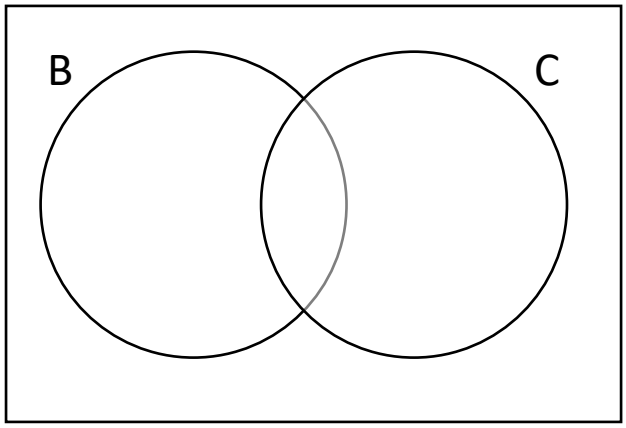


a) Work out:
   i. $P(G')$
   ii. $P(V)$
   iii. $P(G \cap V)$
   iv. $P(G \cup V)$

b) Given that someone has a ticket for Glastonbury, what is the probability they also have a ticket for V Festival?

Test Yourself 2A:

In a class of 30 students, 7 are in the choir, 5 are in the school band and 2 are in both the choir and the band. Draw a Venn diagram to show this information.



a) Work out:
   i. $P(B')$
   ii. $P(C)$
   iii. $P(B \cap C)$
   iv. $P(B \cup C)$

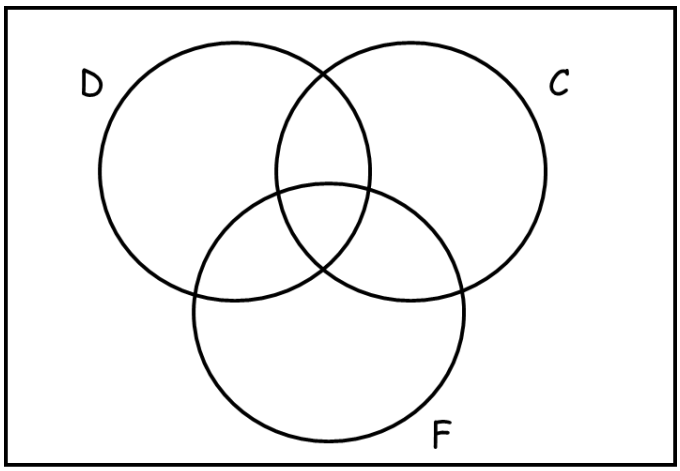
b) Given that someone is in the choir, what is the probability they are in the band?

Probability, Venn diagrams

Test Yourself 2B:

A vet surveys 100 clients. She finds out the following:

25 have dogs
53 have cats
40 have fish
15 have dogs and cats
10 have cats and fish
11 have dogs and fish
7 have dogs, cats and fish



a) Work out:
   i. $P(D)$
   ii. $P(F')$
   iii. $P(D' \cap C' \cap F')$
   iv. $P(C \cup D')$

b) Given that someone has a cat, what is the probability they also have a fish?

Probability, Tree diagrams

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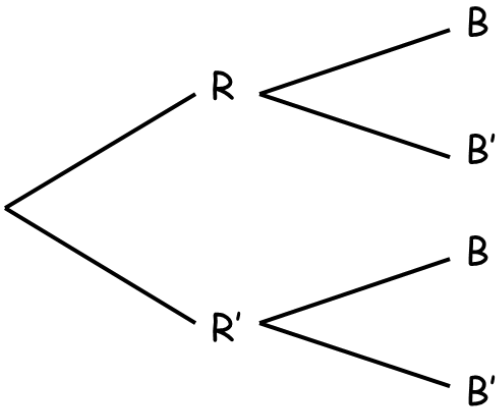
You will have seen **Tree Diagrams** at GCSE level, and they can also be used to represent conditional probabilities.

Example 3:

The number of spectators at an event is dependent on the weather. On a rainy day, the probability of a big turnout is 0.4. However, if it does not rain, there is a probability of 0.9 that there will be a big turnout. The weather forecast gives a 0.75 probability of rain.

a) Show this on the tree diagram:



b) Calculate the following:

i. $P(B \cap R)$
ii. $P(B' \cap R)$
iii. $P(B \cap R')$
iv. $P(B' \cap R')$
v. $P(B)$

Test Yourself 3:

A bag contains 7 green beads and 5 blue beads. A bead is taken at random, the colour recorded and the bead is not replaced. A second is then taken and the colour recorded. Find $P(1\text{ Green} \cap 1\text{ Blue})$.

Probability continued

Starter:

1. How would someone conduct a random sample of people in a block of 100 flats?

2. What is the APR on a loan of £190, repaid in a single instalment of £256 after 9 months?

3. £1 = €1.18. How much would a meal, costing €67.42, be in pounds?

4. The base year for CPI is 2015.
   a) If items have increased in value by 8.5% since then what is the CPI value today?
   b) The index value at the beginning of 2000 was 72.6. What is the percentage change between then and 2015?

5. What is the NI due for someone who earns £221 per week?

- Events are **independent** if one does not affect the other (for example rolling a six on a dice and the weather being sunny)
- Two events are independent if $P(A) \times P(B) = P(A \cap B)$

Example 4A Collins 223:

In a group of 18 sixth formers: 6 are on Facebook, 3 are on Twitter, one student has Facebook and Twitter. 4 students are not on Facebook or Twitter, but are on Snapchat. 6 students are not on any social media.

For this group of students are the events 'being on Facebook' and 'Being on Twitter' independent?

Probability continued

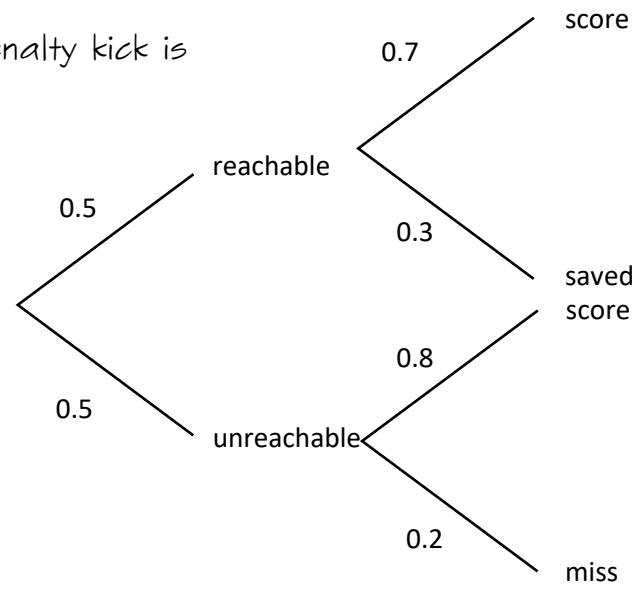
- Events are **independent** if one does not affect the other (for example rolling a six on a dice and the weather being sunny)
- Two events are independent if $P(A) \times P(B) = P(A \cap B)$

Example 4B Oxford 216:

When a penalty kick is taken, a goalkeeper is only able to save the penalty in a reachable zone which covers roughly 75% of the goal. Outside this zone the penalty taker will always score unless they missed the goal entirely. Success rates differ across different leagues and competitions but the probabilities are roughly as shown:

a) What is the probability that that the penalty kick is reachable but a goal is still scored?

b) Is scoring independent of the kick being reachable?



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Probability: Cost-Benefit analysis

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- Questions of this type involve looking at the probability of different events happening and the costs involved in each.
- You are required to work out the various costs and conclude what the best course of action is.

Example 14 Hodder 237: :

Jon has Just bought a new 12 month mobile phone contract for £20 per month.
As part of this he has been provided with a phone with a value of £180.
He has been offered mobile phone insurance for £5 per month that will repair or replace the phone if it gets damaged or lost.
Without insurance he will have to replace the phone himself.
The probability of the phone being damaged or lost is 0.15
Should Jon take out the insurance?

Test Yourself 14 Collins 242

A TV company is producing a new series. The series must be produced on time or the company will be fined £25,000. It is estimated by the TV company that the probability of producing the series on time is 0.84. An insurer offers the TV company insurance costing £5000. If the series is delayed, the insurer will pay the £25,000 fine for the TV company.
a) Should the TV company take out the insurance?
The TV company could buy some improved software at a cost of £8000. This would increase the probability of producing the series on time to 0.95.
a) Should the TV company invest in the improved software?

Probability: Cost-Benefit analysis

- Questions of this type involve looking at the probability of different events happening and the costs involved in each.
- You are required to work out the various costs and conclude what the best course of action is.

Test Yourself 14 Collins 242

When a car is serviced at a garage a courtesy car is often provided for the driver to use while their car is out of use. This is covered by the garage insurance but the driver has to pay the first £500 of any claim on the insurance. The driver has the option of paying £10 to reduce this amount to just £50.
If the probability of the driver having an accident while driving the courtesy car is 0.1, should the driver pay the £10?

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Probability: Cost-Benefit analysis

Μαθσ

- Questions of this type involve looking at the probability of different events happening and the costs involved in each.
- You are required to work out the various costs and conclude what the best course of action is.
- These are similar to what we have done previously – just with more information to work through

Example 15A

An airline has ordered a Boeing 777. The cost of the aeroplane is £390 million.
There is a 45% chance that the aeroplane arrives after the delivery date.
If the delivery date is not met then Boeing will be fined 5% of the value of the aeroplane.

Taking either of the following actions would reduce the risk of delay.

- Paying a different company to deliver the aeroplane would cost £2 million and would reduce the probability of a late delivery to 30%
- Using a more sophisticated tracking system would cost £3.2 million and would reduce the probability of a late delivery to 15%

a) Calculate the expected penalty if no action is taken to reduce the risk of delay

The company wants to reduce the risk of delay.

b) State with justification which one of the actions you would recommend to the company

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Probability: Cost-Benefit analysis

Μαθσ

Example 15B

The Daily Globe newspaper is being taken to court for defamation.
The person who brought the case is asking for £65,000 in damages.
The Daily Globe's in house lawyer has had a few bouts of illness recently, and has had time off work.
You are part of the newspaper's recruitment department.
The Chairman of the paper has asked you to investigate whether it is worth hiring another lawyer.

You estimate the following figures:

- The cost of hiring another lawyer for this case would be £12,000
- The cost of losing the case is £65,000
- Based on the number of sick days recently, the probability the in-house lawyer will be off sick during the trial is 0.4
- If the newspaper appoints a new lawyer the probability they will lose the case is 0.3
- If the in-house lawyer has to take time off during the trial, and another lawyer is not hired the probability that the newspaper will lose the case is 0.85.
- If the in-house lawyer does not take time off sick, the probability the newspaper will lose the trial is 0.3

Based on the expected costs to the paper, advise the Chairman whether the newspaper should hire another lawyer.

Probability: Cost-Benefit analysis

Your Turn 15A

A sports centre is planning a "legends" 5 a side football tournament on the 2nd April, in which former professional footballers will play.
The tournament can either be indoor or outdoor. The indoor arena has a smaller spectator capacity.
The club will charge £5 per ticket for spectators.
The table shows the expected number of spectators and the cost to hire the venue.
In the last 5 years it has rained on 42 out of 150 days in April.

Type of tournament	Cost to hire	Expected number of spectators
Indoor	£1,500	800
Outdoor	£2,100	600 if it rains
		1400 if it does not rain

a) The sports centre wants to maximise the amount of revenue. Should the tournament be outdoor or indoor?
You must show working to justify your answer

b) The sports centre can hire some covered stands for £1,500.
If they hire these stands then they can expect an extra 600 spectators to come to the outdoor tournament in the event of rain
Should they hire the covered stand?

Probability: Cost-Benefit analysis

Your Turn 15B

An actor is hired to take the lead part in a film.
His standard salary is £30,000 a day. If the film shoot overruns then he charges £90,000 for every additional day.
You are in charge of recruiting actors and other staff for the shoot.
The director tells you that the actor will be needed for 8 days. You estimate that there is a 40% chance the film will take longer than 8 days. You estimate that, if the shoot does take longer than 8 days there is a 90% chance the actor will be needed for 1 extra day and a 10% chance that he will be needed for 2 extra days.

You are considering three options:

- **Option A:**
  Hire the actor for 8 days. If the shoot takes longer then pay the salary at the higher rate.
- **Option B:**
  Hire the actor for 9 days. If the shoot is further delayed then pay the salary at the higher rate
- **Option C:**
  Hire the actor for 10 days

a) Which of the three options would you choose? Base your answer on the expected cost of each option
b) The film will not be delayed if some extra staff are employed to ensure it runs smoothly. You estimate that five staff would be needed for this. It would cost £6,400 per member of staff to employ them.

Explain whether you think these people should be hired.

Godalming College

Probability: Cost-Benefit analysis

Μαθσ

Your Turn 15C

A motor racing team is approaching the end of the season, which finishes in 2 weeks, and there are two more races on the calendar.
The car is in need of significant repair, and the team think they can get one more race from it.
The team manager wants the car to have one more race but is concerned about the cost of repairing the car to enable it to enter.

There are two possible races that could be entered – they know for sure that the car will not be able to participate in both.

You are managing the finances for the team and estimate the following:

- The probability of the car being ready to enter any one race is 0.4
- The cost of repairs, staff, garage and equipment hire etc is £10,000 per week
- The cost of entering a race is £6,000
- If the car manages to race again the increased exposure will up their sponsorship money for next season by £40,000
    - **Option A:**
      Do not repair the car, and end the season.
    - **Option B:**
      Take one week to try and get the car ready for the first race. If it is fixed then enter the race, if it isn't then don't try for the next race and end the season.
    - **Option C:**
      Try and repair the car for up to two weeks. If it is ready by the end of the first week then enter that race, and after that end the season.
      If it is not ready by the end of the first week then continue trying to fix it for the second race.

a) Advise the team manager which option she should take, based on the expected costs of each option
b) Give one reason why the manage might not follow your advice

Probability continued

The notation for the probability of event A, given that B has occurred is $P(A|B)$

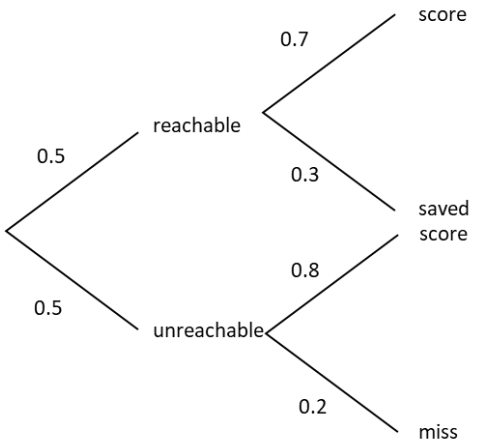
Example 5A Oxford 216:

Write down

a) P(score|reachable)

b) P(miss|unreachable)

c) Calculate the probability that the penalty was reachable, given that the penalty was scored

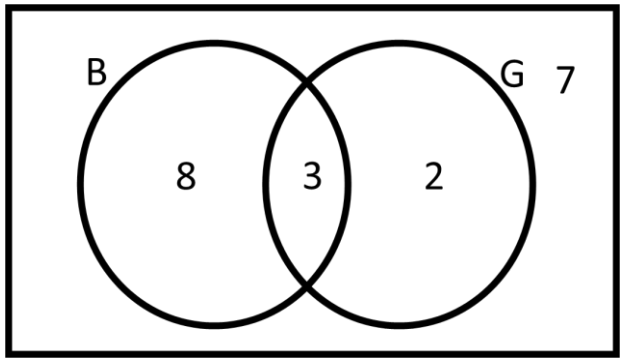


Example 5B:

The Venn diagram represents students in a class, where B is the event that the student is a boy and G is the event that the student wears glasses.

a) Find the probability that that student wears glasses, given that they are a boy.

b) Find $P(B'|B \cup G)$



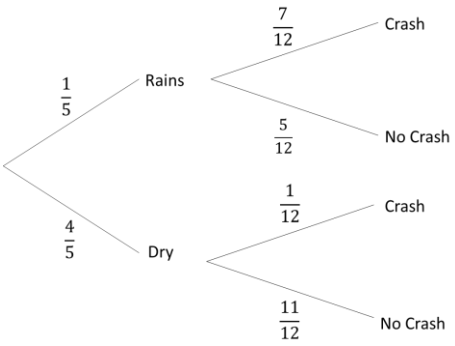
Probability continued

The notation for the probability of event A, given that B has occurred is $P(A|B)$

Test Yourself 5A:

The performance of a cyclist in a race is affected by whether it rains or not that day.

The probability the cyclist crashes is $\frac{7}{12}$ when it's raining and $\frac{1}{12}$ when it's dry.
On any given day, the probability of rain is $\frac{1}{5}$.



a) What is the probability that it the cyclist crashed, given that it was dry?

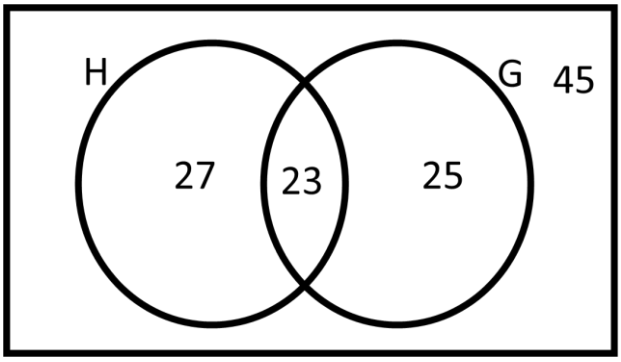
b) What is the probability it was raining given the cyclist crashes?

Test Yourself 5B:

The Venn diagram represents students taking Geography and/or History:

Find

a) $P(H)$
b) $P(G|H)$
c) $P(H'|(H \cup G))$



Probability: Expected Value

- Probability can be used to calculate the winnings/reward you may get in situations involving random events.
- Expected value (usually represented by $E(X)$) can be found by multiplying profit by probability, and adding your results together.

Example 6A:

You develop a game to use at a school fundraising event. The game involves one spin of a 4 sided spinner numbered one to four.
If the side the spinner rests on is prime, the player wins the value in pounds on the spinner.
If the side the spinner rests on is not prime, the player gives the value in pounds on the spinner to the school stallholder.
Will this game make a profit?

Spinner lands on	**1**	**2**	**3**	**4**
Player wins:				
Probability				

Example 6B:

Based upon an analysis of the results of Premier League football matches, the probabilities for the goal difference of the end of the match are estimated to be as follows:

Goal difference	0	1	2	3	4	5
Probability	0.25	0.42	0.23	0.08	0.01	0.01

a) What is the expected goal difference in a match?

b) Given that 1 point is awarded for a draw, and 3 points awarded if a team wins, what is the expected number of points awarded in a premier league match?

Probability: Expected Value

- Probability can be used to calculate the winnings/reward you may get in situations involving random events.
- Expected value (usually represented by $E(X)$) can be found by multiplying profit by probability, and adding your results together.

Test Yourself 6A:

The rules of the game in example 6A are changed as follows:
If the side the spinner rests on is odd the player wins the value in pounds on the spinner
If the side the spinner rests on is even the player gives the value in pounds on the spinner to the stall holder.
What is the expected value that a player will win when playing this game?

Spinner lands on	**1**	**2**	**3**	**4**
Player wins:				
Probability				

Test Yourself 6B:

in a dice game you throw a die and receive £0 for a score of 1 or 2, £1.00 for a score of 3, 4, or 5, and £10 for a score of 6.

If you play the game a large number of times, what would you expect to win per game on average?

Score	**1 or 2**	**3, 4, or 5**	**6**
Prize			
Probability			

Probability: Expected Value

Starter:

1. In a sample the percentage of households with a certain number of television sets is as follows:
   0 (10%), 1 (75%), 2 (10%), 3 (5%). What is the expected number of TV sets per household?

2. What is the AER on an account with 0.25% interest, paid monthly. Compare with an account with nominal interest of 4%, compounded weekly.

3. A shirt costs £46 in a sale where all clothes were reduced to 80% of the original price.
   What was the original price?

4. b) What is the modal class for the size of cricket ball?

   a) Which class contains the median?

Diameter (d mm)	Number of cricket balls
$65 < d \leqslant 66$	5
$66 < d \leqslant 67$	9
$67 < d \leqslant 68$	12
$68 < d \leqslant 69$	15
$69 < d \leqslant 70$	10
$70 < d \leqslant 71$	7
$71 < d \leqslant 72$	2
Total	**60**

- We are continuing with "expected value"
- Expected Value can be represented by $\mathrm{E}(X)$ and is found by multiplying an amount by the probability of getting or losing that amount.

Example 7 Hodder 229:

Joe estimates that, in the year ahead, his business has:
- A 15% chance of making £150,000
- A 40% chance of making £80,000
- A 35% chance of breaking even
- And a 20% chance of losing £80,000

What is his financial expectation, or expected value for the money made, for the year ahead?

Probability: Expected Value

Your Turn 7 Hodder 230:

An advisor tells a business start-up applicant that she has a 45% chance of breaking even, a 45% chance of making £100,000 and a 10% chance of losing £50,000. Calculate her financial expectation for the year.

Example 8 Oxford 219:

A garage is examining a car with a fault. From the manufacturer's manual it is known that the fault will be the result of the failure of one or two equally expensive parts, A or B. The only remedy is to replace one of the parts and, if the fault is not solved, then replace the other part. In such cases, the garage has the common sense policy of always replacing first the part which is most likely to have failed. In this case, from experience it is known that the faults will be the result of part A failing roughly $\frac{3}{5}$ of the time.

Replacing part A takes 2 hours and replacing part B takes 1 hour. Find the expected time that the repair will take. Do you agree with the garage's policy?