16 Turning Points in Physics

PRIOR KNOWLEDGE

The discovery of the electron

- Thermionic emission of electrons occurs when free electrons are emitted from the surface of a heated cathode.
- The work done on an electron which is accelerated through a potential difference, V, is equal to eV.
- The specific charge of a particle is defined as the charge per unit mass. It has units of C kg^{-1}.
- An object falling due to gravity will reach a terminal speed. The speed will depend on the drag force experienced by the object. Drag increases with the size of the object and the speed with which it is falling.
- A charged particle in a uniform electric field will experience a force, F_E, which depends on the magnitude of the field, E.

  $$F_E = EQ$$

  where E is the field strength of the electric field, measured in V m^{-1} (or N C^{-1}), and Q is the charge of the particle in C.
- A charged particle travelling in a direction perpendicular to a magnetic field will experience a force, F_B which depends on the magnitude of the field and the speed at which the particle is moving. The direction of the force is determined by Fleming's left hand rule.

  $$F_E = BQv$$

  where B is the magnetic flux density in T, Q is the charge of the particle in C and v is the velocity of the particle in ms^{-1}. The particle will follow a circular path with radius r.
- A particle which experiences a force, F, perpendicular to its direction of motion will follow a circular path.

  $$F = \frac{mv^2}{r}$$

  where m is the mass of the particle in kg, v is the velocity of the particle in m s^{-1}, and r is the radius of the path in m.

Wave-particle duality

- When light is reflected from a boundary the angle of incidence is equal to the angle of reflection.
- Light is refracted when it crosses the boundary between two materials with different optical density; the light changes speed and direction.
- Waves can undergo constructive or destructive interference.
- In Young's double slit experiment, a single light source may be used with double slits to produce an interference pattern. The fringe spacing is given by

  $$W = \frac{\lambda D}{s}$$

  where λ is the wavelength of the light, D is the distance between the double slits and the screen, and s is the distance between the slits.

- Electromagnetic waves may be treated as transverse waves which are able to propagate through a vacuum. The frequency of the wave determines how the wave is named, giving a spectrum of waves from low frequency radio waves to high frequency gamma rays.
- The de Broglie wavelength of a particle is given by

$$\lambda = \frac{h}{mv}$$

where h is the Planck constant and mv is the momentum of the particle.

Special relativity

- Electromagnetic waves travel at the same speed, $3 \times 10^8\ \mathrm{m s^{-1}}$, through a vacuum.
- A muon is a particle that decays into an electron.

TEST YOURSELF ON PRIOR KNOWLEDGE

1 Calculate the energy of an electron when it moves through a potential difference of 6 V in electron volts, and in joules.

2 Use the data in Table 16.1 to calculate the specific charge of the proton.

	Mass/kg	Charge/C
Proton	1.673×10^{-27}	$+1.60 \times 10^{-19}$

Table 16.1

3 An electron is accelerated in a uniform electric field between two electrodes which are 4.0 mm apart. The p.d. between the electrodes is 20 V. Calculate the force acting on the electron.

4 A particle is travelling with a speed of $1 \times 10^6\ \mathrm{m\,s^{-1}}$ perpendicular to a magnetic field with magnetic flux density of 0.13 T. The particle has a charge of 3.62×10^{-6} C; calculate the force acting on the particle.

5 A student shines monochromatic light of wavelength 580 nm through two slits 0.25 mm apart. The screen is 2.3 m away.

a) What is the frequency of the light?

b) What is the separation of the interference fringes observed on the screen?

6 Explain why a laser is often used to carry out Young's double slit experiment.

7 Calculate the wavelength of an electron moving at 0.1% of the speed of light. The mass of an electron is 9.1×10^{-31} kg.

Isaac Newton, in a letter to Robert Hooke, wrote: 'If I have seen further, it is by standing on the shoulders of Giants'. Physics is often thought of as continually developing so that each new discovery or theory builds on those that happened before it. Each new generation of physicists stand on the shoulders of those who have gone before. Newton was making the point that if it had not been for the work of astronomers such as Copernicus, Brahe, Galileo and Kepler, he would not have discovered the Universal Theory of Gravitation.

However, there are points in the history of Physics where there is a change in our understanding of the world, where there is a theory which is very

Paradigm means the typical pattern or example. In science, a **paradigm shift** refers to a change in the basic assumptions within the main theories.

different from those that were previously accepted. These points are known as **paradigm shifts**. In this chapter, we consider three of these turning points in physics. These are theories and experiments which changed our views and explanations about the world.

TIP

In the exam, you will not be expected to recall the various equations for the specific charge on the electron. You should, however, understand the principles which were used to derive them from the relevant equations from the Data and Formula booklet.

They are:

$$F = \frac{eV}{d},\ F = Bev,\ r = \frac{mv}{Be}$$

and $\frac{1}{2}mv^2 = eV$

TIP

In a question you *may* be given a diagram, a description of apparatus and/or data similar to those described in this section. You will be expected to use the data and one or more of the relevant equations to calculate values such as the speed of the electrons in a beam and the specific charge on the electron.

The discovery of the electron

Cathode rays

In the late 1860s, it was popular to show glass tubes filled with a gas which could be made to glow. These were called Geissler tubes, named after the German scientist and glassblower who developed them. Figure 16.1 shows a Geissler tube.

Figure 16.1 Geissler tubes – a 19th-century scientific curiosity.

When a large voltage was applied to the electrodes at either end of the tube, an electrical current passed through the gas. The gas glowed due to the ionisation and subsequent recombination of the molecules in the gas. Geissler tubes are a form of gas discharge tubes, and the technology was developed further to form neon lighting signs.

By the 1870s, a British scientist, William Crookes, made discharge tubes with a much lower pressure of gas inside. He used a vacuum pump to remove the air inside the tubes.

In these low pressure discharge tubes Michael Faraday observed that the glow appeared part of the way down the tube, with dark space near the cathode. In some tubes, the glass itself would glow. This always occurred at the end of the tube near the anode (positive). It was suggested that this glow was due to rays, named cathode rays, which are emitted from the cathode of the tube.

When there are very few gas molecules, the cathode rays are able to travel the length of the tube and reach the end of the glass. When there are too many gas molecules in the tube, then the cathode rays interact with the gas molecules, causing them to glow. Figure 16.2 shows a diagram of a discharge tube with the glowing column of gas (positive column), as well as a glow near the cathode.

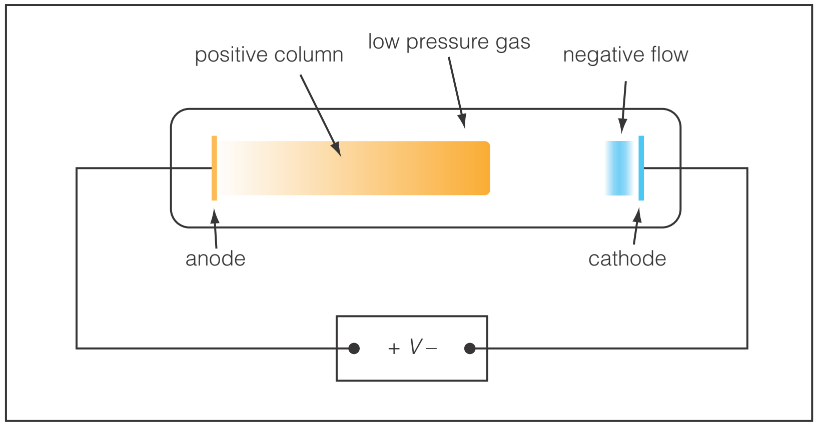


Figure 16.2 A discharge tube showing the glow pattern obtained when atoms in the gas are ionised by the large electric field. The positive ions are attracted towards the negative cathode, and the electrons are attracted towards the positive anode. The movement of the charged particles causes more gas molecules to ionise. When the positive ions and electrons recombine, visible and UV photons are emitted, causing the gas to glow.

In the 1870s, there was disagreement about what the rays were. Some physicists, including Crookes, thought that they were electrically charged particles; other physicists, including Heinrich Hertz, thought that they were a new type of electromagnetic wave which were separate from the current in the gas.

Further experiments showed that the path of the cathode rays could be altered by bringing a magnet near to the discharge tube. Initially, it appeared that the rays were unaffected by electric fields, but when tubes with very low gas pressure were used, the rays were also deflected by electric fields.

From these, and other experiments, it was suggested that cathode rays were negatively charged particles.

Thermionic emission

The early discharge tubes produced cathode rays due to the large electric field between the cathode and anode and are known as cold discharge tubes.

Using a heated cathode to produce cathode rays is a simpler method of creating an electron beam. When a metal filament is heated the conduction electrons in the metal start to move more vigorously and are able gain enough energy to leave the surface of the metal. This effect is known as **thermionic emission**.

Thermionic emission. The release of electrons (or charge carriers) from a heated source.

Figure 16.3 shows a simplified version of a cathode ray tube using thermionic emission to produce an electron beam. Electrons are produced at the heated cathode and accelerate towards the anode due to the high voltage between the cathode and anode. A few electrons pass through the small hole in the anode and travel in a narrow beam towards the screen. The glass tube contains a vacuum so there are no gas molecules for the electrons to interact with and the tube will not glow.

The heated cathode filament and anode arrangement is sometimes called an ‘electron gun’ because the electrons are ‘fired’ with a velocity through the hole in the anode.

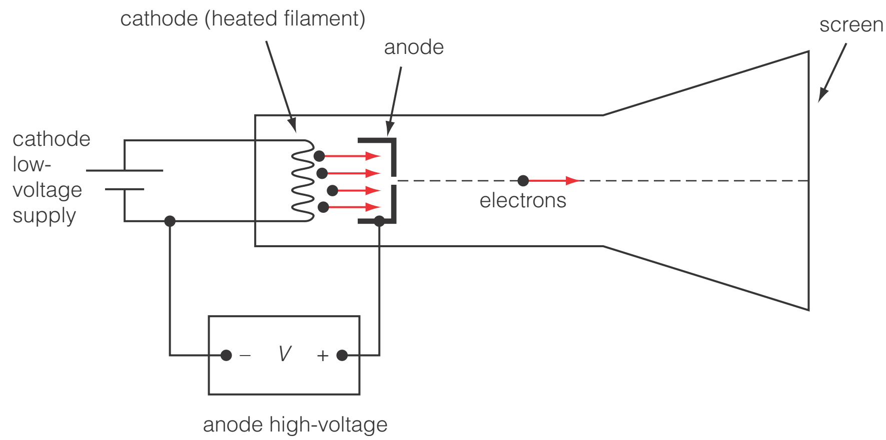


Figure 16.3 A simplified diagram of a cathode ray tube producing a beam of electrons from a heated cathode by thermionic emission.

Using a cathode ray tube, we can calculate the speed at which the electrons are travelling.

The work done on each electron accelerated through the potential difference between the anode and cathode = eV, where e is the charge on the electron.

Each electron will be accelerated from a very low initial speed to a much greater speed, v, as they pass through the hole in the anode. The electrons' initial speed is much less that their final speed, so we treat their initial speed as effectively zero. Their kinetic energy as they pass through the anode will be equal to $\frac{1}{2}mv^2$. This energy is gained from the work done on each electron so we can write:

$$\frac{1}{2}mv^2 = eV$$

Increasing the p.d. across the anode will increase the speed at which the electrons are moving.

Increasing the current in the cathode filament makes the beam of electrons more intense. The increase in current leads to an increase in the temperature of the cathode, which, in turn, increases the number of electrons that have sufficient energy to escape the surface of the metal.

TIP

We usually assume that the electrons are travelling much slower than the speed of light in the cathode ray tube. If the electron speed is close to the speed of light, then relativistic effects must be taken into account.

EXAMPLE

A beam of electrons is produced by thermionic emission from a heated filament. The potential difference between the anode and the filament is 4200 V. The speed of the electrons in the beam was 3.9×10^7 m s^{-1}. Calculate a value for the specific charge of the electron.

$$\frac{1}{2}mv^2 = eV$$

so we can rearrange to obtain an equation for the specific charge of the electron:

$$\frac{e}{m} = \frac{v^2}{2V}$$

Using the data in the question

$$\frac{e}{m} = \frac{(3.9 \times 10^7 \text{ms}^{-1})^2}{2 \times 4200 \text{ V}}$$

specific charge = 1.8×10^{11} C kg^{-1}

TEST YOURSELF

1 A cathode in an electron gun is heated by a small current. Thermionic emission occurs and electrons are emitted from the cathode. The potential difference between the cathode and the anode is 20 V. Calculate the speed of the electrons as they pass through the anode.

2 The work done accelerating an electron between the cathode and the anode of the electron gun in a cathode ray oscilloscope is 1.7×10^{-17} J. What is the potential difference between the cathode and the anode?

3 Calculate the work done in an electron gun to accelerate electrons to a speed of 3.0×10^{7} m s^{-1}.

Measuring the specific charge of the electron

In January 1897, German physicist Emil Wiechert measured the specific charge of the particles in the cathode rays. He was surprised to find that the value of charge / mass was much larger than the value for a hydrogen ion. This meant that either the particles were less massive than the hydrogen ion, or carried a much larger charge. However, there was not enough evidence to know which of these possibilities was the correct one.

History box

Sir Joseph John Thomson, known as J.J. Thomson, a British physicist, carried out a series of experiments, which built on these previous findings, to investigate the nature of particles which formed the cathode rays.

He wrote:

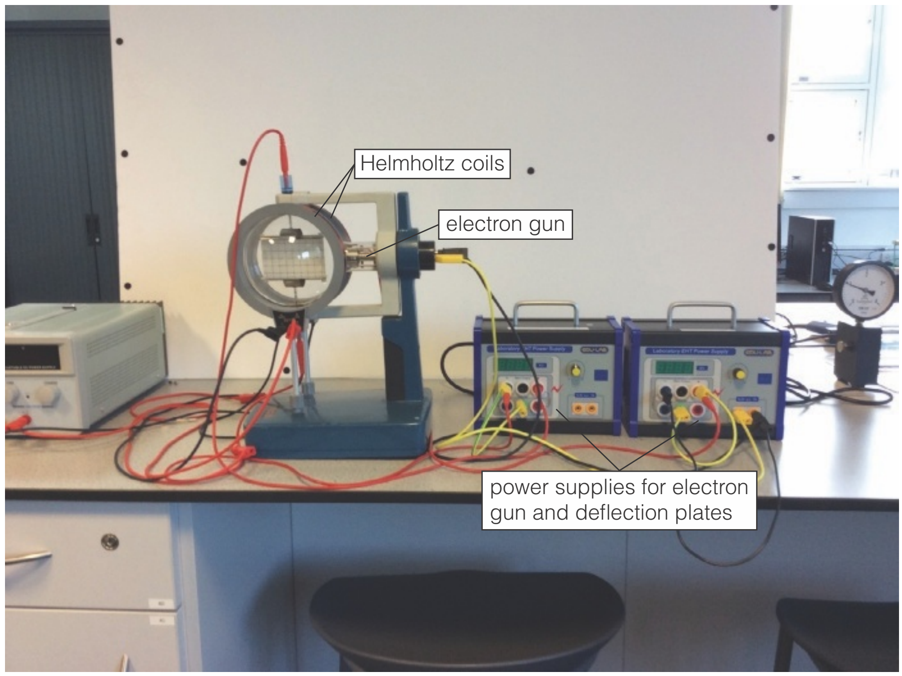
> *What are these particles? Are they atoms, or molecules, or matter in a still finer state of subdivision? To throw some light on this point, I have made a series of measurements of the ratio of the mass of these particles to the charge carried by it.*

He published his findings in a paper published in 1897. Thomson named these particles 'corpuscles'. It was another physicist, Francis FitzGerald, who suggested that these corpuscles were electrons, and although the term didn't quite describe the properties of Thomson's corpuscles, electrons was the name that stuck and that we use today.

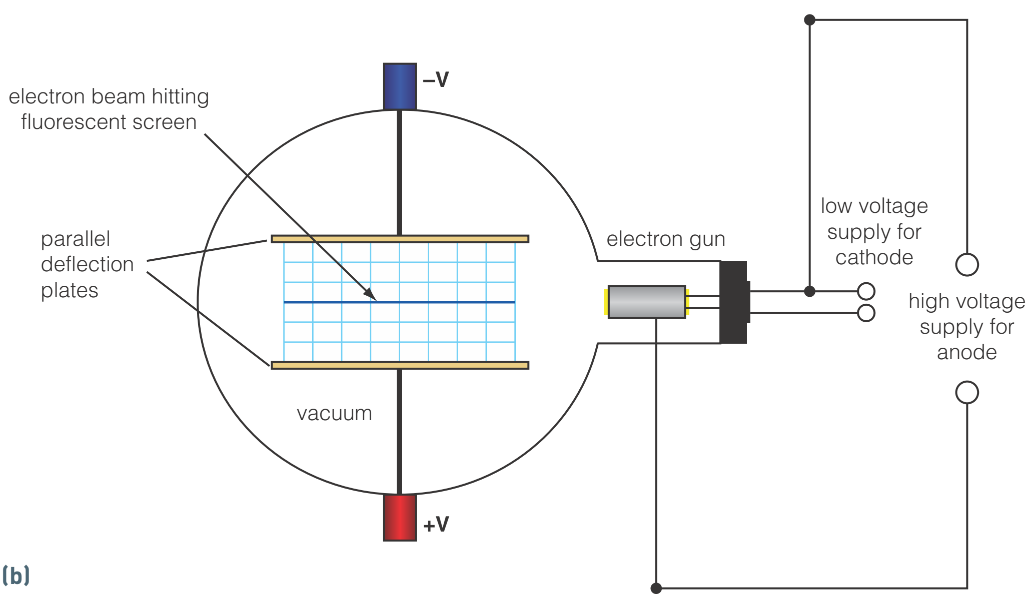
He therefore set up an experiment in which he measured the radius of curvature of the electrons in a magnetic field, and the temperature rise of a metallic thermometer (known as a thermocouple). He assumed that the moving particles would transfer energy to the point where they hit the side of the discharge tube, heating it up. By measuring this temperature rise he could calculate the kinetic energy, and therefore the velocity of the particles.

J.J. Thomson used discharge tubes with a combination of electric and magnetic fields to change the path of the cathode rays. By measuring the deflection with these different combinations of fields, he was able to obtain values for the specific charge of the particles forming the cathode rays. Thomson could not directly measure how fast the electrons were travelling because he was using a cold discharge tube in his experiments.

We can carry out experiments similar to Thomson's work using school equipment today. Figure 16.4 shows a Teltron electron deflection tube which can be used to take measurements of the changes in the path of the electrons.



(a)



(b)

Figure 16.4 (a) An electron deflection tube with Helmholtz coils. The coils provide a uniform magnetic field in the tube (b) Diagram showing the tube in more detail.

Balancing the forces acting on the electrons

The electric field between the parallel plates in the deflection tube deflects the electron beam downwards. The Helmholtz coils are used to produce a magnetic field perpendicular to the electric field so that the beam moves in a straight line again.

Fleming's Left Hand rule can be used to determine the direction of the magnetic field to ensure that there is an upwards force on the electrons.

When the forces acting on the electrons are equal and opposite, then the beam will show no deflection, as shown in Figure 16.5.

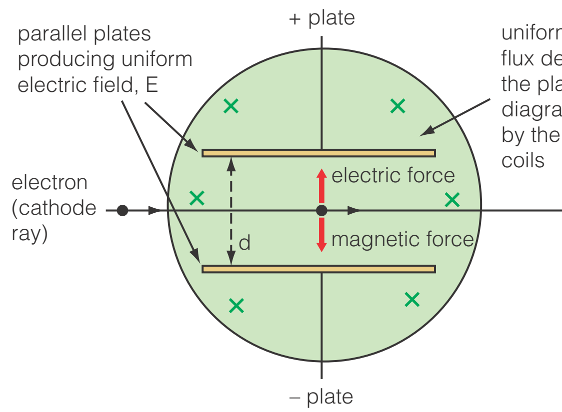


Figure 16.5 Balanced electric and magnetic forces acting on an electron beam.

The electric field can be calculated from the voltage across the parallel plates, V, and the distance between them, d.

$$E = \frac{V}{d}$$

The force on the electrons due to the electric field is given by:

$$F_E = eE = \frac{eV}{d} \qquad \text{equation 1}$$

> **TIP**
> Make sure that you don't get the velocity, v, and the electric field, V, confused when you are using these equations.

The force on the electrons, travelling with velocity, v, due to the magnetic field is given by:

$$F_M = Bev$$

When the forces balance so that the beam is not deflected we can write:

$$F_E = F_M$$

$$\frac{eV}{d} = Bev$$

and so the velocity of the electrons is given by:

$$v = \frac{V}{Bd} \qquad \text{equation 2}$$

TEST YOURSELF

4 An electron is travelling with a speed of 2.2×10^{6} m s^{-1}. It passes through a perpendicular electric field of 2.5×10^{5} V m^{-1}. What is the strength of the magnetic field which will allow it to travel through the fields with no deflection?

5 In a discharge tube an electron passes between two parallel plates which are 10 mm apart. The plates are at a potential difference of 250 V. There is a perpendicular uniform magnetic field of 20 mT in the discharge tube. Calculate the speed of the electrons if they pass through the discharge tube undeflected.

6 In an electron deflection tube, the parallel electric plates are at a potential difference of 300 V. The plates are 6 mm apart. Calculate the electric field between the plates.

Deflecting the electron beam with magnetic field only

When the electric field is removed, the beam of electrons will be deflected by the magnetic field and follow a circular path with radius *r*.

The magnetic force on each electron provides the centripetal force that causes the particle to move in a circular path. The magnetic force is perpendicular to the direction of the velocity of the electrons. This means that the magnetic force does no work on the electrons and their speed remains constant.

We can write:

$$Bev = \frac{mv^2}{r}$$

and rearranging gives an equation for the radius:

$$r = \frac{mv}{Be}$$

and specific charge:

$$\frac{e}{m} = \frac{v}{Br} \qquad \text{equation 3}$$

MATHS BOX

If we do not know the speed of the electrons then we can substitute v, from equation 2, into equation 3.

$$v = \frac{V}{Bd}$$

so

$$\frac{e}{m} = \frac{V}{B} \div \mathrm{Br}$$

giving an equation for specific charge

$$\frac{e}{m} = \frac{V}{B^2 rd}$$

J.J. Thomson was able to measure the values of all of the quantities in this equation, and so was able to obtain a value for the specific charge on the electron without needing to know the speed at which the electrons were travelling.

EXAMPLE

A narrow beam of electrons move in a circular path of radius 25 mm through a uniform magnetic field of flux density 7.3 mT.

The electrons travel at a speed of 3.2×10^{7} m s^{-1}.

Calculate the specific charge of the electron.

Using

$$Bev = \frac{mv^2}{r}$$

we can write:

$$\frac{e}{m} = \frac{v}{Br}$$

$$\frac{e}{m} = \frac{3.2 \times 1^{7}\,\mathrm{m\,s^{-1}}}{7.3 \times 10^{-3}\,\mathrm{T} \times 25 \times 10^{-3}\,\mathrm{m}} = 1.75 \times 10^{11}\,\mathrm{C\ kg^{-1}}$$

TEST YOURSELF

7 A narrow beam of electrons move perpendicularly through an area of magnetic flux density 1.7 mT in a circular path of diameter 3.5 cm. The electrons are moving at a speed of 5.3×10^6 m s^{-1}. What is the specific charge of the electrons?

8 A beam of electrons are travelling at a speed of 2.5×10^7 m s^{-1}. They enter a region of uniform magnetic flux density 2.2×10^{-3} T perpendicular to the beam. The electrons follow a circular path.

(a) Calculate the force experienced by each electron.

(b) Calculate the radius of the circle that the electrons move in.

9 Explain why an electron, moving with a constant speed, *v*, perpendicular to a uniform magnetic flux density, will follow a circular path.

Deflecting the electron beam with electric field only

Using the electron deflection tube without a magnetic field means that the electrons are deflected only by the electric field, as shown in Figure 16.6. As the electrons travel through the electric field they will experience a force which will cause them to accelerate.

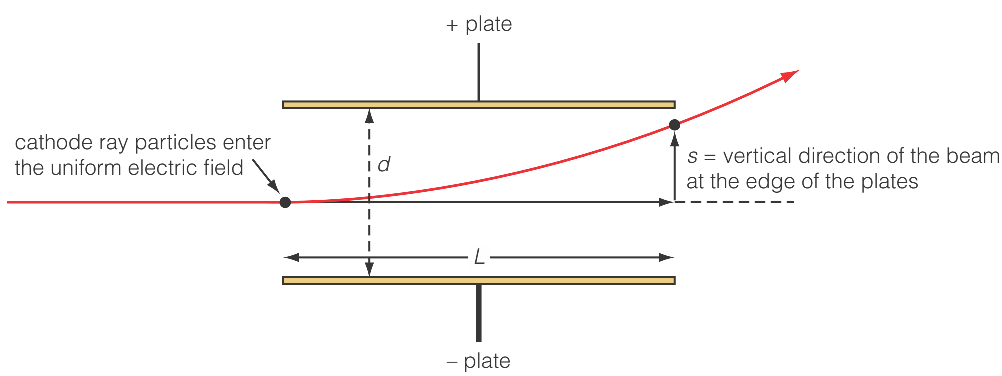


Figure 16.6 Motion of the cathode ray particles (electrons) in a uniform magnetic field.

The amount that the particles are deflected, *s*, can be measured directly and the acceleration of the place can be calculated using the following equation:

$$s = ut + \frac{1}{2}at^2$$

We can use this equation to obtain a value for the acceleration.

The electrons enter this region between the plates moving horizontally, so they have no initial vertical velocity.

The vertical deflection is given by:

$$s = \frac{1}{2}at^2$$

Rearranging gives:

$$a = \frac{2s}{t^2} \qquad \text{equation 4}$$

However, we don't know how long it takes for an electron to travel the length of the plates.

We know that for the horizontal motion the time, t, taken for the electrons to travel the length, L, of the parallel plates

$$t = \frac{\text{distance}}{\text{speed}}$$

From equation 2, using balanced fields, we know that the velocity of the electrons

$$v = \frac{V}{Bd}$$

so

$$t = \frac{LBd}{V}$$

We can now substitute the equation for t into equation 4 to obtain an expression for acceleration:

$$a = \frac{2s}{\left(\frac{LBd}{V}\right)^2} = \frac{2s}{\frac{L^2B^2d^2}{V^2}}$$

giving

$$a = \frac{2sV^2}{B^2d^2L^2} \qquad \text{equation 5}$$

However, we also know the force from the electric field,

$$F_E = m\,a$$

so rearranging for a, and substituting for F_E using equation 1 we obtain

$$a = \frac{F_E}{m} = \frac{eV}{md} \qquad \text{equation 6}$$

We now have two equations for the acceleration of the electrons. We can use equation 5 and 6 to obtain an expression for the specific charge of the electrons.

$$\frac{eV}{md} = \frac{2sV^2}{B^2d^2L^2}$$

$$\frac{e}{m} = \frac{2sV}{B^2L^2d}$$

EXAMPLE

A narrow beam of electrons with a speed of 3.6×10^7 m s^{-1} enters a uniform electric field at right angles to the field. The field is caused by two oppositely charged parallel plates of length 60 mm, with a potential difference across them of 1250 V, separated by a distance of 25 mm.

The electron beam has a deflection of 12.5 mm just as it reaches the edge of the positive plate, as shown in the diagram.

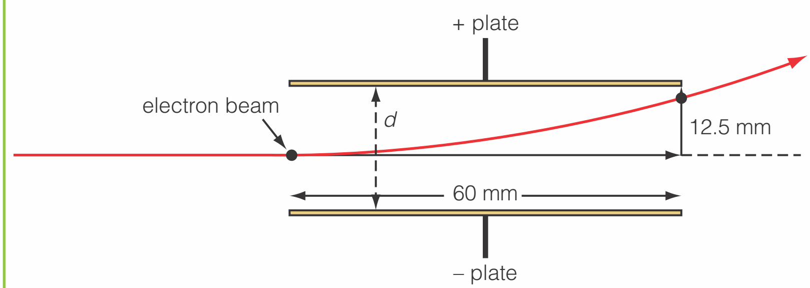


Figure 16.7

Use the data to calculate the specific charge on the electron, *e/m*.

First, we calculate the time that the electrons will spend in the uniform electric field.

$$t = \frac{L}{v} = \frac{60 \times 10^{-3}\ \text{m}}{3.6 \times 10^{7}\ \text{ms}^{-1}} = 1.67 \times 10^{-9}\ \text{s}$$

The displacement is:

$$s = \frac{1}{2}at^2$$

Using $F = ma$ where the force is due to the electric field and

$$F = \frac{eV}{d}$$

we can write

$$ma = \frac{eV}{d}$$

and so acceleration

$$a = \frac{eV}{md}$$

This gives specific charge:

$$\frac{e}{m} = \frac{2sd}{Vt^2}$$

$$= \frac{2 \times 12.5 \times 10^{-3}\text{m} \times 25 \times 10^{-3}\text{m}}{1250\,V \times (1.67 \times 10^{-9}\ \text{s})^2}$$

$$= 1.8 \times 10^{11}\ \text{C kg}^{-1}$$

TEST YOURSELF

10 The deflection plates in a cathode ray tube are 50 mm long. An electron with a speed of 2.5×10^7 m s^{-1} travels parallel to the plates. Calculate how long the electron takes to travel the length of the deflection plates.

11 An electron travels through the deflection plates of an oscilloscope. The electron is moving perpendicularly to the electric field between the plates with a speed of 3.0×10^6 m s^{-1}. The electric field between the plates is 200 N C^{-1} and the plates are 0.1 m long. What is the acceleration of the electron whilst it is in the electric field?

The significance of Thomson's experiments

As a result of the experiments published in his 1897 paper, Thomson came to a number of conclusions:

- The charge could not be separated from the particles, which meant that they were not a new form of electromagnetic wave as had been suggested by Helmholtz.
- The direction of deflection of the particles showed that they were negatively charged.
- The ratio of charge to mass for all the particles is the same. Changing the gas in the discharge tube did not alter the value, so the specific charge was a property of the particles, and not of the measuring equipment.

Thomson's value for the specific charge of the electron was more than 1800 times greater than that of the smallest known ion, hydrogen.

The specific charge of the electron is 1.76×10^{11} C kg^{-1}. The specific charge of a hydrogen ion (or proton) is 9.6×10^7 C kg^{-1}.

Thomson's experiments provided evidence that atoms weren't the smallest part of matter, and that they could in fact be broken into smaller particles.

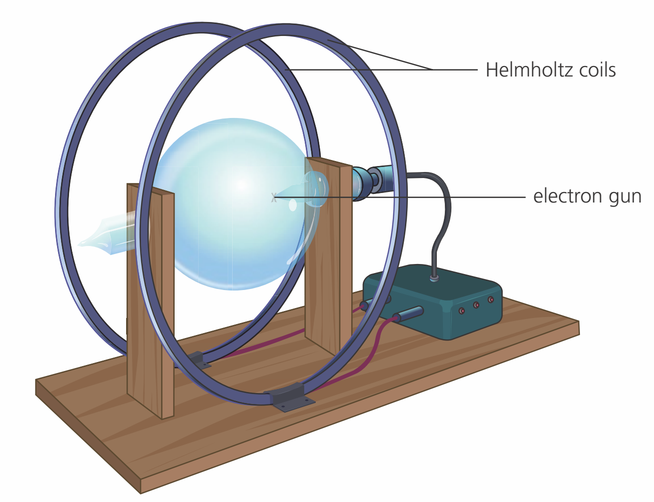
However, he was unable to measure the charge or the mass of the particles separately so could not say what the size of the particles actually was.

ACTIVITY

Fine beam tube

A fine beam tube is another method which can be used to measure the specific charge on electrons. It was developed after thermionic emission was developed so that there was a suitable method of producing a beam of electrons with uniform velocity.

These tubes were originally manufactured by a firm called Teltron, so they are often known as Teltron tubes. Figure 16.8(a) shows a diagram of a fine beam tube. It uses an 'electron gun' which uses thermionic emission to produce two beams of electrons at right angles to each other. This means that, when using the vertical beam of electrons, a suitable magnetic field can form the beam into a complete circle, as can be seen in Figure 16.8(b). The tube is filled with very low pressure helium or hydrogen gas. The gas molecules fluoresce when electrons from the beam collide with them.



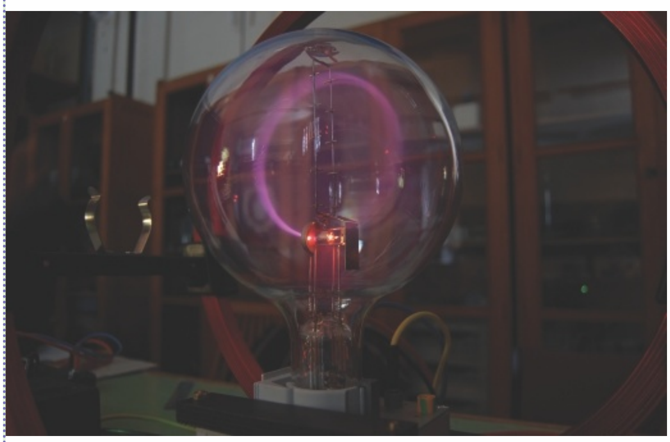


Figure 16.8 Two views of a fine beam tube. (a) Diagram of the tube showing the electron gun and the Helmholtz coils which provide the uniform magnetic flux density. (b) The beam of electrons is deflected into a complete circle by the magnetic field.

A student uses a fine beam tube to investigate the relationship between the anode voltage, V_A, for the electron gun and radius of the circle, *r*, formed by the electron beam. She keeps the magnetic flux density due to the Helmholtz coils constant at a value of 1.5 mT. Table 16.2 shows her results.

Table 16.2

Anode voltage/V	Diameter/cm
5	10.0
20	20.0
45	30.0
79	40.0
123	50.0
177	60.0
208	65.0

(a) Suggest why the student was unable to take measurements of a diameter greater than 65.0 cm.

(b) Plot a graph of r^2 on the *y*-axis against V_A on the *x*-axis.

(c) What does the gradient of the graph represent? (Hint: use the maths box)

(d) Use your graph to calculate the specific charge on an electron.

MATHS BOX

In a fine beam tube, the magnetic force on each electron is given by BeV, and this force provides the centripetal force, so we can write:

$$BeV = \frac{mv^2}{r}$$

The electron beam is produced by thermionic emission in the electron gun, so we know that the kinetic energy of the electrons is:

$$\frac{1}{2}mv^2 = eV$$

where V is the potential difference across the anode.

By combining these two equations, we can obtain the following equation for the specific charge of the electron:

$$\frac{e}{m} = \frac{2V_A}{B^2r^2}$$

TIP

If the data in the question is given to two significant figures, your answer should also be given to two significant figures.

EXAMPLE

A fine beam tube is used to measure the specific charge of the electron. The electrons in the beam are travelling at a speed of 7.4×10^6 m s^{-1} in a magnetic flux density of 0.6 mT. The radius of curvature of the beam is 68 mm.

Calculate the specific charge of the electron.

The magnetic force acts as the centripetal force so we can write:

$$Bev = \frac{mv^2}{r}$$

rearranging gives:

$$\frac{e}{m} = \frac{v}{Br}$$

$$\frac{e}{m} = \frac{7.4 \times 10^6 \text{ms}^{-1}}{6.0 \times 10^{-4} \text{T} \times 68 \times 10^{-3} \text{m}} = 1.8 \times 10^{11} \text{ C kg}^{-1}$$

TEST YOURSELF

12 During thermionic emission, an electron is accelerated from rest through a potential difference of 2000 V between the cathode filament and the anode. Calculate the speed of the electron at the anode. (Ignore relativistic effects.)

13 In an experiment to calculate the specific charge on an electron, a narrow beam of electrons is moving with a velocity perpendicular to a uniform magnetic field of flux density 2.0×10^{-3} T. The beam travels in a circular path with radius 25 mm. Whilst the magnetic field is still present, the path of the beam is straightened using an electric field. This field is applied between two parallel plates 12 mm apart and with a potential difference of 200 V across them. Calculate (a) the velocity of the beam and (b) the specific charge on the electrons in the beam.

14 Electrons are emitted by thermionic emission from a heated filament in an electron gun. The electrons are accelerated through a p.d. of 350 V and are directed into a uniform magnetic field of 8.5 mT. The electrons move in a circular orbit in the magnetic field with a radius of 74 mm. Calculate the specific charge on the electrons.

15 Discuss the historical significance of the value of the specific charge of the electron compared with the specific charge of the H^+ ion.

Measuring the charge of the electron

In 1908, Rutherford and Geiger predicted the charge on the electron from their experiments with helium nuclei. They assumed that the charge on the electron would be half the charge of the helium nuclei. They gave a value for the charge on the electron as 1.55×10^{-19} C.

History box

A number of physicists, including J.J. Thomson, J.S.E. Townsend and H.A. Wilson, realised that a cloud of droplets of water will form around ions in ionised air. By observing the cloud of droplets falling under gravity they could measure the charge on the droplets. This allowed them to obtain values for the charge on the electron.

In 1898 Townsend measured:

- the mass of a cloud of falling droplets
- the speed at which they fell
- the average mass of a single droplet and
- the charge carried by the cloud of droplets.

He then calculated $\frac{e}{m}$ for the cloud of droplets. By using the average mass of a single droplet he was able to obtain a value for the charge on the electron. Townsend's value was approximately 1×10^{-19} C

Stokes' law

For a spherical droplet with radius r, the volume $= \frac{4}{3}\pi r^3$ and so if we know the density, ρ, of the droplet, the mass can be found from the equation:

$$m = \frac{4}{3}\pi r^3 \rho$$

However, measuring the radius of the falling drops directly was not possible experimentally.

In order to calculate the mass of the falling droplets Stokes' law was used.

Stokes' law describes the drag force acting on spherical droplets falling through a viscous medium. In this case, the droplets were falling through air, which, to us, does not appear viscous, but is on the scale of a droplet.

Stokes' law shows that the drag force acting on the droplets depends on the radius of the drop, r, the speed at which it is falling, v, and the viscosity, η, of the medium it is falling through.

The drag force is:

$$F_D = 6\pi\eta rv$$

The speed of the droplet can be calculated by measuring the time it takes to fall through a known distance.

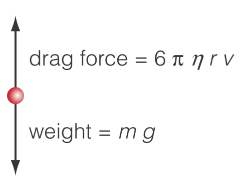


Figure 16.9 The forces acting on a droplet in air falling at its terminal speed.

Figure 16.9 shows the forces acting on a drop falling at its terminal speed through air.

At terminal speed, the two forces acting on the droplet will be equal so we can write:

$$6\pi\eta rv = mg$$

Substituting for mass we obtain:

$$6\pi\eta rv = \frac{4}{3}\pi r^3 \rho g \qquad \text{equation 7}$$

> **TIP**
> We are assuming that buoyancy effects on the drop are negligible. We are also assuming that the drop is falling slowly. At high speeds, Stokes' law does not apply because turbulence in the fluid affects the fall.

The droplet radius can be obtained by rearranging equation 7:

$$6\,\eta\, v = \frac{4}{3}\, r^2 \rho\, g$$

$$\frac{18\eta v}{4\rho g} = r^2$$

$$r = \sqrt{\frac{9\eta v}{2\rho g}}$$

The droplet mass can then be calculated using this value of radius.

EXAMPLE

A raindrop is falling through air with a terminal speed of 2.06 m s^{-1}. The viscosity of air is 1.8×10^{-5} N s m^{-1} and the density of the water in the drop is 1000 kg m^{-3}. Calculate the radius and mass of this raindrop.

Using

$$r = \sqrt{\frac{9\eta v}{2\rho g}}$$

$$r = \sqrt{\frac{9 \times 1.8 \times 10^{-5}\ \text{N s m}^{-1} \times 2.06\ \text{m s}^{-1}}{2 \times 1000\ \text{kg m}^{-3} \times 9.81\ \text{N kg}^{-1}}}$$

$$r = \sqrt{\frac{3.337 \times 10^{-4}}{19620}} = 1.304 \times 10^{-4}$$

$$r = 1.3 \times 10^{-4}\ \text{m}$$

To calculate the mass of the raindrop we use:

$$m = \frac{4}{3}\pi r^3 \rho$$

$$m = \frac{4}{3}\pi\,(1.3 \times 10^{-4}\ \text{m})^3 \times 1000\ \text{kg m}^{-3}$$

$$m = 9.2 \times 10^{-9}\ \text{kg}$$

The radius of the raindrop is 0.13 mm and the mass of the raindrop is 9.2×10^{-9} kg.

History box

In 1903, Charles Wilson (inventor of the cloud chamber) extended the earlier falling water droplet experiments to measure the electron charge by using a 2000 V battery to provide an electric field. The falling motion of the cloud could be stopped because the force due to the electric field balanced the force due to gravity. Wilson also obtained a value for the charge on the electron of approximately 1×10^{-19} C

Robert Millikan developed Wilson's experiment by using larger voltage batteries, initially 4000 V, but in 1909 he used a 10 000 V battery. He was trying to measure the rate at which the droplets evaporated. However, he found that with the 10 000 V battery, he could hold individual drops in place for up to 60 seconds.

TIP

Millikan's original paper is available online at www.aip.org/history/gap/PDF/millikan.pdf where you can read in more detail the improvements he made and the results that he obtained.

Millikan's oil drop experiment

Early measurements of the charge on an electron used charged clouds of water. However, there were problems with this approach:

- air currents affected the motion of the drops
- the drops tended to evaporate in the apparatus
- the electric field wasn't very uniform
- Stokes' law wasn't valid because the drops were falling too quickly.

However, in 1913, Robert Millikan published the results of his experiments to measure the charge on falling droplets of oil. He had made a number of improvements to the experiment, including better optics and reduced convection currents in the air within the apparatus. He published data from 58 different drops of oil measured over the space of 60 days. Figures 16.10 and 16.11 show Millikan's oil drop apparatus and a simplified diagram of the apparatus.

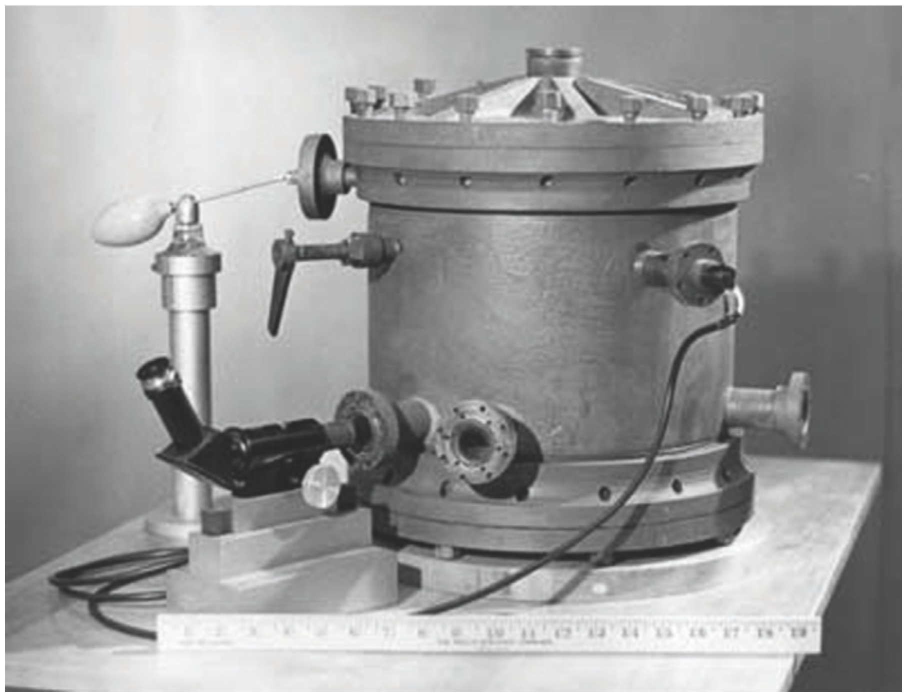


Figure 16.10 Millikan's oil drop experimental apparatus.

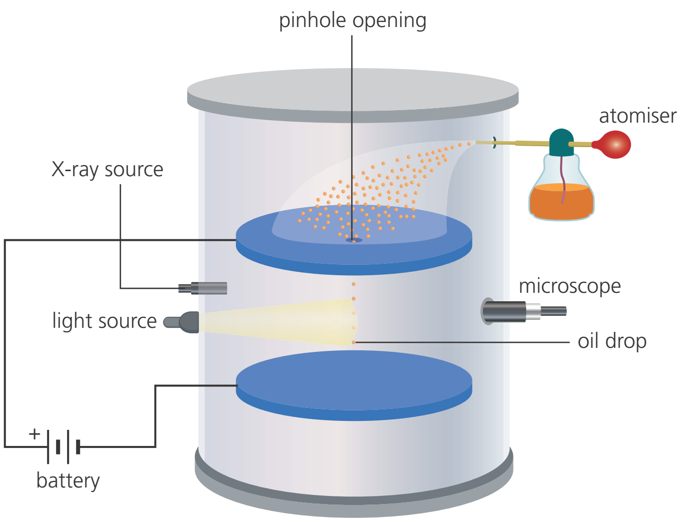


Figure 16.11 Simplified diagram of Millikan's apparatus.

Millikan sprayed oil into the body of the apparatus using an atomiser to produce a cloud of droplets. A small number of the droplets fell through the pinhole opening in the anode. These droplets became negatively charged as they were ionised by X-rays from the X-ray source. Using the microscope eyepiece, Millikan was able to measure the speed of drops as they fell due to gravity. He could then create an electric field between the anode and cathode. The drops would then stop falling under gravity, and even move back up towards the anode if the field was strong enough.

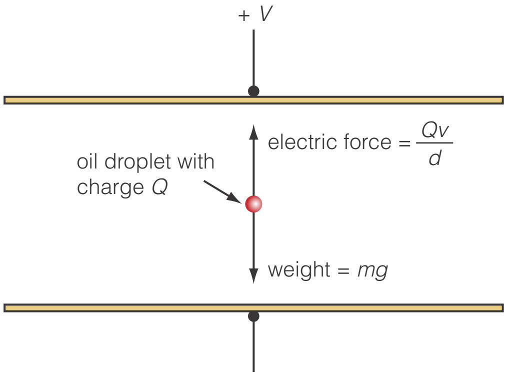


Figure 16.12 The forces acting on a stationary drop held in a uniform electric field.

Figure 16.12 shows the forces acting on an oil drop which is held stationary in an electric field.

When the drop is stationary, the upward force due to the electric field is equal to the downward force due to gravity acting on it.

If the drop has a charge, Q, and a mass, m, then we can write:

$$\frac{Qv}{d} = mg$$

and the charge on the droplet is given by:

$$Q = \frac{mgd}{v}$$

where the mass of the droplet is found using the value of the radius calculated by the drop falling under gravity.

EXAMPLE

Charged droplets of oil were observed falling between two oppositely charged parallel plates. The spacing between the plates was 6.0 mm. When the potential difference between the places was 5700 V, one particular droplet with mass 6.2×10^{-14} kg was observed to be stationary between the plates.

Calculate the charge on this droplet.

When stationary, the forces on the droplet are balanced.

$$\frac{Qv}{d} = mg$$

so

$$Q = \frac{mgd}{v}$$

$$Q = \frac{6.2 \times 10^{-14} \times 9.8 \times 6.0 \times 10^{-3}}{5700}$$

$$Q = 6.4 \times 10^{-19}\ \text{C}$$

History box

When collecting his experimental data, Millikan chose drops which took between 10 and 40 seconds to fall past the window of his apparatus. If a drop was moving slower or quicker he didn't use the measurements from them in calculations. By choosing drops in this way, Millikan was able to reduce the statistical error on his measurements. However, other physicists suggested that choosing drops could be seen as 'cherry picking' results reducing the validity of Millikan's results.

Significance of Millikan's oil drop experiment

Millikan assumed that the charge on each drop would be an integral multiple of the charge on the electron. By working out the common factor from his measurements he obtained a value for the charge of the electron.

The value for the charge on the electron that Millikan published was $1.592 \pm 0.003 \times 10^{-19}$ C.

This is smaller than the accepted value of the charge on the electron which is 1.602×10^{-19} C. This was because Millikan was using a value for the coefficient of viscosity which was too low.

Millikan, and his experiment, supported the hypothesis that charge was quantised in integer multiples of the charge on the electron and he was awarded a Nobel prize in 1923, in part, as a consequence of his work on measuring the charge on an electron.

TEST YOURSELF

16 A charged droplet is held stationary between two electrically charged plates. The potential difference between the plates is reduced to 0 V. Describe and explain what happens to the motion of the drop.

17 A student is using a set of apparatus to carry out Millikan's oil drop experiment. An electric field of 5.86×10^4 V m^{-1} is required to hold a particular drop stationary. The parallel plates in the experiment are 1.50 cm apart. What is the required potential difference between plates?

18 A student carried out an experiment to measure the charge on oil droplets held stationary between two charged plates. Table 16.3 shows the measured values of mean charge for each drop.

Table 16.3

Oil drop	Mean charge on the drop/C
1	1.53×10^{-19}
2	6.29×10^{-19}
3	1.58×10^{-19}
4	3.13×10^{-19}

Explain why the student did not obtain any values of charge on the drop less than value obtained for drop 1.

19 In a Millikan oil drop experiment, when the pd on the plates is zero a charged oil droplet is observed to fall with a steady speed of 3.3×10^{-5} m s^{-1}. The mass of the droplet is 6.0×10^{-15} kg. The density of the oil is 924 kg m^{-3} and the viscosity of air is 1.8×10^{-5} N s m^{-2}.

(a) Calculate the radius of the droplet.

(b) The drop is charged with 10 electrons. The plates are a distance of 10.0 mm apart. What p.d. across the plates is required to hold the drop stationary?

Wave-particle duality

Theories of light

From the time of the early Greek philosophers, people had been trying to explain the properties of light. A number of different models were suggested to explain the properties of reflection and refraction of light. By the late 17th century, the two main models were a particle model and a wave model. The different models were able to explain some, but not all, of the properties shown by light.

Newton and the corpuscular theory of light

Although Newton wasn't the first scientist to propose that light was formed from small particles (called 'corpuscles' meaning 'little bodies') he was perhaps the most influential.

Newton proposed that light was formed from corpuscles that were continuously given out in all directions by luminous objects. One consequence of this theory was that luminous objects should be constantly losing mass.

The corpuscular theory could explain reflection, refraction and dispersion.

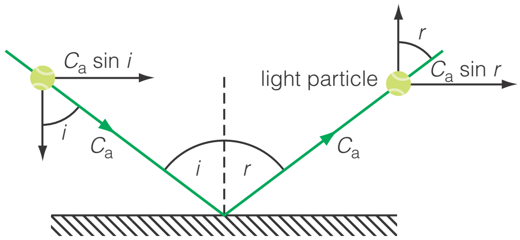


Figure 16.13 A ball being reflected from a solid surface. This is a model of how corpuscular theory could be used to explain the reflection of light.

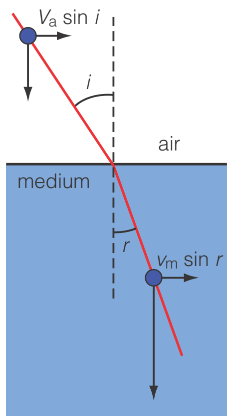


Figure 16.14 The change in velocity for a corpuscle entering a medium. The speed in the vertical direction increases as the corpuscle crosses the boundary.

Maths box

We can obtain an expression for the refractive index of the medium by equating the two components of velocity parallel to the surface of the medium:

$$v_a \sin i = v_m \sin r$$

$$\frac{\sin i}{\sin r} = \frac{V_m}{V_a} = {}_a n_m$$

TIP

Newton originally wrote that there were five colours in a spectrum: red, yellow, green, blue and violet. He later added orange and indigo to the spectrum to make it analogous to the number of notes in a musical scale. However, it can be hard to see the difference between indigo and violet.

Reflection

Imagine throwing a bouncy ball onto a hard surface. The ball bounces back at the same angle that it was thrown at. Figure 16.13 shows the change in velocity that occurs as a ball bounces from the surface. The velocity has been resolved into horizontal and vertical components both before and after the ball has been reflected from the surface.

As the ball hits the surface its vertical component of velocity is reversed, but the horizontal component remains unchanged. This means that the ball is reflected at the same angle at which it hit the surface.

Corpuscular theory treated light corpuscles as if they were solid elastic balls. The change in the vertical component of velocity means that the law of reflection can be explained. The angle of incidence is equal to the angle of reflection.

Refraction

Newton assumed that there was a force of attraction between matter and light. When light corpuscles were in the middle of the air or a transparent medium, such as glass or water, then the forces acting on the corpuscles acted on all sides of them and there was no resultant force. However, at the boundary between air and a medium, the forces were unbalanced and there was a greater force of attraction on the corpuscle. This meant that the vertical component of velocity increased. Figure 16.14 shows the change in velocity when a corpuscle entered a medium.

The horizontal component of velocity wasn't altered because there were equal numbers of air and medium particles on either side, so the horizontal forces were still balanced.

The change in the vertical component explained why the light ray changed direction towards the normal on entering the medium. It could also explain the increase in angle of the light when leaving the medium.

Corpuscular theory required the speed of light to be faster in a medium than in air.

Dispersion

Dispersion occurs when different colours of light are refracted by different amounts as they travel through a medium. Newton explained this phenomenon by assuming that different coloured light corpuscles had different mass, with red being the most massive. This meant that their velocities were affected slightly differently when entering the material. Red was least affected and so changed direction the least, as shown in Figure 16.15.

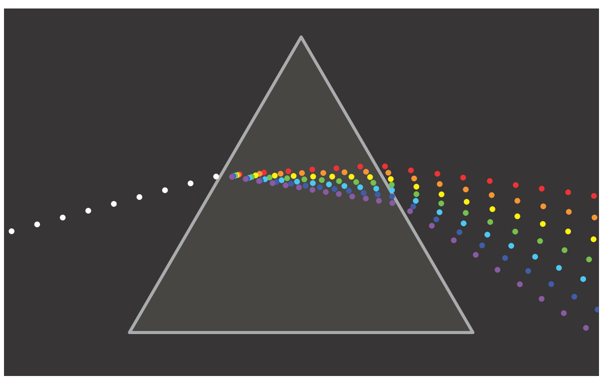


Figure 16.15 Dispersion of white light into a spectrum. Red corpuscles show least dispersion.

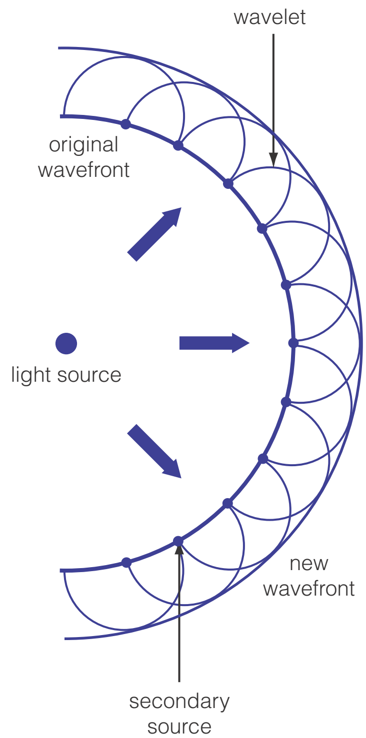


Figure 16.16 The propagation of a light wave according to Huygens' wave theory of light.

Huygens and the wave theory of light

Huygens was a Dutch astronomer. He proposed that light was a longitudinal wave similar to sound. As longitudinal waves require a medium to travel through, Huygens proposed that space was filled with a substance called æther. This was transparent and had no inertia (or mass).

Waves travelled through different materials by the propagation of wave fronts. Figure 16.16 shows a wavefront moving outwards from a light source. Each point on the original wavefront acts as a new point source, and wavelets spread out from them. These wavelets then combined to form a new wavefront.

Huygens' theory could also be used to explain reflection and refraction of light.

Reflection

When a wavefront was incident on a reflective surface, the point at which the wavefront is reflected acts as a secondary source and new wavelets are formed.

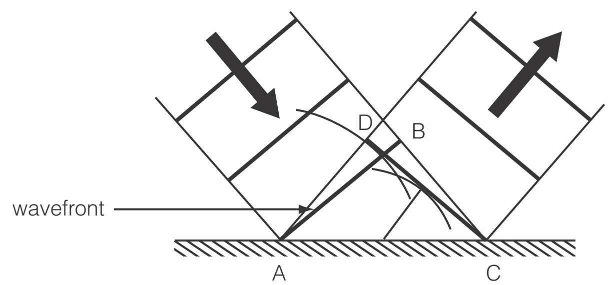


Figure 16.17 The reflection of light explained using Huygens' wave theory of light.

In Figure 16.17 the wavefront, AB, is reflected from the surface. As each part of the wavefront reaches the surface it acts as a secondary source and a new wavefront is created. This is shown as CD. The shape of the wavefront is not affected by reflection, and the angle of incidence is equal to the angle of reflection.

Refraction

Figure 16.18 shows how Huygens wave theory could be used to explain refraction of light. As the wavefront, AB, reaches the boundary, it acts as a secondary source of waves. The wavefront is travelling at an angle to the boundary so it takes the wavefront a finite amount of time to pass over the boundary. The wavelets formed as each point of the wave is incident on the boundary spread out in the medium and form a new wavefront, CD, inside the material.

Huygens' wave theory predicted that the speed of light would be slower inside a medium compared with in air.

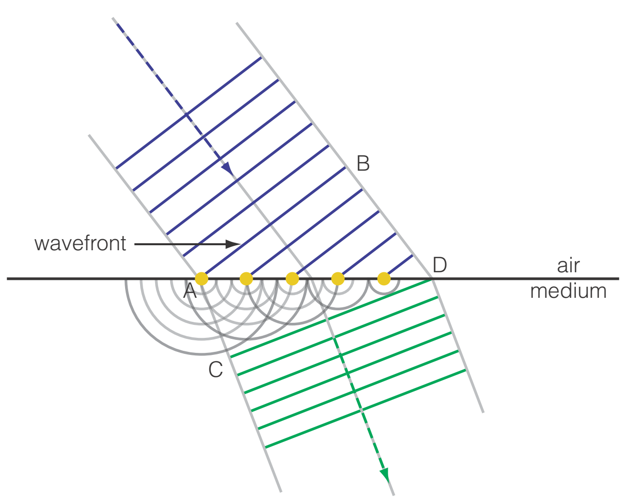


Figure 16.18 Refraction of light using Huygens' wave theory of light.

Comparing corpuscular and wave theories of light

Neither theory could fully explain all the properties of light. Table 16.4 shows a comparison of the theories, and the phenomena associated with light.

> **TIP**
> *Physicists now use wave theory to explain dispersion and colour. However, Huygens did not use his version of wave theory to do so as colour was not well understood at the time.

Table 16.4

	Newton's corpuscular theory	Huygens' wave theory
Reflection	✓	✓
Refraction	✓	✓
Dispersion	✓	✓*
Diffraction	✗	✓
Interference	✗	✓
Polarisation	✗	✗

Although Huygens' wave theory was able to explain some of the phenomena, it was not widely accepted at the time. There was, in the 1690s, no way to measure the speed of light so there was no straightforward way of testing each theory.

By describing light as a longitudinal wave, Huygens had to propose the aether in space as a medium for the wave to travel through. Again, at the time there was no way of testing if the aether actually existed.

Both theories of light had their merits but also some weaknesses. There was no way to measure the speed of light. Newton's theory became accepted because he was viewed to be the greater authority, particularly in England.

TEST YOURSELF

20 Light enters a glass block from the air. What do Newton's corpuscular theory and Huygens' wave theory predict about the speed of light in the glass compared with the speed of light in the air?

21 Define polarisation and discuss why neither Newton's or Huygens' theory of light could explain polarisation.

22 Give two reasons why Newton's theory was accepted as the best description of light rather than Huygens' theory.

Young's double slit experiment

The corpuscular theory of light was accepted for over 150 years. However, experiments carried out by Thomas Young at the beginning of the 19th century provided more evidence that Huygens' wave theory was a better explanation for the behaviour of light.

> **TIP**
> Young's paper 'Experiments and Calculations Relative to Physical Optics' can be read online at http://rstl.royalsocietypublishing.org/content/94/1.1

History box

In a paper published in 1804 Young described his experiments to observe the fringes formed when light goes round very small objects or through very small gaps. The first experiment was quite simple and Young writes:

'I made a small hole in a window-shutter, and covered it with a piece of thick paper, which I perforated with a fine needle. I brought into the sunbeam a slip of card about one-thirtieth of an inch in breadth, and observed its shadow.'

History box

Young described his observations of destructive interference by writing:

'we may … infer, that homogeneous light, at certain equal distances in the direction of its motion, is possessed of opposite qualities, capable of neutralising or destroying each other, and of extinguishing the light, where they happen to be united …'

Although Young couldn't fully explain why interference happened, his observations provided additional support to Huygens' wave theory of light.

TIP

In the exam you may be asked to explain why interference fringes were formed in Young's double slit experiment. Recall that dark fringes occur where there is destructive interference: the waves are 180° out of phase and the path difference is $(n+\frac{1}{2})\lambda$. Bright fringes occur where there is constructive interference: the waves are in phase and the path difference between the two slits is $n\lambda$. There is more detail on Young's double slit experiment in Book 1, Chapter 6.

Figure 16.19 shows the experimental set-up used for Young's double slit experiment. This experiment allows fringes to be seen more clearly because the light passing through the double slit is monochromatic and coherent.

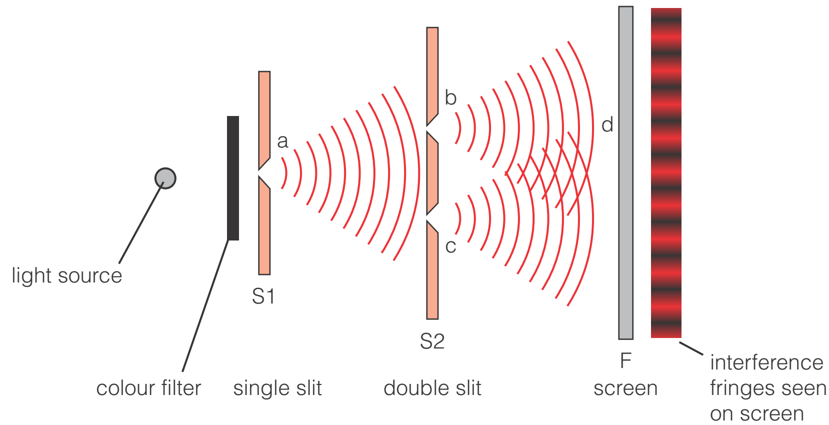


Figure 16.19 Schematic diagram of Young's double slit experiment showing the interference pattern seen on the screen.

Through his experiments, Young found that the spacing of the fringes depended on:

- the separation of the double slits
- the distance from the double slits to the screen
- the colour (wavelength) of the light.

Newton's corpuscular theory was unable to explain the interference patterns that Young observed. In Newton's theory, the corpuscles would go through each slit and only two bright fringes would be seen.

Two French scientists Augustin-Jean Fresnel and François Arago, whilst studying polarised light, developed a mathematical explanation of interference. They realised that if light was a transverse wave, not a longitudinal one, they could explain polarisation.

Fresnel and Arago's work provided the evidence that Newton's corpuscular theory was not correct and the wave theory of light was finally widely accepted.

TEST YOURSELF

23 Explain why Young's double slit experiment did not provide support for Newton's corpuscular theory of light.

24 Suggest why Young used monochromatic light in his experiment.

25 Discuss the advantages of using a laser as a light source compared with the experimental set-up shown in Figure 16.19.

Fizeau's speed of light experiment

In 1849, a French physicist, Hippolyte Fizeau, carried out a measurement of the speed of light which overcame the problem of experimenter reaction time. Figure 16.20 shows a diagram of Fizeau's apparatus. Using a lens, L_1, light from a source was focused onto the edge of a toothed wheel with 720 teeth. The wheel had regular notches cut into the circumference. The

History box

Galileo suggested that the speed of light could be measured over a number of miles using lanterns to produce timed flashes of light. His method was tried out in 1667, but there was no obvious delay in the time it took light to travel the distance of 1 mile (1609 m). In part, this was because the reaction time of the experimenters was greater than any possible measurable delay.

Maths box

Fizeau's calculations

The toothed wheel has N teeth, and is rotating at an angular velocity of n revolutions per second.

The distance travelled by the light is $2d$.

From circular motion we know that the time taken for the wheel to rotate through an angle θ is given by

$$t = \frac{\theta}{\omega}$$

where ω is the angular velocity of the wheel.

When the light is a minimum the light passes through one gap, but is blocked by the next tooth, so the wheel must rotate through an angle of

$$\frac{1}{2N}$$

The time it takes for the light to travel

$$t = \frac{1}{2nN}$$

$$\text{The speed of light} = \frac{\text{distance}}{\text{time}}$$

$$= \frac{\frac{2d}{1}}{2nN} = 4dnN$$

For Fizeau's experiment $N = 720$ teeth, $n = 12.6$ revs s^{-1}, $d = 8633$ m.

This gives a value for the speed of light of 3.13×10^8 m s^{-1}

light shone through one of the gaps on the edge of the wheel and passed through another lens, L_2 to form a parallel beam of light. The light then travelled to another lens, L_3, where it was focused onto a curved mirror, M, and reflected back towards L_2 and through the toothed wheel. The light was then observed using the eyepiece, E.

In Fizeau's experiment the two ends of the experiment were on two different hills on the outskirts of Paris, a distance of over 8 km apart. The light from the source therefore travelled over 16 km.

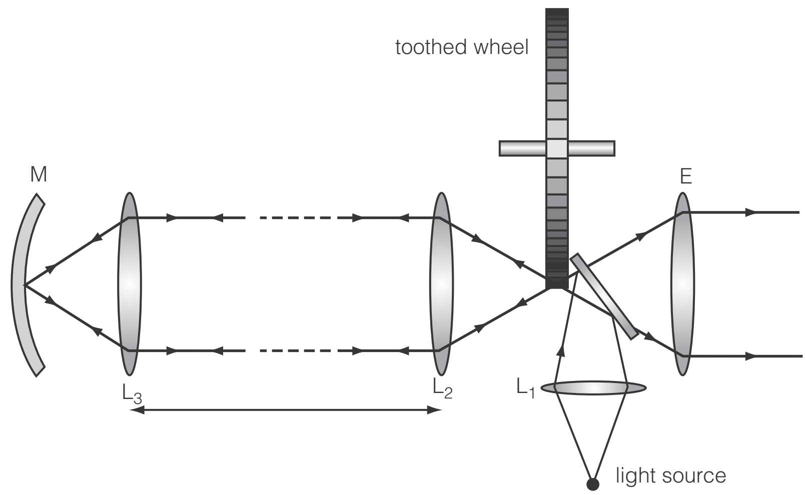


Figure 16.20 Fizeau's experiment to measure the speed of light.

The experiment was set up so that the light was correctly focused and could be seen in the eyepiece. Then the wheel was rotated, and the speed of rotation increased. The intensity of the light seen in the eyepiece varied. The light intensity was a minimum when the gap had moved and a tooth was in its place. The intensity was a maximum when the returning light passed through a gap.

Using his measurements of the variation in light Fizeau obtained an initial value for the speed of light as 313 274 000 ms^{-1}.

Fizeau also carried out further experiments to measure the speed of light in different materials, such as water. He found that the speed of light in water was slower than the speed of light in air.

Fizeau's experiments showed:

- light travels at a finite, but very fast, speed
- light travels more slowly in materials.

The wave theory of light predicted that light would slow down when travelling through a material, and so Fizeau's findings provided additional support that the wave theory provided the best description of light.

Maxwell's prediction of electromagnetic waves

TIP
Magnetism, electromagnetism, and the magnetic effect of a current, are covered in more detail in Chapter 7 of AQA A-level Physics Student Book 2. It would be helpful to review them as part of your revision for this module.

From the late 18th century, physicists had suspected that there was a relationship between electricity and magnetism. In 1820, physicist Hans Oersted gave a lecture on electricity and magnetism.

For one of the demonstrations in the lecture, he had a wire connected to a battery. He noticed that a compass needle near the wire spontaneously moved away from magnetic north when the battery was connected or disconnected from the wire. This demonstration showed that there was a magnetic field associated with a changing current. However, the nature of relationship between the two was not clear.

In 1864, James Clark Maxwell published a paper in which he gave a mathematical description of the relationship between electric and magnetic fields. His equations predicted that waves of oscillating electric and magnetic fields existed which would travel at a speed that could be calculated from simple experiments.

Maxwell predicted that the speed of these waves was given by:

$$c = \frac{1}{\sqrt{\varepsilon_0 \mu_0}}$$

Both ε_0 and μ_0 are constants which had already been measured experimentally by the time Maxwell developed his theory. ε_0 is the permittivity of free space and determines the strength of electric fields. μ_0 is the permeability of free space and determines the strength of magnetic fields.

TIP
You can read Maxwell's paper online at http://rstl.royalsocietypublishing.org/content/155/459

Using data for these constants from experiments, Maxwell obtained a value of 310 740 000 m s^{-1}. Comparing this value with measured values for the speed of light, such as Fizeau's, Maxwell wrote:

> *The agreement of the results seems to show that light and magnetism are affections of the same substance, and that light is an electromagnetic disturbance propagated through the field according to electromagnetic laws.*

Maxwell's equations also predicted the existence of electromagnetic waves other than visible light and ultraviolet. These predictions were shown to be correct when X-rays and radio waves were discovered.

Electromagnetic waves are formed by accelerating charged particles, often electrons. A charged particle produces an electric field. However, a moving charge causes a changing electric field perpendicular to the direction of motion. A changing electric field produces a changing magnetic field perpendicular to the electric field. The changing magnetic field produces a changing electric field, and so on. These oscillations in the magnetic and electric field do not require a medium to propagate through, and once they begin they will continue to travel through space until they interact with matter. Figure 16.21 shows the orientation of the electric and magnetic fields relative to the direction of motion of the electromagnetic wave.

The wavelength of an electromagnetic wave is the shortest distance between two points on the wave where the electric field and the magnetic field are in phase. This can be seen in Figure 16.21.

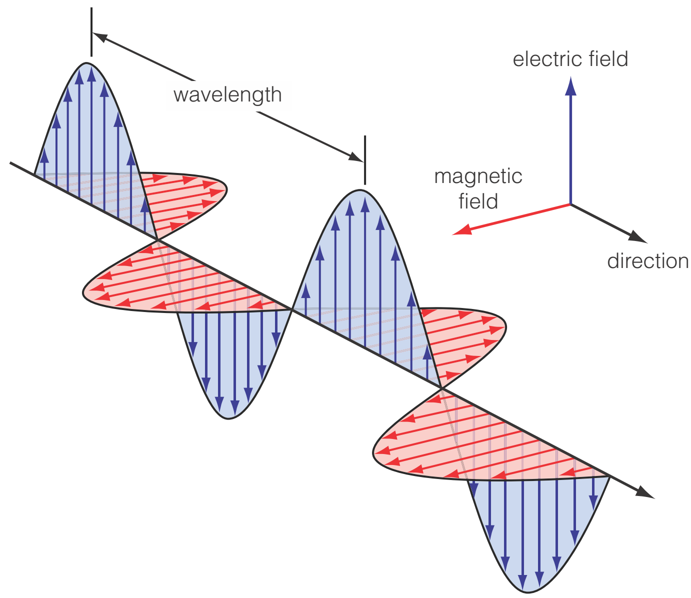


Figure 16.21 An electromagnetic wave. The electric and magnetic fields oscillate at right angles to each other.

EXAMPLE

The permittivity of free space, $\varepsilon_0 = 8.85 \times 10^{-12}$ F m^{-1}. The permeability of free space μ_0 is $4\pi \times 10^{-7}$ H m^{-1}. Use these values to calculate the predicted value for the speed of electromagnetic waves, such as light.

$$c = \sqrt{\frac{1}{\varepsilon_0 \mu_0}}$$

$$c = \sqrt{\frac{1}{8.85 \times 10^{-12}\,\text{F m}^{-1} \times 4\pi \times 10^{-7}\,\text{H m}^{-1}}}$$

$$c = \sqrt{\frac{1}{1.112 \times 10^{-17}}} = \frac{1}{3.33 \times 10^{-9}} = 2.998 \times 10^{8}\,\text{m s}^{-1}$$

$$c = 3 \times 10^{8}\ \text{m s}^{-1}$$

The close agreement between the values for the speed of light obtained by two different methods shows the importance of reproducibility in physics. Fizeau and Maxwell obtained similar values for the speed of light using two very different methods. Fizeau's method involved experimentation and direct measurement, whilst Maxwell's value was a consequence of the development of a mathematical description for electromagnetic waves. This provided support that Maxwell's equations for electromagnetism described the observed properties of light (an electromagnetic wave) and were a correct description of the nature of electromagnetic waves.

Hertz's discovery of radio waves

Heinreich Hertz was the first person to generate radio waves in 1887. In doing so he demonstrated that Maxwell's equations describing electromagnetic waves were correct.

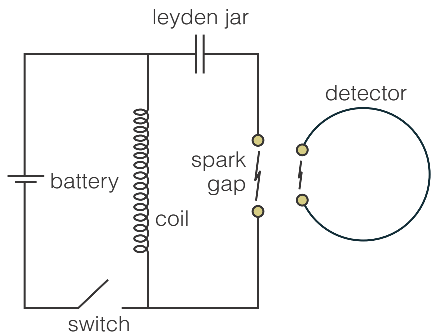


Figure 16.22 A simplified circuit diagram showing the equipment Hertz used to generate and detect radio waves.

Figure 16.22 shows the equipment that Hertz used to generate radio waves. The transmitter consisted of a circuit containing a Leiden jar (an early type of capacitor), an induction coil and a loop of wire with small copper spheres on each end. The detector was an incomplete loop of copper wire, also with small copper spheres at each end.

When the switch was closed, the induction coil produced a high voltage across the spark gap. This spark (moving charge) caused radio waves to be produced. The waves propagated through the air towards the detector. These electromagnetic waves then induced a voltage in the copper wire detector which caused a small spark to jump across the detector gap.

Hertz found that a metal plate placed between the transmitter and detector prevented the waves from reaching the detector. However, insulators didn't appear to affect the radio waves. Placing a concave-shaped metal plate behind the detector made the detector sparks stronger because the radio waves were focused by the plate back onto the detector.

The waves produced by the spark gap transmitter are **polarised**. If a receiving aerial is used to detect the waves produced, then the intensity is a maximum when the receiver is parallel to the transmitter spark gap. This is shown in Figure 16.23. The intensity decreases as the aerial is rotated through 90°, until it is zero when the aerial is parallel to the magnetic field of the electromagnetic wave.

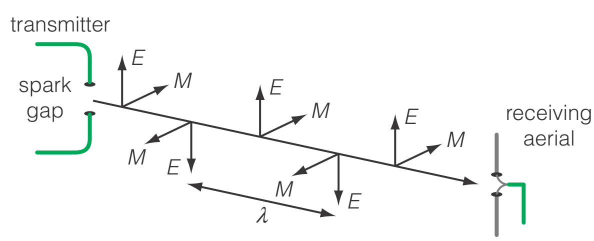


Figure 16.23 The generation of vertically polarised electromagnetic waves using a spark gap transmitter.

TIP

Polarisation was covered in Book 1, Chapter 5. You may wish to review that material when revising for the exam in this topic.

The reason that the waves are polarised is because the motion of the electrons in the spark is in one plane (vertical in the diagram). This sets up electromagnetic waves which have the E field oscillating in one plane only.

When the electromagnetic wave reaches the receiving aerial, then the oscillating electric field causes electrons in the aerial to move, and the signal to be detected. As the angle between the electric field and aerial is altered, the electric field causes less motion of electrons in the aerial and the intensity of the signal decreases. When the receiving aerial is at right angles to the electric field, the aerial will not be affected by the electric field.

Measuring the speed of radio waves

If the waves that Hertz was producing were the electromagnetic waves predicted by Maxwell, then they should travel at the same speed. To calculate the speed of the radio waves, Hertz created standing waves by reflecting radio waves using a flat metal plate, as shown in Figure 16.24. As he moved a dipole detector between the transmitter and the plate, Hertz found that there were points where the signal was a maximum and points where it was a minimum. A pattern of antinodes and nodes had been produced.

TIP

In standing waves, the distance between two nodes, or two antinodes, is equal to half a wavelength.

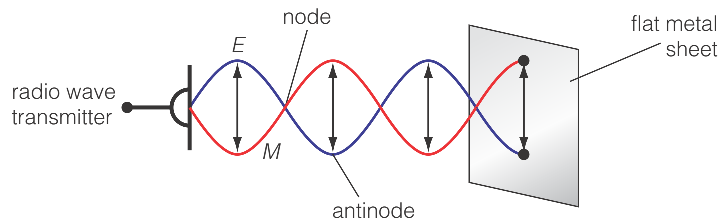


Figure 16.24 Creating standing waves using a flat metal sheet to reflect the waves.

By measuring the distance between two adjacent maxima (antinodes), Hertz was able to deduce the wavelength of the radio waves. He calculated the frequency of the waves by assuming that the generation of the radio waves was due to the electrical characteristics of the transmitter circuit.

Hertz's calculations gave a value for the speed of radio waves which was very similar to that predicted by Maxwell. He had shown that radio waves were a type of electromagnetic radiation.

TEST YOURSELF

26 How did the measurement of the speed of radio waves support Maxwell's theory of electromagnetic radiation?

27 Explain how stationary radio waves can be formed using a radio wave transmitter and a flat metal sheet.

Wave theory provided a good description of some of the properties of light and other electromagnetic waves, such as reflection, refraction and interference. However, towards the end of the 19th century, there were some experimental observations which the theory couldn't explain.

Blackbody radiation, Planck and the ultraviolet catastrophe

All objects with a temperature above absolute zero will emit electromagnetic waves in the form of infrared radiation. In general, the frequency of the emitted wave will depend on the temperature of the object and the surface of the object. You will recall that dark coloured matt surfaces absorb and emit radiation better than shiny white surfaces.

We can't always perceive the infrared radiation emitted. However, very hot objects emit electromagnetic waves with a frequency in the range of visible light, as can be seen in the picture of a blacksmith's forge in Figure 16.25.

In 1860, Gustav Robert Kirchoff introduced the term **black-body** radiation whilst studying the electromagnetic radiation absorbed and emitted from objects. Kirchoff described a theoretical object that absorbed or emitted all radiation which was incident on it. To obtain black bodies, Kirchoff covered objects with black soot from burning lamps.

Figure 16.25 Blacksmiths can tell the temperature of the metal by observing its colour. Visible and infrared radiation are being emitted from the surface.

A **black-body** is an object that absorbs or emits all wavelengths of electromagnetic radiation falling on the object. It does not reflect any wavelengths.

Maths box

The amount of radiation that should be given off by a black body, according to classical theory, was calculated using an equation called the Rayleigh-Jeans law. This links the radiance of an object with its temperature and wavelength. The radiance is the energy given off by an object per second measured over a spherical volume. This is also known as the energy density (the amount of energy in a given volume).

$$radiance = \frac{2ckT}{\lambda^4}$$

where c is the speed of light, k is the Boltzmann constant, T is the temperature of the object and λ is the wavelength of electromagnetic wave.

As the wavelength gets smaller, then the value of radiance gets much larger.

As $\lambda \rightarrow 0$ then radiance $\rightarrow \infty$

Treating electromagnetic radiation as a wave led to a theory which predicted that the energy density of wavelengths given off from a black-body would rapidly become infinite at short wavelengths (see maths box).

When physicists measured the spectra from black-bodies, this large increase in energy density at shorter wavelengths did not happen. Figure 16.26 shows the energy density as a function of wavelength for black-body objects at different temperatures and compares it with the value predicted by the theory. The difference between the theoretical values and the experimental values were particularly large at short wavelengths. In 1911, this problem was nickname the **ultraviolet catastrophe** by Paul Ehrenfest.

Ultraviolet catastrophe. The term used to describe the prediction (made by classical wave theory) that the energy emitted by a black-body would continue to increase as wavelength decreased. This did not describe the behaviour that was observed.

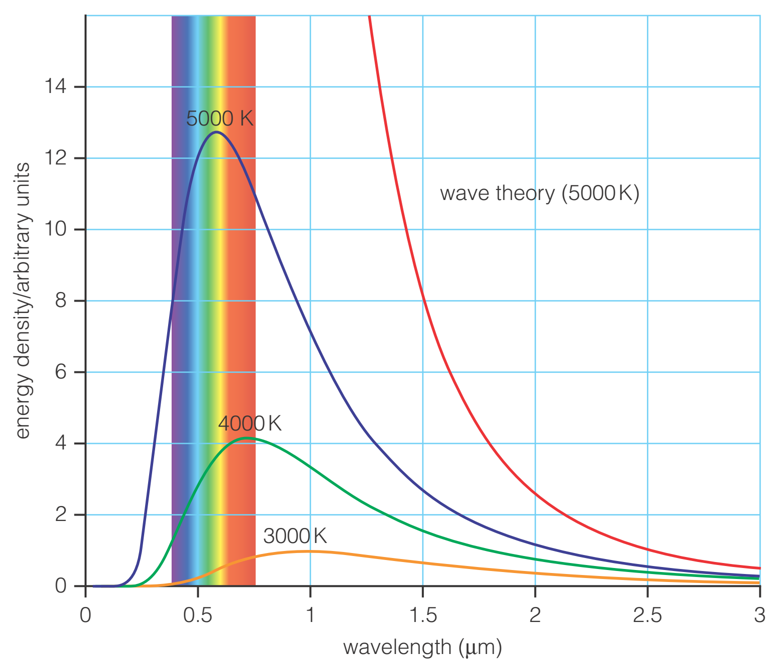


Figure 16.26 Graph showing the energy density of wavelengths emitted from black-body objects at different temperatures. Wave theory predicted that the energy density becomes infinite at short wavelengths.

In 1900, Max Planck worked to describe the behaviour of black-bodies mathematically. His equations contained a number of constants which were derived from experiments, rather than from the theory. In order to make his theory match the experimental observations, Planck made an assumption about the oscillators that were producing the electromagnetic radiation within the black-body. He assumed that the energy was quantised and is given by the equation:

$$E = nhf$$

where

n is an integer (1, 2, 3, …)

h is Planck's constant and

f is the frequency of the oscillator.

Planck initially assumed that this quantisation was a mathematical technique, and wasn't something which could be explained physically. However, it did mean that the theoretical predictions matched the experimental results.

The discovery of photoelectricity

TIP

The photoelectric effect is covered in Book 1, Chapter 4.

History box

Photoelectricity was first observed by Hertz during his experiments on radio waves. In order to observe the sparks produced more clearly, he put the spark gap receiver into a darkened box. He noticed that when he did this the maximum spark length reduced. After further experimentation, he concluded that when UV radiation was shone onto the receiver the maximum spark length was produced. However, he didn't pursue this any further. We now know that the UV radiation releases electrons from the surface of the metal and increases the spark current.

Following Hertz's publication of his research, other scientists extended his work and investigated the effect of UV and other light on metal plates. One, Alexander Stoletov, developed apparatus to measure the photoelectric effect. He found that, by putting a metal plate into a circuit and shining UV light onto the plate, he could measure a current due to the photoelectric effect. This photo-current was directly proportional to the intensity of the UV light and due to the emission of electrons from the surface of the metal plate. The electrons emitted are called photoelectrons.

When ultra-violet light is shone onto a zinc plate attached to a negatively charged gold leaf electroscope, the electroscope is seen to discharge. By contrast, shining visible light onto the zinc plate does not cause any change to the electroscope.

This was explained (in Book 1), by the idea that the UV photons have sufficient energy to knock out an electron from the zinc. But, the photons of visible light do not have sufficient energy to dislodge an electron from the metal surface.

Photoelectric effect. This occurs when light shining on the surface of a material, particularly a metal, releases electrons from the material.

Intensity. The light energy per second incident on the surface of the metal.

Further investigations into the **photoelectric effect** were made and the following features were observed:

- No photoelectrons are emitted if the incident light is below a threshold frequency, f_0. The threshold frequency varies for different metals. Visible light can be used to emit electrons from alkali metals such as sodium and potassium, but other metals require ultraviolet light which has a higher frequency.
- Photoemission starts to occur as soon as light of the appropriate frequency is shone onto the metal surface.
- As the **intensity** of incident light increases the number of photoelectrons emitted also increases.
- Photoelectrons are emitted with a range of kinetic energies. The maximum kinetic energy, KE_{max}, depends on the frequency of the incident light (as long as it is above the threshold frequency).

Classical wave theory and the photoelectric effect

In order to be emitted from the surface of a metal, the electrons must gain energy.

Classical wave theory made the following predictions about the photoelectric effect:

- Photoelectrons would be emitted for all frequencies of incident light. The energy transferred could 'build up' and so photoelectrons should be emitted for all frequencies of light. There was no threshold frequency.
- There would be a delay between shining light onto the metal surface and the emission of photoelectrons. At lower frequencies or intensities, it would take a longer time before emission started.
- The intensity of the light should be proportional to the maximum kinetic energy of the electrons.

When we compare the predictions made by wave theory, and the experimental observations of photoelectricity, we can see that wave theory does not explain any of the observations.

Explaining photoelectricity

The explanation of photoelectricity was developed by Albert Einstein. In 1905, he published a paper in which he proposed that electromagnetic radiation was composed of discrete quanta of energy rather than being a continuous wave. By making this assumption, the experimental observations of the photoelectric effect could be explained.

This concept of light being composed of discrete quanta follows from Planck's work on black-body radiation, and that is why the constant is called Planck's constant.

However, Einstein's view of quanta was that they were real physical phenomena, not just a mathematical convenience. These quanta were later named photons.

Each photon carries an amount of energy, E, which is directly proportional to the frequency, f, of the electromagnetic radiation:

$$E = hf$$

where h is Planck's constant, 6.63×10^{-34} J s.

The photon theory of light is used to explain the experimental observations for the photoelectric effect:

- In order to leave the surface of a metal, each electron must obtain sufficient energy through an interaction with a single photon. Photons with a low frequency do not transfer enough energy and so no electron is emitted. There is a threshold frequency below which no electrons are emitted.
- Electrons need less energy to leave the surface of some metals. The threshold frequency for these metals is lower. The amount of energy required is called the work function, $\boldsymbol{\Phi}$, of the metal.
- As the intensity of light increases, the number of photons increases so more electrons can be released from the surface. However, each photon still carries the same amount of energy, so if the light is below the threshold frequency no electrons will be emitted, regardless of intensity.
- The energy transferred by a photon provides the electron with energy to escape the surface of the metal. If the photon energy is greater than this, then the additional energy is seen as the kinetic energy of the electron. This means that the maximum kinetic energy will increase with frequency.

For each photon we can say:

$$E = hf$$

In order to be emitted from the surface of a metal, an electron must gain energy at least equal to the work function of the metal. If the photon carries more energy than the work function, then the electron will gain kinetic energy *KE*. We can write:

$$hf = \boldsymbol{\Phi} + KE$$

where

hf is the energy of the photon in J

$\boldsymbol{\Phi}$ is the work function of the metal in J

KE is the kinetic energy of the electron in J.

The maximum kinetic energy of the electron will be equal to the energy of the photon less the work function of the metal:

$$KE_{\text{max}} = hf - \boldsymbol{\Phi}$$

TIP

The work function of a metal may be given in either joules (J) or electron volts (eV). Remember that 1 eV = 1.6×10^{-19} J.

EXAMPLE

Light of wavelength 408 nm is incident on a calcium metal surface. Calculate the maximum kinetic energy of the emitted photoelectrons.

The work function of calcium is 4.64×10^{-19} J, Planck's constant = 6.63×10^{-34} J s and $c = 3 \times 10^{8}$ m s^{-1}.

$$f = \frac{c}{\lambda}$$

Using:

$$KE_{max} = hf - \Phi$$

$$KE_{max} = \frac{hc}{\lambda} - \Phi$$

$$KE_{max} = \frac{6.63 \times 10^{-34}\,\text{J s} \times 3 \times 0^{8}\,\text{m s}^{-1}}{408 \times 10^{-9}\,\text{m}} - 4.64 \times 10^{-19}\,\text{J}$$

$$KE_{max} = 4.875 \times 10^{-19}\,\text{J} - 4.64 \times 10^{-19}\,\text{J}$$

$$KE_{max} = 2.4 \times 10^{-20}\,\text{J}$$

Einstein's description of electromagnetic radiation as being quantised and his explanation of the photoelectric effect led to him being awarded the Nobel prize for Physics in 1921. He showed that light, and other electromagnetic radiation, could be thought of as massless particles carrying energy, rather than waves.

ACTIVITY

Measuring Planck's constant

The photoelectric effect can be investigated using a photocell. A photocell is a circuit component which contains a metal-plate anode and a cathode. When light of an appropriate frequency is shone onto the anode, photoelectrons are emitted. The photoelectrons move towards the cathode and the current can be measured using a sensitive pico ammeter. Figure 16.27 shows the circuit used in this experiment.

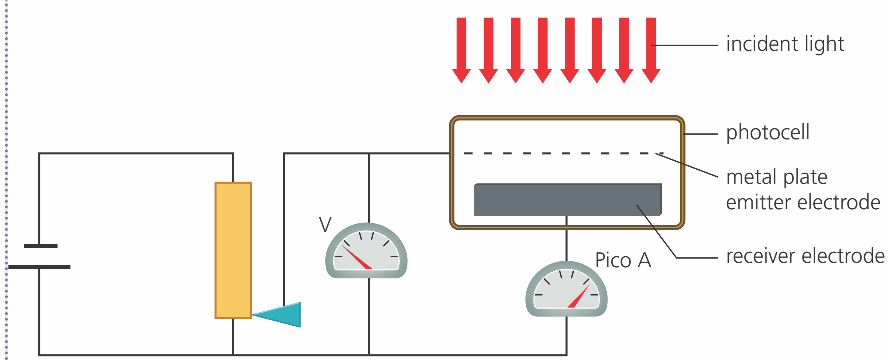


Figure 16.27 Measuring Planck's constant using a photocell.

Initially, the metal plate emitter electrode is at 0 V. The photoelectric current is measured for different frequencies of light. The electrons are produced at the emitter electrode, and move towards the receiver electrode.

For each frequency of light, the potential divider is used to make the emitter electrode increasingly positive compared with the receiver electrode. The photoelectrons have to do extra work to move away from the metal surface if it is positively charged. The number of electrons emitted decreases as the voltage of the emitter electrode increases and so the photoelectric current decreases. The stopping voltage, V_s, is the voltage at which the photoelectric current just becomes zero.

At the stopping voltage, the work done by each electron, eV_s, will be equal to the maximum kinetic energy so

$$eV_s = KE_{max}$$

We know that

$$KE_{max} = hf - \Phi$$

and we can therefore write:

$$eV_s = hf - \Phi$$

Comparing this with the equation for a straight line, $y = mx + c$, plotting a graph of eV_s on the y-axis against frequency on the x-axis will give a gradient equal to Planck's constant and a negative y-intercept equal to the work-function of the metal (in J).

The data in Table 16.5 were taken using the method described using two different photocells each containing a different metal.

Table 16.5

	Metal 1	Metal 2
Frequency/ 10^{14} Hz	V_s/V	V_s/V
5.66	0.05	0.36
5.88	0.14	0.44
6.37	0.36	0.64
6.98	0.61	0.89
8.11	1.06	1.36

(a) Describe the relationship between stopping voltage and the maximum kinetic energy of the photoelectrons.
(b) Plot a graph of eV_s against frequency for both metals. Use the same axes for both sets of data.
(c) Using your graph, calculate a value for Planck's constant.
(d) Using your graph, calculate the work function for each metal.

TEST YOURSELF

28 The threshold frequency of copper is 1.1×10^{15} Hz. Describe what is observed when a sheet of copper is exposed to light of frequency (a) 1.0×10^{15} Hz and (b) 1.2×10^{15} Hz.

29 Light of 486 nm is incident on the surface of a sodium metal plate. Sodium has a work function of 3.8×10^{-19} J. Calculate:
(a) the energy of a photon of light at this wavelength
(b) the maximum kinetic energy of the emitted photoelectrons.

30 The work function of a metal surface is 5.6×10^{-19} J. Light of wavelength 460 nm is incident on the surface.
Determine whether photoelectrons will be emitted from the metal surface.

31 State two of the features of the photoelectric effect that wave theory could not explain. Describe how the photon theory can explain these features.

Wave-particle duality

In his PhD thesis in 1924, Louis de Broglie proposed the hypothesis that all matter particles have a wave associated with them. In doing so, he drew on the theories of light which suggested that light had wave-like properties when considering phenomena such as reflection, interference and polarisation, but particle-like properties when considering phenomena such as the photoelectric effect.

History box

De Broglie's hypothesis remained a theoretical model for many years, although early in the 1920s it was suggested that matter waves could be investigated by looking at the scattering of electrons as they travelled through crystalline solids. Following this, two physicists, Clinton Davisson and Lester Germer, carried out electron diffraction experiments that confirmed that particles could display wave-like properties.

De Broglie thought that the wavelength of the matter waves was related to the momentum of the particle.

A particle with mass, m, travelling at a speed, v, has a momentum, p, where $p = mv$.

De Broglie's hypothesis states that the relationship between its momentum, p, and its wavelength is

$$p = \frac{h}{\lambda}$$

where h is Planck's constant in J s

p is momentum in kg m s^{-1}

λ is wavelength in m.

TEST YOURSELF

32 An electron has a de Broglie wavelength of 2.3×10^{-8} m. Calculate its speed.

33 An electron and a proton are moving at the same speed. Are their de Broglie wavelengths the same? Explain your answer.

34 Calculate the wavelength of an electron moving at 1% of the speed of light.

Electron diffraction

A beam of electrons with uniform speed is directed through a thin metal foil, as shown in Figure 16.28. When a fluorescent plate is placed on the other side of the foil and developed, a pattern of concentric rings is observed.

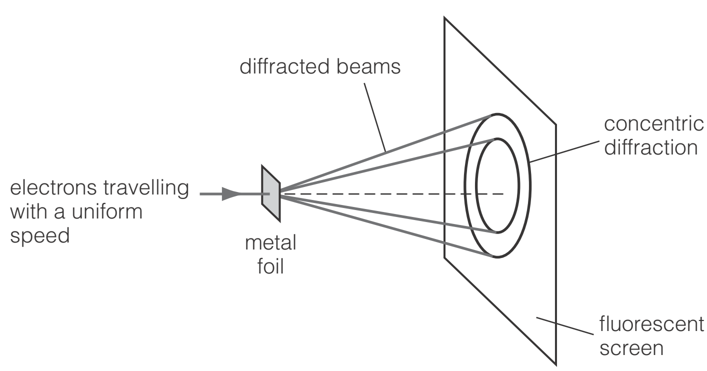


Figure 16.28 A simplified diagram showing experimental measurement of electron diffraction through a thin metal foil. The apparatus is contained in an evacuated tube (not shown).

The pattern of diffraction rings seen in this experiment is very similar to those obtained during X-ray diffraction experiments.

In the electron diffraction tube, each plane of atoms in the metal foil behaves like a diffraction grating. This causes constructive and destructive interference leading to the pattern of dark and bright rings.

The angle, θ, through which the waves are scattered is related to the wavelength, λ:

$$\lambda \propto \sin \theta$$

Experiments with electron diffraction show that as the momentum of the electrons increases the angle through which they are scattered decreases.

De Broglie's hypothesis predicted that the wavelength of particles was inversely proportional to their momentum. As momentum increases, wavelength decreases.

Electrons show the same behaviour as the X-rays: increasing the momentum leads to a decrease in the wavelength which led to a corresponding decrease in the scattering angle.

These experiments confirmed to physicists that particles could have wave-like properties and that de Broglie's hypothesis was correct.

The electron diffraction experiment can be carried out in school science using an electron diffraction tube, as shown in Figure 16.29. Figure 16.30 shows an electron diffraction tube in use.

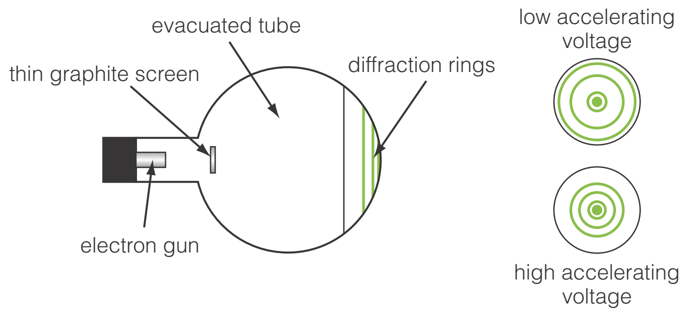


Figure 16.29 An electron diffraction tube showing the diffraction pattern obtained by altering the accelerating voltage between the anode and cathode in the electron gun.

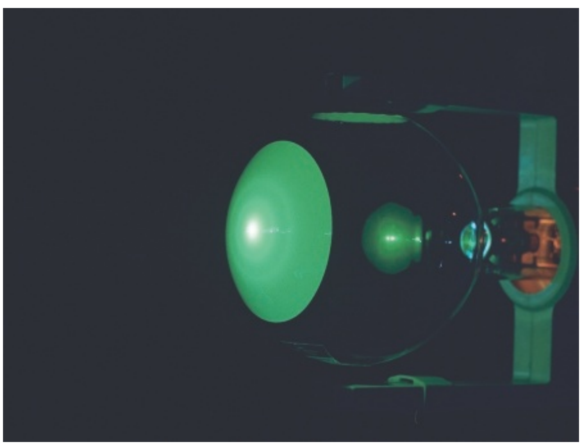


Figure 16.30 An electron diffraction tube showing the concentric circles of bright and darker patches on the fluorescent screen at the end of the tube.

MATHS BOX

For any number, n, the square root of that number can be written as a fractional index:

$$\sqrt{n} = n^{\frac{1}{2}}$$

and the square root of the inverse of the number can be written:

$$\sqrt{\frac{1}{n}} = n^{-\frac{1}{2}}$$

We can therefore write the de Broglie wavelength as:

$$\lambda = \frac{h}{m^1 \, m^{-\frac{1}{2}} \sqrt{2eV}}$$

To multiply numbers with indices we add the indices, so

$$m^1 \times m^{-\frac{1}{2}} = m^{\left(1-\frac{1}{2}\right)} = m^{-\frac{1}{2}} = \sqrt{m}$$

so

$$\lambda = \frac{h}{\sqrt{m}\,\sqrt{2eV}}$$

or

$$\lambda = \frac{h}{\sqrt{2meV}}$$

TIP

The equations we have used for the de Broglie wavelength are valid when the speed of the particles is much smaller than the speed of light. If the electrons are travelling closer to the speed of light, then relativity must be taken into account.

As the electrons are accelerated towards the anode through a potential difference, V, their kinetic energy will be equal to the work done.

$$\frac{1}{2}mv^2 = eV$$

and

$$v = \sqrt{\frac{2eV}{m}}$$

From earlier, the de Broglie wavelength for each electron will be:

$$\lambda = \frac{h}{mv}$$

Substituting for v into this equation we obtain:

$$\lambda = \frac{h}{m} \div \sqrt{\frac{2eV}{m}}$$

and we can simplify this to

$$\lambda = \frac{h}{\sqrt{2meV}}$$

Increasing the anode voltage decreases the wavelength of the matter waves. This gives less scattering, as can be seen in Figure 16.29.

EXAMPLE

In an electron gun, electrons are released due to thermionic emission. Calculate the de Broglie wavelength of an electron accelerated from rest through a potential difference of 2500 V.

$$\lambda = \frac{h}{\sqrt{2meV}}$$

$$\lambda = \frac{6.63 \times 10^{-34}\,\text{J s}}{\sqrt{2 \times 9.11 \times 10^{-31}\,\text{kg} \times 1.6 \times 10^{-19}\,\text{C} \times 2500\,\text{V}}}$$

$$\lambda = \frac{6.63 \times 10^{-34}}{\sqrt{7.288 \times 10^{-46}}} = \frac{6.63 \times 10^{-34}}{2.699 \times 10^{-23}}$$

$$\lambda = 2.5 \times 10^{-11}\text{ m}$$

This wavelength is much smaller than that of visible light.

TEST YOURSELF

35 Calculate the wavelength of an electron if the accelerating voltage is 1500 V in an electron diffraction tube.

36 An electron has a wavelength of 2.05×10^{-11} m. Calculate the potential difference through which it has been accelerated.

Electron microscopes

Resolving power: The ability of an optical instrument to distinguish between two features close together on an object under examination. The resolving power is limited by the diffraction of light.

In 1873 two physicists, Hermann von Helmholtz and Ernst Abbe, showed that the **resolving power** of a microscope was related to the wavelength of the light being used. The resolving power depends on how much diffraction occurs when the light passes through the objective lens of the microscope. A shorter wavelength, such as blue light, increases the resolving power and allows finer details on the specimen to be seen.

Following de Broglie's hypothesis and electron diffraction experiments which showed that electrons have wave-like properties, from 1932, physicists and engineers began to develop prototype electron microscopes. Ernst Ruska, one of these physicists, was jointly awarded a Nobel prize in 1986 for his work on the development of electron microscopy.

Electrons with sufficient momentum can have much shorter wavelengths than light which means that a microscope using the wave-like properties of electrons will have much greater resolving power than an optical microscope.

EXAMPLE

The diameter of an atom is of the order of 10^{-10} m. Calculate the accelerating voltage needed for electrons to have a wavelength of this order of magnitude.

$$\lambda = \frac{h}{\sqrt{2meV}}$$

Squaring both sides of the equation:

$$\lambda^2 = \frac{h^2}{2meV}$$

Rearranging to make V the subject:

$$V = \frac{h^2}{2me\lambda^2}$$

Substituting the data given in the question:

$$V = \frac{(6.63 \times 10^{-34}\ \text{J s})^2}{2 \times 9.11 \times 10^{-31}\ \text{kg} \times 1.6 \times 10^{-19}\ \text{C} \times (1 \times 10^{-10}\ \text{m})^2}$$

$$V = \frac{4.3957 \times 10^{-67}}{2.9152 \times 10^{-69}} = 150.78\ \text{V}$$

An accelerating voltage of approximately 150 V will provide electrons with a wavelength of the order of the atomic diameter.

Electron microscopes have lenses, just as optical microscopes do. However, rather than being formed from glass, the 'lenses' in an electron microscope alter the path of the electron by the use of a magnetic or electric field. Figure 16.31 shows an electron micrograph (picture) of a graphite surface taken with a scanning tunnelling microscope.

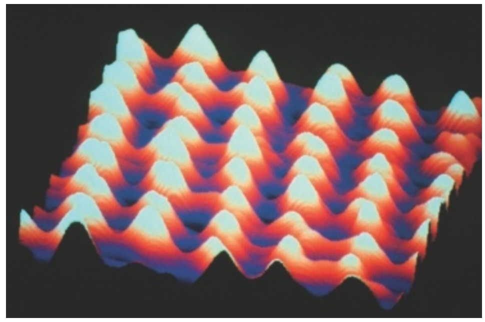


Figure 16.31 The surface of graphite as seen by a scanning tunnelling microscope showing the atomic structure of the specimen.

The transmission electron microscope (TEM)

The first electron microscopes to be developed were transmission electron microscopes (TEM).

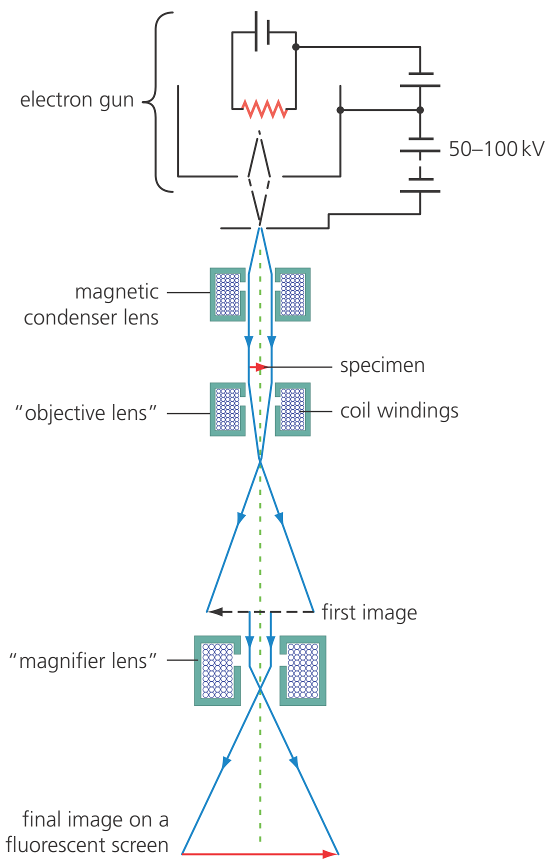


Figure 16.32 Transmission electron microscope. The lenses are formed from electromagnets that are used to alter the path of the electrons.

In a TEM a beam of electrons passes through an extremely thin specimen. As the electrons interact with structures within the specimen (e.g. cell walls, grain boundaries in metals) the electrons are scattered. The scattered electrons are focused by a number of magnetic lenses and a magnified image of the sample is formed. The image may be formed onto photographic film, but other methods of recording the image may be used including CCDs.

In order to reduce collisions of the electrons with air molecules, the inside of the TEM is reduced to a very low pressure; it is effectively an evacuated tube. Figure 16.32 shows a schematic diagram of a TEM and indicates the path of the electrons through the microscope.

The electron gun produces a beam of electrons. The speed, and therefore the wavelength, of the electrons is controlled using the anode voltage. The condenser lens is used to form the electrons into a parallel beam. This beam then passes through the specimen which is held in place on a specimen stage, often a fine gold wire grid. The stage can be moved and rotated so that different sections of the specimen can be examined.

The scattered beam of electrons then passes through the objective lens which forms a magnified, inverted image of the sample. Figure 16.33 compares the action of a magnetic lens and the action of an optical lens.

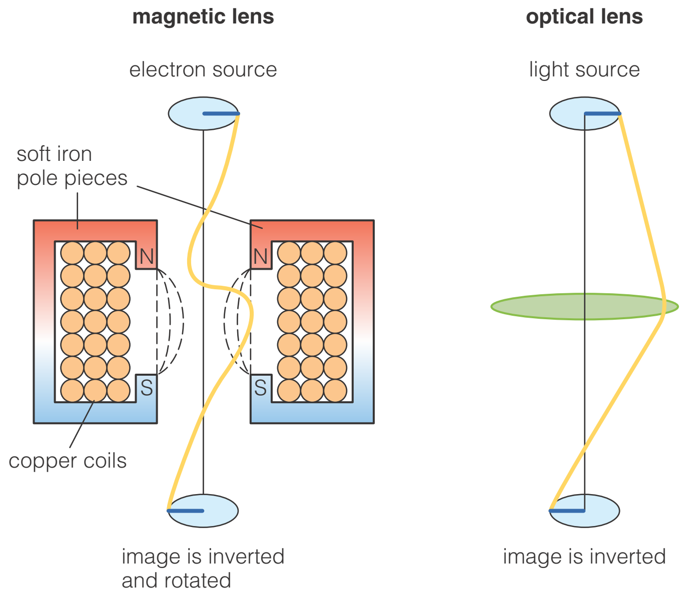


Figure 16.33 Comparing magnetic lenses with optical lenses. In each case, electrons or light rays which travel through the centre of the lens do not change direction. Light rays or electrons which pass through the edge of the lens are more affected – either by refraction in the case of light, or by the magnetic flux in the case of electrons.

Finally, the magnifier lens focuses the electrons from the intermediate image to form a final magnified image on the screen or recording device.

The resolving power of the TEM can be increased by increasing the anode p.d. This also enlarges the image on the screen and increases the magnification and detail which can be seen.

The detail that can be seen using TEM is limited by the thickness of the specimen used and by lens aberrations.

The specimen used in TEM must be very thin, of the order of hundreds of nanometres. As the electrons pass through the material they slow down slightly. This causes their de Broglie wavelength to increase and therefore the resolving power to decrease.

Lens aberration occurs because the scattered electrons will be travelling at slightly different speeds. This means that the magnetic lenses may be unable to focus electrons from the same part of the sample onto the same point on the screen.

Recent improvements in correcting lens aberration mean that some transmission electron microscopes can have a resolution of approximately 50×10^{-12} m, giving magnifications of over 50 million.

The scanning tunnelling microscope (STM)

The scanning tunnelling microscope uses a very fine tipped probe which moves across, or scans, a small area of the surface of a specimen. It makes use of the wave-like nature of electrons. Although it is called a microscope, it has no lenses and works in a different way from both light microscopes and transmission electron microscopes.

The scanning tunnelling microscope was developed in 1981 by Gerd Binning and Heinrich Rohrer. They were jointly awarded a Nobel prize for their work in 1986, along with Ernst Ruska.

> **TIP**
> Piezoelectric materials produce a small p.d. when stretched or compressed. They also produce a small change in length when a p.d. is applied to them. Piezoelectric transducers are components which use these small changes to move the probe tip a very small amount in the STM.

Figure 16.34 shows the general principle by which a scanning tunnelling microscope works on the large scale and the atomic scale.

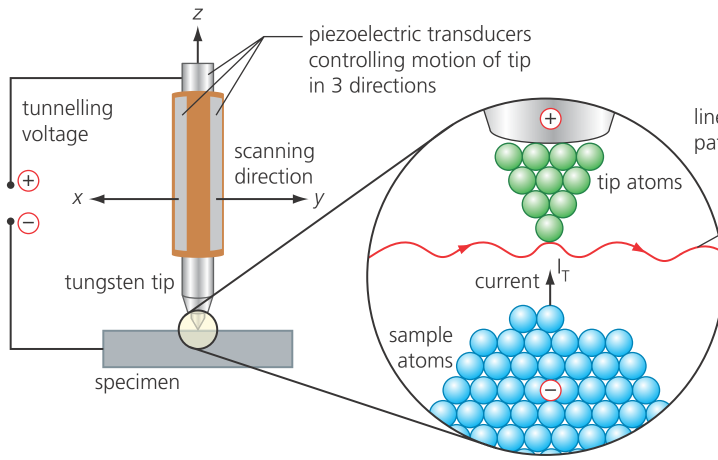


Figure 16.34 The scanning tunnelling microscope makes use of the current which flows from the specimen to the fine tip of the probe.

The probe tip is held very close to the surface of the specimen, typically no more than 1 nm above the surface. The tip is slightly positive compared with the surface of the specimen. At this small distance, electrons can 'tunnel' across the gap, causing a tunnelling current to flow. This current increases measurably as the tip gets closer to the surface and decreases if the distance between the tip and surface increases.

A STM may be used in two different modes:

- Constant current mode. The tip is moved across the surface and piezoelectric transducers move the tip up or down so as to keep the measured tunnelling current constant. The up and down movement can be imaged as a map of the peaks and troughs of the surface.
- Constant height mode. The tip is moved across the surface at a fixed vertical position. The change in the tunnelling current can be used to map the height of the surface of the specimen.

> **TIP**
> Angstrom, Å, is a unit of measurement often used by scientists working at atomic scales. 1 Å = 1×10^{-10} m.

In both modes, the resolving power of a scanning electron microscope provides a map of the surface on the scale of individual atoms. The vertical resolution is of the order of 0.1 Å, which is smaller than the smallest atom. Figure 16.35 shows an STM image of the surface of graphite. Each peak is a carbon atom and the spacing between two atoms is 2.5 Å.

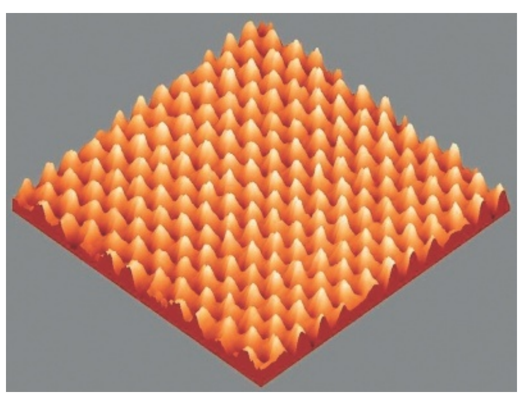


Figure 16.35 Scanning tunnelling microscope image of a graphite surface. The resolution of the microscope is such that individual atoms can be seen.

The tunnelling current arises due to the wave-like nature of electrons. Figure 16.36 shows a simplified version of the wave-like electron travelling between the tip and the surface of the specimen. The probe tip is at a slightly lower energy level because of the positive potential difference applied to it in the STM.

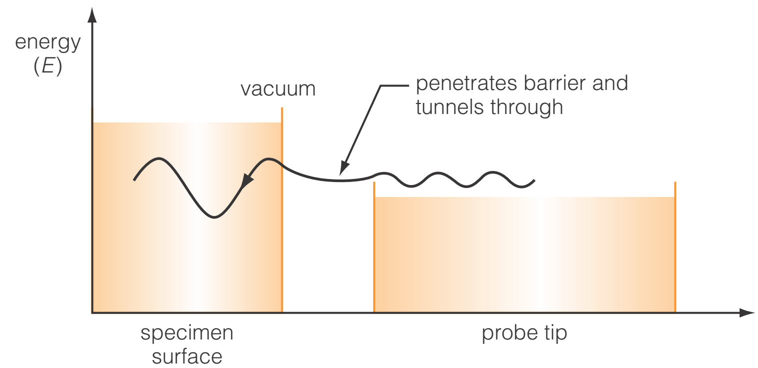


Figure 16.36 Electron matter wave crossing the gap between the specimen and the probe.

In classical physics (treating electrons as particles) it is not possible for the electron to cross the gap.

However, because the electron has wave-like properties, the amplitude of the wave reduces as it crosses the barrier, but does not reduce to zero. This means that there is a finite probability that the electron can cross to the probe tip causing a tunnelling current.

The electrons don't travel across the gap in the same way as they do in photoelectric currents. Rather, they disappear from the specimen and immediately reappear in the probe.

This strange quantum behaviour only occurs at very small scales and we do not notice it in classical physics. However, the fact that scanning tunnelling microscopes can be built and produce images provides evidence that wave-particle duality is a real phenomenon. Quantum theory does provide an accurate explanation of the behaviour of the world of the very small.

Special relativity

In search of the luminiferous æther

When Huygens proposed his wave theory of light, he assumed that light would need a medium to travel through. The waves which were known at the time, including water waves and sound waves, all travelled through a medium. However, light was known to travel through space and Huygens therefore suggested that space was filled with something called luminiferous æther. At the end of the 19th century, many physicists carried out experiments to measure the properties of this luminiferous æther.

History box

As physicists learned more about the properties of light waves and their nature, the properties of the luminiferous æther became more unusual.

The æther was thought to be:

- transparent
- incompressible
- a fluid, but one that was rigid enough to allow a light wave to travel through it
- massless
- without viscosity, so that it didn't affect the motion of planets.

Although physicists were aware that these properties were extremely unlikely to exist in one matter, the æther was part of the theory of light. Many experiments were carried out to try to measure the properties of the æther. The most well known of these experiments is the Michaelson-Morley experiment.

As the Earth travelled round the Sun, one theory of the æther was that there would be an 'æther wind' as the Earth moved through the æther. According to this theory, the movement through the æther would mean that light would take longer to travel through a known distance depending on its direction. By measuring the speed of light in different directions, it was thought that the presence of the æther, and the motion of the Earth through it, could be detected.

Physicists thought that the change in time would be very small because the Earth travels round the Sun at approximately 3×10^4 m s^{-1}, which is only 0.01% of the speed of light.

The Michelson–Morley experiment

The experiment developed by Albert Michelson and Edward Morley was published in 1887. It uses a type of interferometer that allows light from a single source to undergo constructive and destructive interference to produce an interference fringe pattern. The apparatus is shown in Figure 16.37 and consists of a white light source, a semi-silvered mirror, a compensating plate and two full mirrors. The semi-silvered mirror acts as a beam splitter and allows both a reflected and transmitted beam of light. A compensating plate, a plane glass block that is the same thickness as the semi-silvered mirror, is included to ensure that the light beams in the parallel and perpendicular arms of the interferometer travel the same distance in total. Figure 16.38 shows a photograph of the apparatus.

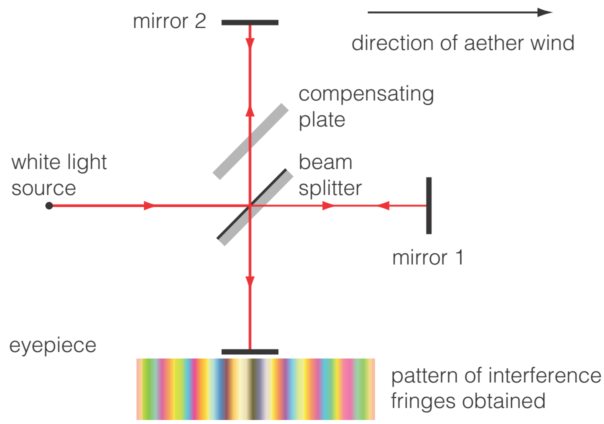


Figure 16.37 Schematic diagram of the Michelson–Morley interferometer experiment.

Figure 16.38 A Michelson–Morley interferometer. Although this apparatus is shown on a brick pillar, in the original experiment the interferometer took place in the basement of the laboratory. It was also placed on a heavy stone slab floating on a bath of mercury. This was to prevent any vibrations from affecting the measurements.

The two beams from the beam splitter travel towards the mirrors in the parallel and perpendicular arms of the interferometer. They are then reflected back to the beam splitter and travel through it to the eyepiece of a viewing telescope. The two beams are initially from the same light beam, so they are coherent. This means that there will be an interference pattern seen because of the difference in path lengths of the two beams.

- Bright fringes will be seen when the beams are in phase with each other.
- Dark fringes will be seen when the beams are 180° out of phase with each other.

If the Earth is moving through the luminiferous æther in the direction shown in the diagram, then the time it took for the light to travel to mirror 1 and back would be affected by the æther and one particular pattern of interference fringes would be obtained.

If the apparatus is turned through 90°, then there would be a difference in the time it takes for the light to travel between the mirrors because the æther would affect the light differently. The interference pattern should change position.

The æther theory predicted that the change in orientation would make the fringes shift by about 0.4 of a fringe width. Michelson and Morley made their apparatus as sensitive as possible and could detect a 0.05 of a fringe shift.

However, after many attempts, they were unable to detect the predicted fringe shift. They had obtained a null result which suggested that the æther did not exist.

Failing to detect absolute motion

Relative motion is the movement of one object in comparison to another. Relative motion may appear different to different observers.

You may have had the experience of sitting on a stationary train watching another stationary train through the window. Sometimes you think that your train has started to move, but then realise that it was actually the other train which started moving. There was **relative motion** between the two trains. You can observe relative motion easily in some cases, for example when balls are thrown towards or away from you.

However, other motion is not so easy to observe. As you are reading this book, the Earth is travelling at a speed of 30 km s^{-1} through space. However, you probably don't feel like you are moving at that speed.

Absolute motion is motion which is independent of where it is being observed from. We now know that everything is in motion and absolute motion does not exist.

Classical physics, described by scientists such as Galileo and Newton, depends on the concept of absolute time and space. In other words, time and space do not depend on the motion of any observer and are the same throughout the universe. In the train example above, in classical physics there should be some viewpoint that would allow you to measure the motion of each train exactly. This would be **absolute motion** and wouldn't depend on where it was observed.

The Michelson–Morley experiment should have been able to detect the absolute motion of the Earth through space, due to the effect that this motion would have on the motion of light.

In the arm of the interferometer which was travelling parallel to the direction of motion of the Earth through space, it should have taken the light slightly longer to travel to the mirror and back than in the arm which was perpendicular to the motion of the Earth.

Although the difference in travel time would be very small, the Michelson–Morley experiment was able to measure such small differences as changes in the interference pattern. This would have enabled physicists to measure the absolute motion of Earth.

However, there was no measurable difference in the experiment. The speed of light was invariant (remained the same) regardless of the motion of the observer. This was an unexpected result and it appeared that classical physics didn't hold true for light.

The Michelson–Morley experiment showed that there was no way to measure the absolute motion of the Earth.

TEST YOURSELF

37 Why are fringes produced in the Michelson–Morley experiment?

38 The Michelson–Morley experiment produced a null result. Explain what this null result was.

39 What was the significance of the null result in the Michaelson–Morley experiment?

Einstein's theory of special relativity

In 1905, Einstein published his paper 'On the electrodynamics of moving bodies' which included his theory of special relativity. Special relativity is used to describe the relative motion between two observers (or frames of reference).

Inertial frame of reference. Any situation where the motion is in a straight line and with a constant velocity.

The theory is called 'special relativity' because it is valid only where different observers are in relative motion with a constant velocity. These are known as **inertial frames of reference**.

Einstein's general theory of relativity (published in 1916) is used to describe the more complicated theory in frames of reference which are accelerating.

Imagine two people at an airport. Anna is standing next to a moving walkway watching her friend Beth who is standing on the moving belt, as shown in Figure 16.39.

Anna, standing by the walkway, will see her friend move past her from right to left at a steady velocity of 0.5 m s^{-1}. Beth, on the moving walkway, however, will see Anna move in the opposite direction from left to right at a steady 0.5 m s^{-1}. Anna and Beth have different frames of reference.

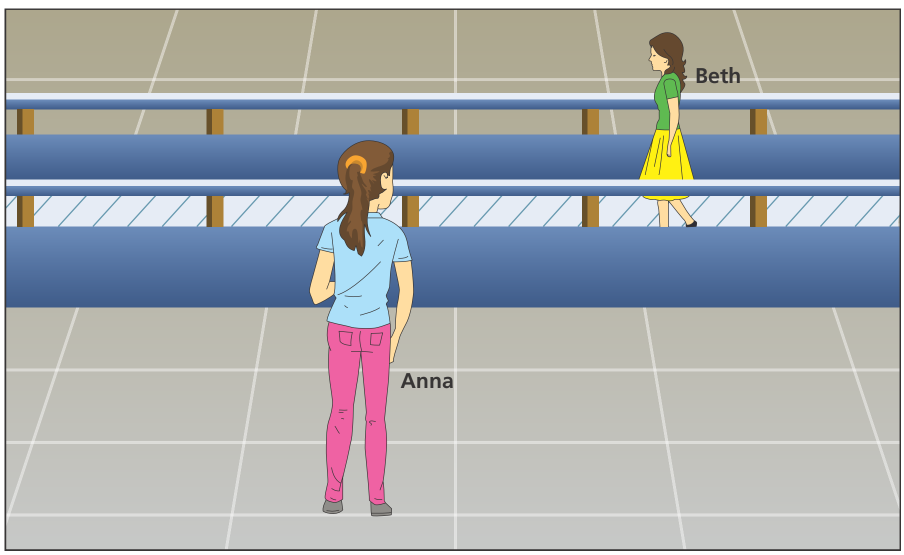


Figure 16.39 Anna and Beth have different frames of reference. They are moving relative to one another.

Special relativity is based on two postulates:

- The laws of physics are the same for all observers in all parts of the universe. If we watch another person in a different inertial frame we know that the laws of physics work exactly the same for them as for us.
- The speed of light is invariant, i.e. light always travels at the same speed in a vacuum.

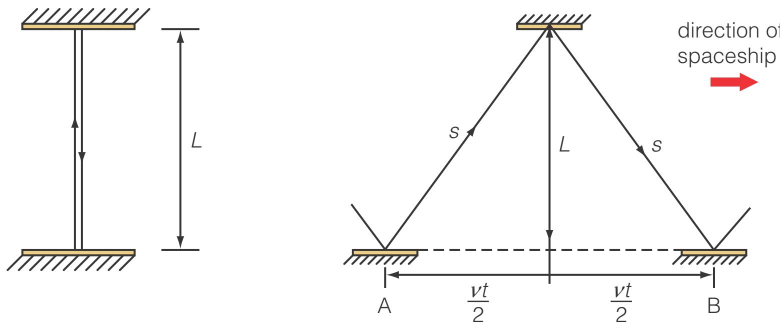
From these two postulates, Einstein developed the theory of special relativity.

This has a number of surprising outcomes.

Time dilation

We are used to thinking of time as moving at a constant rate. However, special relativity shows that there is no measure of absolute time that we can measure events against.

Imagine Beth is in a moving spaceship with a clock. She measures the time it takes for a light pulse to be reflected between two horizontal mirrors in the spaceship a vertical distance, L, apart. This is shown in Figure 16.40(a). The spaceship is travelling past a space station where Anna is standing on a viewing platform watching Beth and the light. Anna also times how long it takes for the light pulse to travel from one mirror and back again, as shown in Figure 16.40(b). The spaceship is moving with a speed v.



(a) Beth's view inside the spaceship (b) Anna's view from the space station

Figure 16.40 Beth and Anna both have stop clocks to measure how long it takes for the pulse of light to travel between two mirrors.

Proper time is the time, t_0, measured by the observer in the same frame of reference as the event that is being timed.

Beth, inside the spaceship, sees that the light has travelled a distance $2L$. She measures the time taken as $\frac{2L}{c}$, where c is the speed of light in vacuum.

This is called the **proper time**, t_0.

$$t_0 = \frac{2L}{c}$$

However, Anna who is on the space station viewing platform, sees that the light has travelled further. Using Pythagoras' theorem, the distance the light has travelled is $2s$ where

$$s = \sqrt{L^2 + \frac{v^2t^2}{4}}$$

Rearranging (see maths box) we can obtain an equation which relates the proper time, t_0, measured by Beth and the time, t, measured by Anna.

$$t = \frac{t_0}{\sqrt{1 - \frac{v^2}{c^2}}}$$

This is known as **time dilation**.

Time dilation: The observation that time in a moving frame of reference runs more slowly than the time for the same event in a stationary frame of reference.

Maths box

The distance travelled is equal to speed × time

so

$$2s = ct = 2\sqrt{\frac{4L^2 + v^2t^2}{4}}$$

Squaring both sides of the equation and rearranging for t:

$$c^2t^2 = 4L^2 + v^2t^2$$

$$t^2\,(c^2 - v^2) = 4L^2$$

$$t = \sqrt{\frac{4L^2}{(c^2 - v^2)}}$$

$$t = \frac{2L}{(c^2 - v^2)^{\frac{1}{2}}}$$

It would be better if this equation was not dependent on the length being measured, so using the equation for time from Beth's point of view we obtain:

$$t_0 = \frac{2L}{c}$$

or

$$2L = ct_0$$

substituting into the equation for t above:

$$t = \frac{ct_0}{\left(c^2 - v^2\right)^{\frac{1}{2}}}$$

or

$$t = \frac{t_0}{\sqrt{1 - \frac{v^2}{c^2}}}$$

TIP

If you study special relativity beyond A-level, you will come across the Lorentz factor, γ. This is given by

$$\gamma = \sqrt{1 - \frac{v^2}{c^2}}$$

and it is often used to simplify the equations.

EXAMPLE

A space station is travelling at a speed of 7.2×10^7 m s^{-1} relative to an observer on Earth. An astronaut on the station measures the journey as taking 360 minutes. How long did the journey take when measured from Earth?

$t_0 = 360$ minutes

$v = 7.2 \times 10^7$ m s^{-1}

$c = 3 \times 10^8$ m s^{-1}

$t = ?$

$$t = \frac{360}{\sqrt{1 - \frac{(7.2\times10^7)^2}{(3\times10^8)^2}}} = \frac{360}{\sqrt{1 - 0.05}}$$

$t = 371$ minutes

For the observer on Earth, the journey took longer than the time that the clock on board the space station measured for the journey.

The effect of time dilation is very small at speeds much lower than the speed of light. This is why we do not notice its effects in everyday life.

TEST YOURSELF

40 A spring–mass system on a space ship oscillates with a period 5 s as measured by an observer on the ship. The ship is travelling at a steady speed of 0.5 *c* past the Earth. What will the period of the spring–mass system be as measured by an observer on Earth?

41 An astronaut is on a rocket moving past a space station at a speed of 0.8 *c*. The heartbeat of an astronaut is measured by a doctor on the space station as 43 beats per minute. The astronaut also measured her own heartbeat. What is her heartbeat?

Length contraction

As well as time running more slowly when you are moving, another consequence of special relativity is that moving objects get shorter when they move fast.

Let's return to space. Imagine Anna is standing on the space station viewing platform measuring the length of a long rod on the spaceship as it travels past moving at a speed v. Beth is still inside the spaceship and measures the length of the rod from inside the ship. This is shown in Figure 16.41.

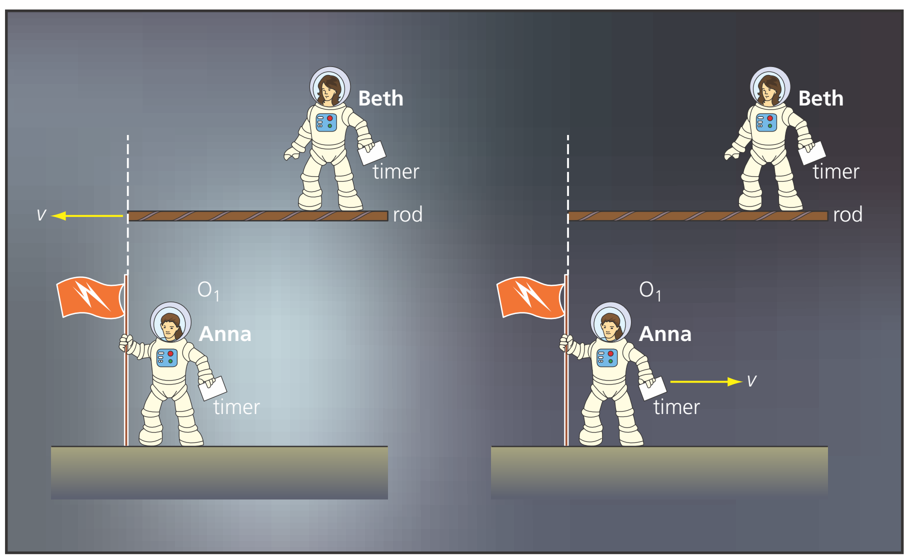


Figure 16.41 Anna and Beth are measuring the length of a rod in the spaceship.

TIP
We use t_0 for the time Anna measures because she is stationary relative to her stop clock.

The proper length, L_0, of the rod is its length as measured by an observer at rest relative to the rod.

In our example, this is the length of the rod that Beth measures on board the space ship.

On the space station platform Anna measures the time taken, t_0, for the rod to pass by her at a velocity, v. To Anna, the rod has length $L = vt_0$.

For Beth, the proper length of the rod, L_0, is equal to vt.

We can therefore write:

$L = vt_0$ and $L_0 = vt$

Combining these equations to remove v:

$$\frac{L}{t_0} = \frac{L_0}{t}$$

or

$$\frac{L}{L_0} = \frac{t_0}{t}$$

But we know that

$$\frac{t}{t_0} = \frac{1}{\sqrt{1-\frac{v^2}{c^2}}}$$

Substituting this into the equation above we get:

$$\frac{L}{L_0} = \sqrt{1-\frac{v^2}{c^2}}$$

Therefore we can write:

$$L = L_0\sqrt{1-\frac{v^2}{c^2}}$$

Length contraction occurs only on distances measured parallel to the direction of travel. Distances at right angles will not change. So if we were looking at a tennis ball travelling at speeds near to the speed of light we would see something similar to the pictures shown in Figure 16.42.

Figure 16.42 Length contraction of a tennis ball travelling at different speeds.

TEST YOURSELF

42 A space ship has a length of 1000 m. What length would it have, as measured from a planet, if it were travelling with a speed of 0.7*c* past the planet?

43 The nearest star to Earth is Alpha Centauri which is about 4 light years away. A space ship, travelling at 0.99*c*, travels to Alpha Centauri. How far does the space ship measure the distance to be?

Evidence for time dilation and length contraction

Cosmic rays are passing through the Earth's ionosphere all the time. As they do, muons may be formed. These muons travel at speeds of up to 99.6% of the speed of light. Muons are unstable and decay into an electron and a neutrino.

In the lab, the half-life of muons is measured as 1.5 μs. Using classical physics the muons would only travel about 650 m on average.

However, more muons than expected are detected on the surface of Earth, more than 20 km below where they are produced.

The muons are moving close to the speed of light, and so we must take relativistic effects into account. We can explain the discrepancy using either time dilation or length contraction.

(a) From the viewpoint of an observer on Earth, the 'clock' for the muon runs more slowly and the lifetime is much longer.

$$t = \frac{t_0}{\sqrt{1-\frac{v^2}{c^2}}}$$

$$t = \frac{1.5\times10^{-6}}{\sqrt{1-0.996^2}} = \frac{1.5\times10^{-6}}{0.089}$$

$$t = 1.68 \times 10^{-5}\, s$$

With this half-life, the muons can travel a distance of 5000 m before half of them will decay.

(b) We can also consider the muon discrepancy because the distance that the muons are travelling, in their frame of reference, is less than the 20 km that we measure.

For muons moving at 0.996 c, a distance of 20 km will contract to:

$$L = 20\sqrt{1-996^2} = 20\times0.089 = 1.79\,\text{km}$$

In the muon's frame of reference, the distance they are travelling is only 1.79 km, so more of them will survive to reach the surface of the Earth.

The number of muons observed at different heights in the atmosphere can only be explained if special relativity is used. These measurements were first made by Bruno Rossi and David Hall in 1941 and provided evidence for Einstein's theory of special relativity. They observed that approximately 80% of the muons produced in the atmosphere were able to travel 2 km. Non-relativistic calculations predict that far fewer muons would be able to travel that distance.

EXAMPLE

Figure 16.43 shows a sketch of cosmic rays being produced in the atmosphere and detected at an observatory 2 km below. The intensity of muons at the observatory is 80% of the intensity produced in the atmosphere. The half-life of muons at rest is 1.5 μs. The muons produced in the atmosphere are travelling at 0.996 c.

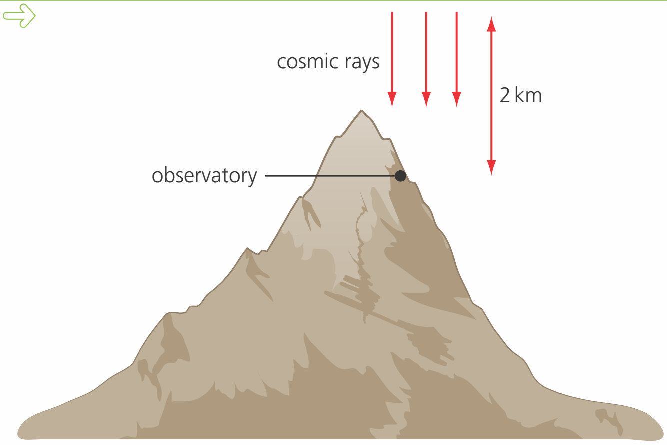


Figure 16.43 Cosmic rays produced in the atmosphere are measured at a mountain observatory.

Calculate:

(a) The time taken for a muon to travel the 2 km to the observatory.
(b) The number of rest half-lives that will take place in this time.
(c) The expected intensity at the observatory compared with the atmosphere.
(d) The intensity of muons at the observatory if time dilation occurs.

(a) $$\text{time} = \frac{\text{distance}}{\text{speed}}$$

$$\text{time} = \frac{2000\text{ m}}{0.996 \times 3 \times 10^{8}\text{ m s}^{-1}}$$

time = 6.69×10^{-6} s

(b) Number of half-lives = $\frac{6.69 \times 10^{-6}\text{ s}}{1.5 \times 10^{-6}\text{ s}}$ = 4.46 half lives

(c) $$\frac{\text{intensity}}{\text{initial intensity}} = 2^{-4.46}$$

$$\frac{I}{I_0} = 0.045$$

or 4.5 % of initial intensity of muons should be observed 2 km

(d) Half-life of moving muons

$$t = \frac{t_0}{\sqrt{1-\frac{v^2}{c^2}}}$$

$$t = \frac{1.5 \times 10^{-6}}{\sqrt{1-0.996^2}} = \frac{1.5 \times 10^{-6}}{0.089}$$

$t = 1.68 \times 10^{-5}$ s

This is $\frac{6.69 \times 10^{-6}}{1.68 \times 10^{-5}} = 0.398$ half-lives

$$\frac{\text{intensity}}{\text{initial intensity}} = 2^{-0.398} = 0.7589$$

intensity = 0.7589 initial intensity

Around 80% of the muons produced in the atmosphere reach the observatory. This matches the numbers measured by Bertozzi and Hall in 1941.

Maths box

After 1 half-life

$$\frac{I}{I_0} = 2^{-1}$$

after 2 half-lives

$$\frac{I}{I_0} = 2^{-2}$$

so after 4.46 half-lives:

$$\frac{I}{I_0} = 2^{-4.46}$$

Relativistic mass

Rest mass. The mass of an object measured in its rest frame. It is not moving relative to an observer.

Einstein showed that mass also has relativistic effects. The proper mass, m_0, of an object is the mass of the object at rest (or in its rest frame of reference). This is also called its **rest mass**.

The relationship between the mass of an object measured by an observer (in a different frame of reference) and its speed, v, is given by the equation:

$$m = \frac{m_0}{\sqrt{1 - \frac{v^2}{c^2}}}$$

This means that as an object gets faster:

- its mass increases (as measured by an observer)
- it requires more energy to increase the speed of the object.

Maths box

We can derive the equation for relativistic mass by considering an experiment carried out by Anna and Beth in space again.

Beth is in her spaceship travelling past Anna on the space station. Each woman has a ball of mass, m, as shown in Figure 16.44. The balls' masses were measured when they were in the same frame of reference, and were the same. (They are made of the same material and have the same number of atoms.) Since the laws of physics hold in all reference frames, it means that Anna measures the mass of her ball to be m_A and Beth measures the mass of her ball to be m_B, and therefore $m_A = m_B$.

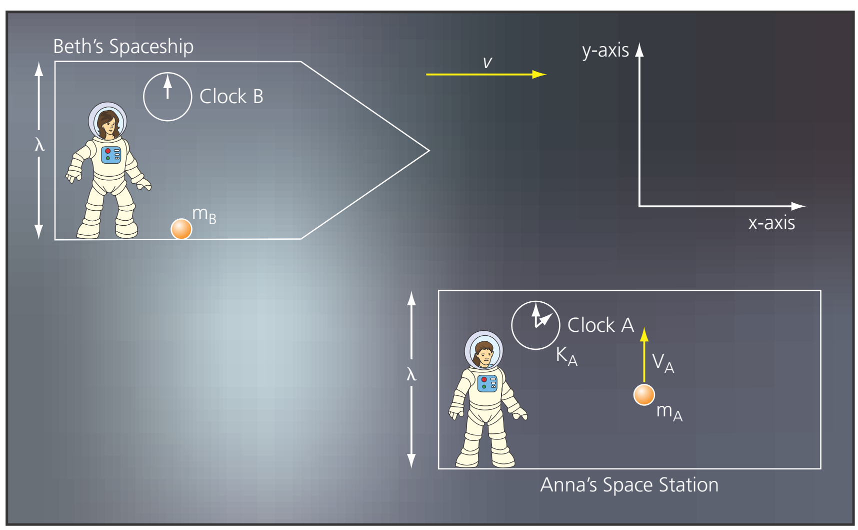


Figure 16.44 Beth in her space ship is travelling past Anna at a speed of *v*.

In the first part of the experiment, Anna launches her ball across the space station. It has a momentum $m_A\ v_A$, (as measured by Anna) and a speed $v_A = \frac{l}{t_A}$ because it takes time, t_A, to cross the space station.

The second part of the experiment is when the balls collide as Beth passes Anna. The two balls collide elastically, as shown in Figure 16.45. Momentum is conserved and ball A stops and transfers all its momentum to B.

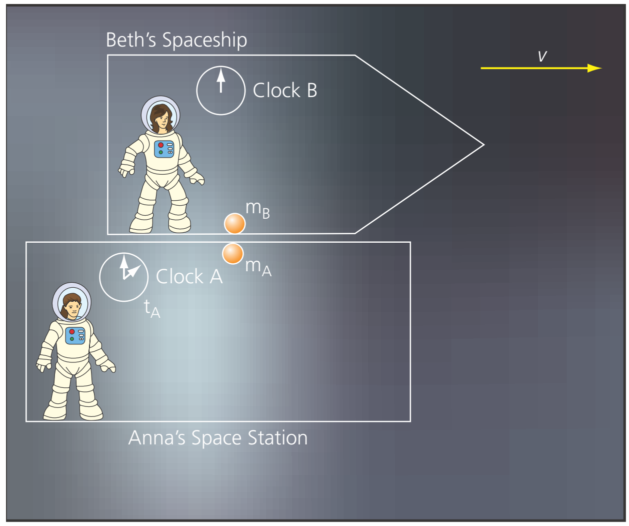


Figure 16.45 The balls collide as Beth flies past Anna.

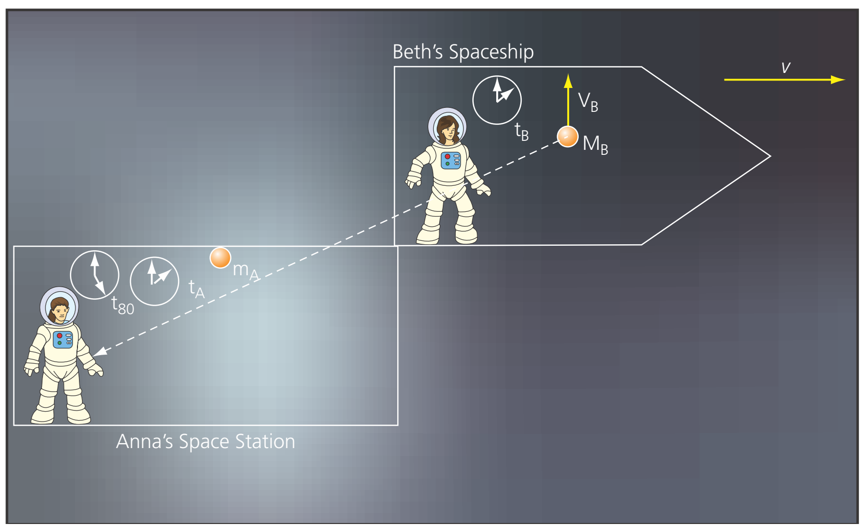


Figure 16.46 Ball A is stationary, and ball B moves upwards with a velocity v_A.

In the third part of the experiment, Beth's ball now moves across her spaceship. It has momentum $m_B v_B$ (as measured by Beth) and speed $v_B = \frac{l}{t_B}$ because it takes time, t_B, to cross her spacecraft.

But Anna also looks at Beth's spaceship to measure the speed that Beth's ball is moving at. Anna measures:

- the width of Beth's spaceship – which is still length, l, because there is no length contraction perpendicular to motion. Anna observes length contraction along the x-axis, but not along the y-axis.
- the time, t_{B_O}, the ball took to travel across the distance l. (The subscript $_{B_O}$ is used in show that it is the time in Beth's spacecraft *observed* by Anna). Due to time dilation Anna measures a time of:

$$t_{B_O} = \frac{t_B}{\left(1 - \frac{v^2}{c^2}\right)^{\frac{1}{2}}}$$

So Anna calculates that the speed of Beth's ball, v_{B_O} is:

$$v_{B_O} = \frac{l}{t_{B_O}} = \frac{\frac{l}{t_B}}{\left(1-\frac{v^2}{c^2}\right)^{\frac{1}{2}}}$$

$$v_{B_O} = \frac{l\left(1-\frac{v^2}{c^2}\right)}{t_B}$$

Anna knows that momentum is conserved, so working in her frame of reference she writes:

$m_A v_A = m_{BO} v_{BO}$

and therefore:

$$m_A \frac{l}{t_A} = m_{so} \frac{l\left(1-\frac{v^2}{c^2}\right)^{\frac{1}{2}}}{t_B}$$

But, $t_A = t_B$ and $m_A = m_B$

$$m_{B_O} = \frac{m_B}{\left(1-\frac{v^2}{c^2}\right)^{\frac{1}{2}}}$$

Beth measures the mass of her ball to be m_B (which is equal to m_A), but Anna measures Beth's ball (m_{BO}) to be more massive.

Figure 16.47 shows a graph of the ratio of mass/relativistic mass against speed (in units of c). You can see that at low speeds ($v << c$) the effects of relativity are very small and mass/relativistic mass = 1.

As the speed increases, from about $v = 0.5c$, then the relativistic mass starts to increase relative to the rest mass. At $v = 0.9c$, the relativistic mass is about 2.5 times larger than the rest mass.

Another feature of the graph is that there is a limit on the speed of an object. Nothing can travel faster than the speed of light. As v nears c, the relativistic mass gets rapidly larger and tends to infinity.

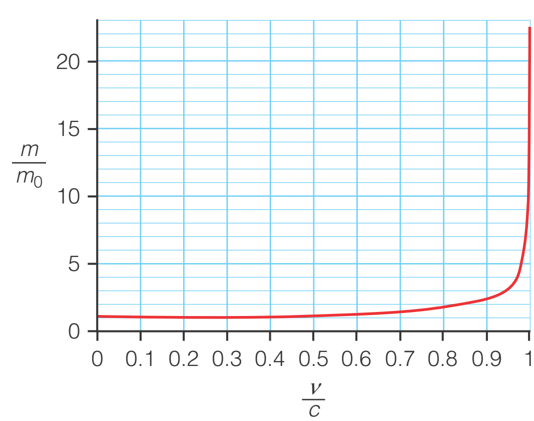


Figure 16.47 Graph showing the variation of relativistic mass as the speed of an object increases.

EXAMPLE

At the Large Hadron Collider in CERN the protons in the collider are moving at approximately 0.999999991c. Calculate their relativistic mass in terms of their rest mass.

$$m = \frac{m_0}{\sqrt{1-\frac{v^2}{c^2}}} = \frac{m_0}{\sqrt{1-\frac{(0.9\,999\,991c)^2}{c^2}}} = \frac{m_0}{\sqrt{0.000\,000\,017\,999\,999}}$$

$$m = \frac{m_0}{1.3\,416\,41 \times 10^{-4}}$$

$$m = 7454\ m_0$$

The protons are 7000 times more massive than if they were stationary. A large amount of the energy at CERN is used to accelerate the protons to this speed.

The huge energy equivalence of the protons means that when they collide together a shower of particles can be produced – as long as the total energy equivalence of the new particles is less than the original energy equivalence of the protons.

TEST YOURSELF

44 The rest mass of a proton is 1.673×10^{-27} kg. What is the relativistic mass of a proton when it is travelling at 0.8c?

45 A space probe has a rest mass of 722 kg. It is travelling at a speed of about 1.7×10^8 m s^{-1}.

(a) What is its relativistic mass at this speed?

(b) Explain why it would it not be possible to double the speed of the space probe.

Relativistic energy

In classical physics, kinetic energy is determined by the mass and velocity of an object.

However, we have seen that the mass of an object has relativistic effects. If the velocity of an object remains the same and its mass increases, then it will gain kinetic energy.

In his 1905 paper, Einstein showed that mass and energy are interchangeable as given by the equation

$$E = mc^2$$

Transferring energy *to* an object increases its mass, transferring energy *from* an object decreases its mass.

Rewriting Einstein's equation to include the rest mass we obtain:

$$E = \frac{m_0c^2}{\sqrt{1-\frac{v^2}{c^2}}}$$

TIP

Using $E = mc^2$, 1 kg of mass is equivalent to 9.0×10^{16} J.

Rest energy. The energy equivalent to the rest mass of an object.

If the object is stationary, $v = 0$ and the **rest energy** of the object $E_0 = m_0 c^2$.

If the object is moving with a speed v, then the difference between its total energy E, and the rest energy, E_0, will be its kinetic energy, E_k.

We can write:

$$E_k = mc^2 - m_0c^2$$

The total energy of the object can be written as:

$$mc^2 = E_k + m_0c^2$$

EXAMPLE

An electron is travelling at a speed of $0.99c$. What is its kinetic energy?

The mass of the electron will be:

$$m = \frac{m_0}{\sqrt{1-\frac{v^2}{c^2}}}$$

$$m = \frac{m_0}{\sqrt{1-\frac{(0.99c)^2}{c^2}}} = \frac{m_0}{\sqrt{1-0.9801}}$$

$$m = 7.1\ m_0$$

$$E_k = m\ c^2 - m_0\ c^2$$

so

$$E_k = 7.1\ m_0\ c^2 - m_0\ c^2$$

$$E_k = 6.1\ m_0\ c^2$$

Figure 16.48 shows the change in energy (in terms of rest mass) as the speed of an object increases. At $v = 0$, the energy is the energy equivalent of the rest mass, $E = mc^2$.

The line on the graph shows the increase in kinetic energy as the object speeds up.

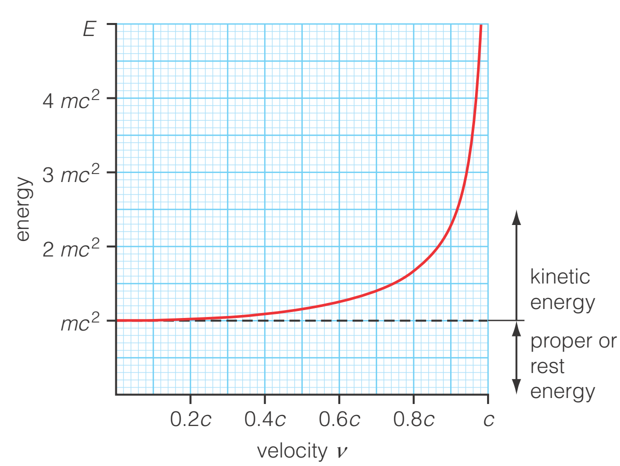


Figure 16.48 Graph showing the increase of energy of an object as its speed increases.

At speeds much lower than the speed of light ($v << c$), the equation for relativistic kinetic energy tends towards $\frac{1}{2}m_0v^2$.

TEST YOURSELF

46 An electron travels with a speed of 0.85 c. What are (a) its relativistic total energy and (b) its kinetic energy in MeV?

47 A proton is accelerated to an energy of twice its rest mass. Calculate its kinetic energy.

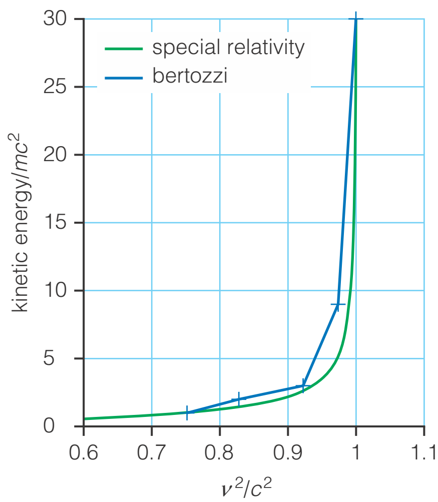


Figure 16.49 Graph showing the agreement between Bertozzi's results and the predictions of kinetic energy from special relativity.

Evidence for relativistic kinetic energy

In 1964 William Bertozzi published a paper which reported experiments aimed at measuring relativistic effects directly.

Using the electron accelerator at MIT in America, he accelerated electrons to speeds close to the speed of light. He used five different repeats of his experiment with speeds between 0.752c and c. In each repeat, the electrons travelled a distance of 8.4 m until they crashed into an aluminium disc.

The time of flight of the electrons in each repeat was measured and the velocity data agreed with relativistic predictions.

Bertozzi also measured the heat produced as the electrons hit the aluminium disc, which provided a direct measurement of their kinetic energy. Figure 16.49 shows a comparison between the predictions from special relativity and the results from Bertozzi's experiment. Bertozzi's measurements were within 10% of the predictions.

Accelerating particles

Previously in this chapter, we have considered accelerating electrons. However, we have not included relativistic effects in our calculations.

When a charged particle is accelerated from rest through a potential difference its kinetic energy is equal to the work done accelerating it.

$$W = QV = E_k$$

Including relativistic effects, its total energy is therefore:

$$E = m_0c^2 + QV$$

TEST YOURSELF

48 An electron is accelerated through a potential difference of 20 kV. Calculate (a) its energy if relativistic effects are ignored (b) its relativistic energy.

49 Describe how Bertozzi's experiment provided support for the theory of relativity.

Exam style questions

1 Figure 16.50 shows an electron gun which is part of discharge tube. The electrons are produced with a kinetic energy of 8×10^{-16} J.

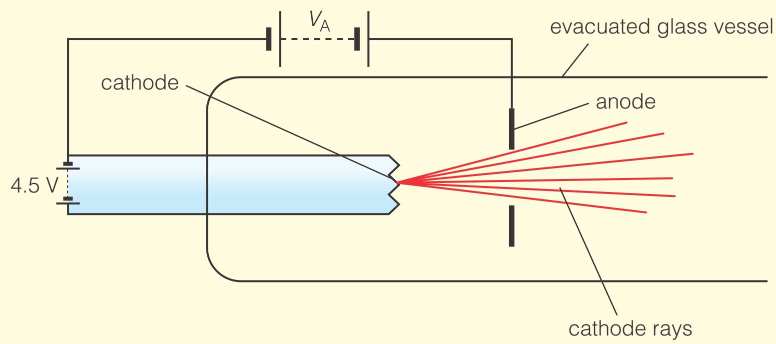


Figure 16.50

(a) Explain why the glass vessel is evacuated. *(1)*

(b) Explain the significance of the 4.5 V supply in producing cathode rays. *(2)*

(c) Calculate the potential difference between the cathode and the anode. *(2)*

(d) State and explain the effect on the cathode rays of increasing the anode potential. *(2)*

2 A beam of electrons travels at right angles to a uniform magnetic field, as shown in Figure 16.51.

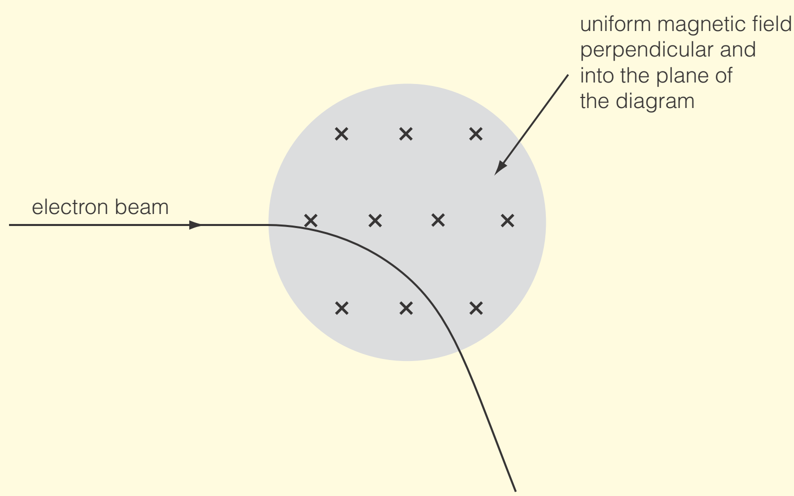


Figure 16.51

(a) The electrons move at a constant speed in a circular path. Explain why. *(2)*

(b) The speed of the electrons in the beam is 8.3×10^{6} m s^{-1}. When the magnetic flux density is 0.6 mT, the radius of curvature of the beam is 79 mm.

Calculate the specific charge of the electron. *(3)*

3 An electron beam passes between two parallel plates which are separated by 2.0 cm. There is a potential difference across the plates of 300 V. The deflection of the beam is just cancelled by a magnetic field of flux density 6.0×10^{-4} T applied at right angles to the electric field. When the electric field is removed, the radius of curvature of the path of the electrons is 25 cm.

What is the value of the specific charge $\frac{e}{m}$? *(4)*

4 Two parallel plates each 6.0 cm long produce a uniform electric field. The plates are at a p.d. of 3 kV and are 50.0 mm apart. A beam of electrons moving horizontally at constant speed enters the electric field and is deflected a vertical distance of 22.0 mm as a result. When a uniform magnetic field of flux density 2.05×10^{-3} T is applied at right angles to the beam and electric field, the beam is no longer deflected and becomes straight.

Calculate the specific charge $\frac{e}{m}$ of the electron. *(6)*

5 In an experiment to measure the charge of on an oil drop, a small droplet of oil was observed between two horizontal charged metal plates, as shown in Figure 16.52.

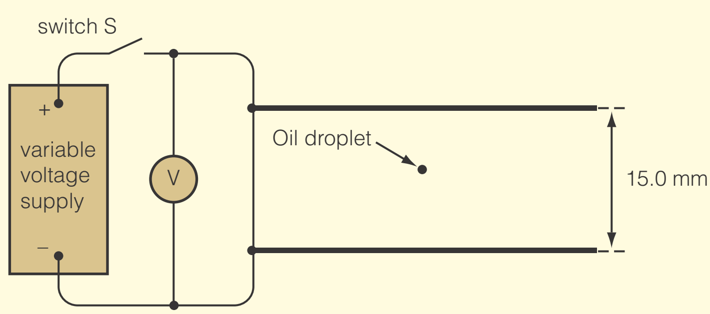


Figure 16.52

(a) With the switch S open, the droplet fell vertically at a steady speed of 1.1×10^{-4} ms^{-1}.

Calculate

(i) the radius of the drop and

(ii) the mass of the drop. *(4)*

Density of oil, $\rho = 880$ kg m^{-3}

viscosity of air, $\eta = 1.8 \times 10^{-5}$ N s m^{-2}

(b) When the switch was closed the potential difference between the plates gradually increased. When the potential difference was 1130 V, the drop stopped moving and remained stationary between the plates.

(i) Explain, in terms of the forces acting, why the droplet stopped moving and remained stationary when the potential difference was increased. *(2)*

(ii) Calculate the charge on the droplet. *(3)*

6 Figure 16.53 shows the refraction of a light ray as it passes from air into glass.

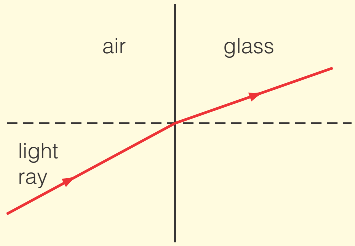


Figure 16.53

(a) Use Newton's theory of light to explain the refraction of light rays at the air–glass boundary. *(4)*

(b) After a long time, Huygens' theory of light was accepted by the scientific community in place of Newton's corpuscular theory. State one piece of evidence that supported Huygens' theory and explain why it supports his theory. *(3)*

7 Young's double slit experiment provided evidence for the wave nature of light. Figure 16.54 shows a simplified version of the experiment.

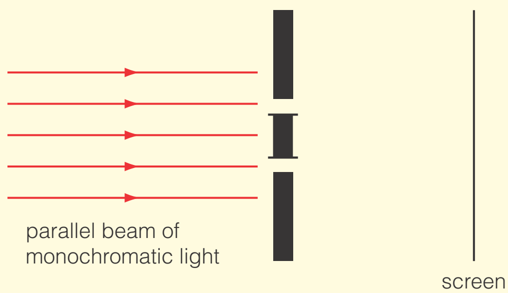


Figure 16.54

(a) Describe the pattern seen on the screen and explain why the pattern was formed. *(5)*

(b) Discuss why the pattern described in (a) cannot be explained using Newton's corpuscular theory of light. (2)

8 Describe, in terms of electric and magnetic fields, the nature of electromagnetic waves travelling in a vacuum. You may wish to draw a labelled diagram. *(3)*

9 (a) Explain what is meant by the term photon. *(1)*

(b) Calculate the energy of a photon of light with a wavelength of 650 nm. *(1)*

(c) The work function of three different metals are given in Table 16.6.

Table 16.6

Surface	Work function / eV
Copper	4.53
Calcium	2.87
Magnesium	3.66

Explain what is meant by the term work function and why the metals each have a different value for the work function. *(2)*

(d) A beam of light with wavelength of 650 nm is shone in turn onto each of the different metal surfaces. Determine which, if any, of the metal surfaces will undergo photoelectric emission. Explain your answer. *(3)*

10 In a transmission electron microscope, an electron gun is used to provide a beam of electrons which is directed through a thin specimen of biological material, as shown in Figure 16.55. The electrons scattered by the specimen are focused onto a fluorescent screen where an image of the specimen is formed.

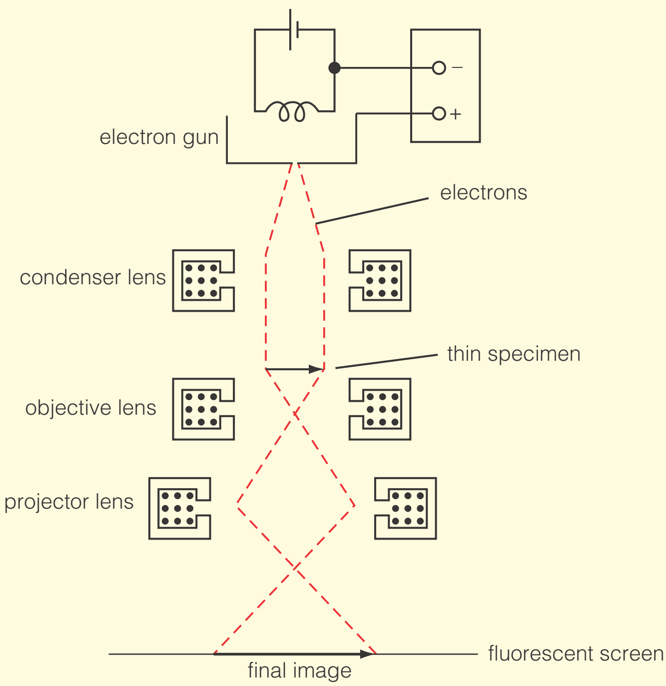


Figure 16.55

(a) Describe how the electron gun provides a beam of electrons that are all travelling with the same speed. *(3)*

(b) State, and explain, one reason why it is important that the electrons in the beam travel at the same speed. *(2)*

(c) State how the amount of detail of the image can be increased. *(1)*

11 Whilst investigating radio waves, Hertz created stationary waves using a large flat metal sheet to reflect radio waves, as shown in Figure 16.56.

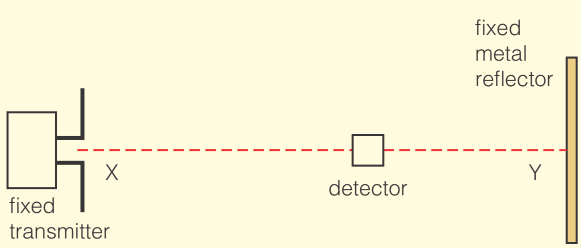


Figure 16.56

(a) Describe and explain the pattern of intensity of radio waves detected by the detector as it is moved from X to Y. *(3)*

(b) Explain how the wavelength and speed of the radio waves can be determined using this equipment. Assume that the frequency of the radio waves is known. *(4)*

12 In a scanning tunnelling microscope (STM) a very fine-tipped metal probe is scanned across a surface, as shown in Figure 16.57.

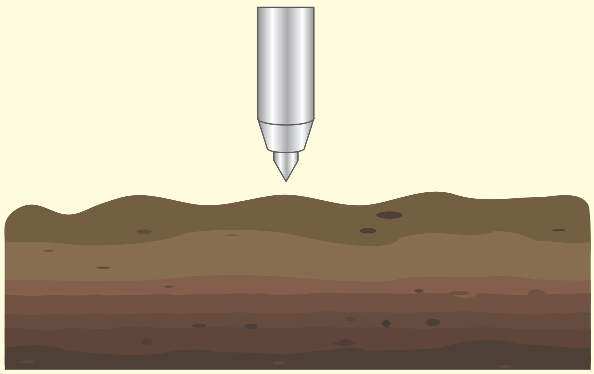


Figure 16.57

(a) Explain why electrons can transfer between the surface and the tip of the probe when the gap between the two is very narrow and there is a small p.d. between the tip and the surface. *(3)*

(b) STM may be used in two different modes. State the name of one of these modes and describe how the STM is used to obtain an image of the surface in this mode. *(3)*

13 (a) An electron of mass m has kinetic energy E_k. Show that the de Broglie wavelength λ of this electron is given by *(2)*

$$\lambda = \frac{h}{\sqrt{2mE_k}}$$

(b) An electron is at rest. Calculate the potential difference through which the electron must be accelerated so that its de Broglie wavelength is equal to 0.40 nm. *(2)*

14 A parallel beam of electrons is incident on a thin metal film. The electrons are scattered by the film and are then observed on the fluorescent screen, as shown in Figure 16.58. The electrons are all travelling with the same speed.

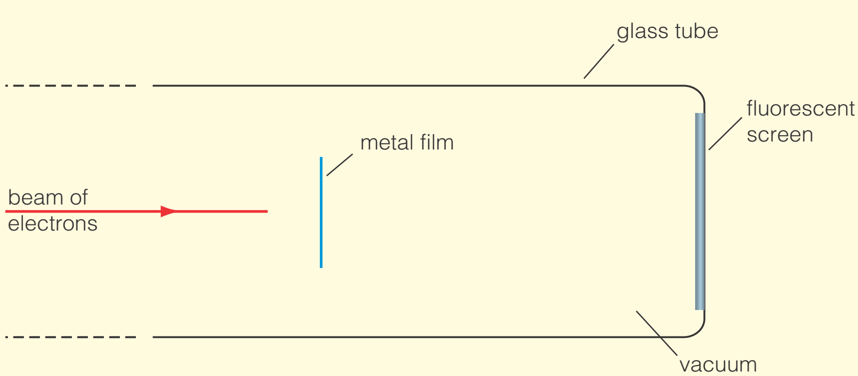


Figure 16.58

(a) Suggest why the glass tube contains a vacuum. *(1)*

(b) The apparatus shown in Figure 16.58 can be used to provide evidence for the wave-like nature of particles. Describe the pattern that will be seen on the fluorescent screen and explain how this provides evidence for the wave-like nature of particles.

(Quality of written communication should be taken into account when answering this question.) *(6)*

(c) Describe what will happen to the pattern if the speed of the electrons is increased. *(2)*

15 Light of wavelength 590 nm is shone onto a metal surface. Electrons are emitted from the surface causing a photoelectric current. The surface is then positively charged and the photoelectric current decreases. It becomes zero. The stopping potential is +0.35 V.

(a) Calculate the work function of the surface. *(5)*

(b) The experiment is repeated using a different metal. Using the same wavelength of light there is no photoelectric emission from the uncharged surface.

Explain why the wave theory of light cannot explain this observation. *(3)*

16 Einstein proposed a theory of special relativity in 1905.

(a) State, and explain, the two postulates on which special relativity is based. *(4)*
One consequence of special relativity is time dilation.

(b) What is meant by the proper time of an object. *(1)*

(c) A clock is flown on board a rocket. The rocket is flying at a speed of 0.93c. The flight lasts 10 minutes according to the pilot of the rocket. Calculate the time that passes for the ground crew waiting for the rocket to return. *(3)*

17 Epsilon Eridani is a star 10.49 light years away from Earth. In 2008, an exoplanet was discovered orbiting Epsilon Eridani. The light from the exoplanet has taken 10.49 years to reach the Earth, as measured by scientists on Earth.

A rocket leaves for the exoplanet at a speed of 0.9c relative to Earth.

Assume that the Earth and the exoplanet are stationary compared with one another.

(a) What is the distance, in m, to the exoplanet as measured by scientists on Earth? *(1)*

(b) In the frame of reference of the astronauts, how far did they travel? *(2)*

(c) One the rocket, one of the astronauts measured its external dimensions. The rocket is 82 m long and has a diameter of 21 m. The rocket is flying with its long axis parallel to the direction of motion. Calculate the dimensions of the spacecraft, as measured by a scientist on Earth. *(3)*

Multiple choice questions

1 In the photoelectric effect, which of the following statements is false?

(a) UV light is needed for the photoelectric effect to occur in most metals.

(b) A faint light contains very little energy so it takes time for the photoelectric current to start flowing.

(c) An intense light will cause more electrons to be emitted than a faint light.

(d) Electrons released by higher frequency light will have higher kinetic energy.

2 A clock, which ticks once per second when at rest, is moving past you with a uniform speed. From your frame of reference is the clock:

(a) ticking once per second

(b) ticking quickly

(c) ticking slowly

(d) running backwards?

3 A photon has a wavelength of 700 nm. Which of the following lines is correct?

Table 16.7

	Frequency / Hz	Energy / eV
(a)	4.29×10^{14}	2.84×10^{-19}
(b)	2.3×10^{-15}	1.78
(c)	4.29×10^{5}	0.56
(d)	4.29×10^{14}	1.78

4 Which of the following can be explained with a wave theory of light?

(a) polarisation

(b) photoelectric effect

(c) quantisation of energy

(d) none of the above

5 In a vacuum all photons have the same:

(a) frequency

(b) wavelength

(c) energy

(d) speed

6 A cathode ray tube has a potential difference of 10 kV between the cathode and anode. What will be the energy of the cathode rays in J?

(a) 1.6×10^{-14} J

(b) 1.6×10^{-15} J

(c) 1.6×10^{-16} J

(d) 1.6×10^{-17} J

7 Milikan's oil-drop experiment showed that:

(a) charges are continuous

(b) electrons have a very small mass

(c) only the electric field interacts with electrons

(d) charges come in integer multiples of the elementary charge e

8 What does a changing electric field induce?

(a) charge

(b) magnetic field

(c) light

(d) electrons

9 Monochromatic light of wavelength 4.80×10^{-7} m is shone onto a metal surface. The work function of the metal surface is 1.20×10^{-19} J. What is the maximum kinetic energy, in J, of an electron emitted from the surface?

(a) 2.94×10^{-19} J

(b) 4.14×10^{-19} J

(c) 5.34×10^{-19} J

(d) 6.25×10^{14} J

10 What is the wavelength of an electron that has been accelerated through a potential difference of 9.0 kV?

(a) 1.2×10^{-26} m

(b) 1.7×10^{-22} m

(c) 5.2×10^{-21} m

(d) 1.3×10^{-11} m

Answers

Activity: Fine beam tube

1 (a) The diameter of the path of the electron beam is limited by the diameter of the tube. If the diameter of the path is greater than the diameter of the tube, the beam with hit the glass wall and will not form a complete circle.

(b) You need to convert the diameter into metres and divide by 2 to obtain the radius. Your graph should look something like Figure A16.1.

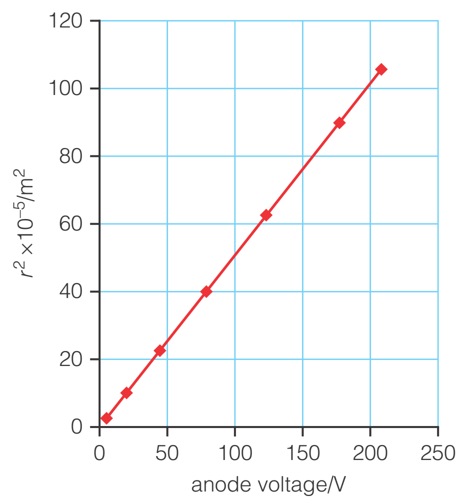


Figure A 16.1

(c) using the equation from the maths box

$$\frac{e}{m} = \frac{2V_A}{B^2r^2}$$

so rearranging to get an equation of the form $y = mx + c$:

$$r^2 = \frac{2m}{eB^2V_A}$$

with no y-intercept.

so the gradient $= \dfrac{2m}{eB^2}$

(d) The measured value of the gradient of the graph $= 0.51 \times 10^{-5}$ kg/C T^2 using gradient $= \dfrac{2m}{eB^2}$

$$\frac{e}{m} = \frac{2}{\text{gradient} \times B^2}$$

$$\frac{e}{m} = \frac{2}{0.51 \times 10^{-5} \times (1.5 \times 10^{-3})^2} = 1.74 \times 10^{11}\,\text{C kg}^{-1}$$

Activity: Measuring Planck's constant

(a) The stopping voltage is equal to the maximum kinetic energy of the photoelectrons.

(b) Your graph should look similar to that shown in Figure A16.2.

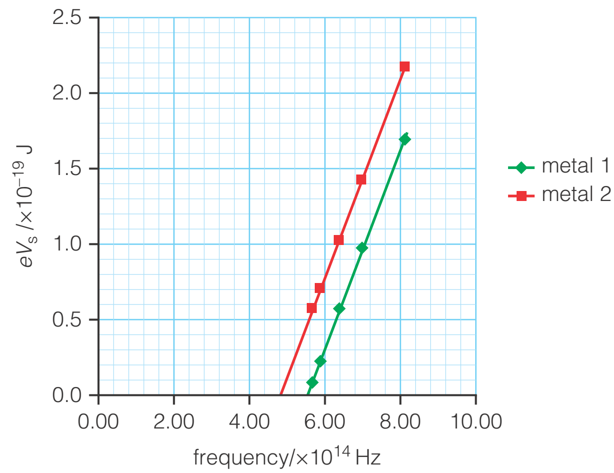


Figure A16.2

(c) Planck's constant is the gradient of the graph.
gradient = 0.66×10^{-33} J s
Therefore Planck's constant = 6.6×10^{-34} J

(d) Metal 1 gradient, m = 0.66×10^{-33} J s
Substitute for point (5.5×10^{14}, 0) in the equation of the line, $y = mx + c$
$0 = 0.66 \times 10^{-33} \times 5.5 \times 10^{14} + c$
$0 = 3.63 \times 10^{-19} + c$
$c = -3.63 \times 10^{-19}$

Metal 1 work function = 3.63×10^{-19} J

Metal 2. Gradient, m = 0.66×10^{-33} J s

Substitute for point (4.8×10^{14}, 0) in the equation of the line, $y = mx + c$
$0 = 0.66 \times 10^{-33} \times 4.8 \times 10^{14} + c$
$0 = 3.17 \times 10^{-19} + c$
$c = -3.17 \times 10^{-19}$

Metal 2 work function = 3.17×10^{-19} J

Answers to Test yourself on prior knowledge

1 work done = 6 eV
or 9.6×10^{-19} J

2 Specific charge = $\frac{\text{charge}}{\text{mass}} = 9.56 \times 10^{7}$ C kg^{-1}

3 $E = \frac{V}{d} = 5000$ V m^{-1}
$F = EQ$
$F = 8.0 \times 10^{-16}$ N

4 $F_B = BQv$
$F_B = 0.13 \text{ T} \times 3.62 \times 10^{-6} \text{ C} \times 1 \times 10^{6} \text{ m s}^{-1}$
$F_B = 0.47$ N

5 (a) $v = f\lambda$
$f = 5.2 \times 10^{14}$ Hz

(b) $w = \frac{\lambda D}{s}$

$w = 5.3$ mm

6 The light from a laser is coherent and intense. The light which is incident on the two slits must be coherent for interference effects to be seen.

7 $\lambda = \dfrac{h}{mv}$

$\lambda = 2.4 \times 10^{-9}$ m

Answers to Test yourself questions

1 $\dfrac{1}{2}mv^2 = eV$

$$v^2 = \frac{2eV}{m}$$

$$v^2 = \sqrt{\frac{2eV}{m}} = \sqrt{\frac{2 \times 1.602 \times 10^{-19}\text{C} \times 20\text{ V}}{9.109 \times 10^{-31}\text{kg}}} = \sqrt{7.035 \times 1012}$$

$v = 2.65 \times 10^6$ m s^{-1}

2 Work done = eV

$$V = \frac{\text{work done}}{e}$$

$$V = \frac{1.7 \times 10^{-17}\text{ J}}{1.602 \times 10^{-19}\text{ C}}$$

$V = 106$ V

3 Work done will be equal to $\frac{1}{2}mv^2$

work done = $\frac{1}{2} \times 9.109 \times 10^{-31}$ kg $\times (3.0 \times 10^7$ m s$^{-1})^2$

work done = 4.1×10^{-16} J

4 $eE = Bev$

$$B = \frac{E}{v}$$

$B = 0.11$ T

5 $v = \dfrac{V}{Bd}$

$$v = \frac{250\ V}{20 \times 10^{-3} \times 10 \times 10^{-3}} = 1.25 \times 10^6\text{ m s}^{-1}$$

6 $\text{E} = \dfrac{V}{d}$

$E = 5 \times 10^4$ V m^{-1}

7 $Bev = \dfrac{mv^2}{r}$

$$\text{so } \frac{e}{m} = \frac{5.3 \times 10^6\text{ m s}^{-1}}{1.7 \times 10^{-3}\text{T} \times 1.75 \times 10^{-2}\text{ m}}$$

specific charge of the electrons = 1.78×10^{11} C kg^{-1}

8 (a) $F = Bev$

$F = 2.2 \times 10^{-3}$ T $\times 1.602 \times 10^{-19}$ C $\times 2.5 \times 10^7$ m s^{-1}

$F = 8.81 \times 10^{-15}$ N

(b) $F = \dfrac{mv^2}{r}$

$$r = \frac{mv^2}{F}$$

$$r = \frac{9.11\times10^{-31}\ \text{kg}\times(2.5\times10^{7}\ \text{m s}^{-1})^2}{8.81\times10^{-15}\ \text{N}}$$

$r = 0.065$ m

9 The force on the electron is always perpendicular to the direction of motion and so the electron undergoes circular motion.

10 $v = \frac{\text{distance}}{\text{time}}$

$$\text{time} = \frac{50\times10^{-3}\ \text{m}}{2.5\times10^{7}\ \text{m s}^{-1}}$$

time between the plates = 2×10^{-9} s

11 Use $F = ma$
Force due to electric field = eE
$eE = ma$
so

$$a = \frac{eE}{m} = \frac{1.602\times10^{-19}\times200}{9.11\times10^{-31}} = 3.51\times10^{13}\ \text{m s}^{-2}$$

12 Work done = $eV = 1.6 \times 10^{-19}$ C $\times$ 2000 = 3.2×10^{-16} J

Work done = kinetic energy = $\frac{1}{2}mv^2$

$$v = \sqrt{\frac{2E_k}{m}} = \sqrt{\frac{2\times3.2\times10^{-16}\ \text{J}}{9.11\times10^{-31}\ \text{kg}}}$$

$v = 2.7 \times 10^{7}$ m s^{-1}

13 When the path of the electrons is straight then

$$Bev = \frac{eV}{d}$$

$$v = \frac{V}{Bd} = \frac{200\ \text{V}}{2.0\times10^{-3}\ \text{T}\times12\times10^{-3}\ \text{m}} = 8.3\times10^{6}\ \text{m s}^{-1}$$

From the motion of the beam in the magnetic field

$$Bev = \frac{mV^2}{r}\ \text{or}\ \frac{e}{m} = \frac{v}{Br}$$

$$\frac{e}{m} = \frac{8.3\times10^{6}\ \text{m s}^{-1}}{2.0\times10^{-3}\times25\times10^{-3}} = 1.66\times10^{11}\ \text{C kg}^{-1}$$

specific charge = 1.7×10^{11} C kg^{-1}

14 Using

$$Bev = \frac{mv^2}{r}$$

and

$$\frac{1}{2}mv^2 = eV$$

we obtain an equation:

$$\frac{e}{m} = \frac{2V}{B^2r^2}$$

$$\frac{e}{m} = \frac{2 \times 350\text{ V}}{(8.5 \times 10^{-3}\text{ T})^2 \times (7.4\ 10^{-3}\text{ m})^2} = 1.77 \times 10^{11}\text{ C kg}^{-1}$$

15 Specific charge for electron = 1.76×10^{11} C kg^{-1}. The specific charge of a hydrogen ion is 9.6×10^7 C kg^{-1} which was the largest known specific charge measured at the time of Thomson.

This could mean that the electron had either a much greater charge or a much smaller mass.

16 Drop would start to move downwards, accelerating until it reached its terminal speed.

When p.d. reduced to zero, there will no longer be an electric force on the drop. The forces are unbalanced and the weight downwards will cause the drop to accelerate. As the drop speeds up, the drag force increases with increasing speed. When the drag force is equal and opposite to the weight the forces will be balanced and the drop will move at a steady (terminal) speed.

17 $E = \frac{V}{d}$

$V = Ed$

$V = 5.86 \times 10^4\text{ V m}^{-1} \times 1.5 \times 10^{-2}\text{ m}$

$V = 879\text{ V}$

18 Charge is quantised, so will not obtain a value of charge less than that on the electron which is 1.6×10^{-19} C (allowing for experimental uncertainties).

19 (a) When the forces are balanced

$$6\pi\eta rv = mg$$

$$r = \frac{mg}{6\pi\eta v}$$

$$r = \frac{6.0 \times 10^{-15}\text{ kg} \times 9.81\text{ N kg}^{-1}}{6\pi \times 1.8 \times 10^{-5}\text{ N s m}^{-2} \times 3.3\ 10^{-5}\text{m s}^{-1}} = 5.26 \times 10^{-6}\text{ m}$$

(b) To hold the drop stationary

$$\frac{QV}{d} = mg$$

$$V = \frac{mgd}{Q}$$

$$V = \frac{6.0 \times 10^{-15}\text{ kg} \times 9.81\text{ N kg}^{-1} \times 10 \times 10^{-3}\text{ m}}{10 \times 1.6 \times 10^{-19}\text{ C}} = 368\text{ V}$$

20 Newton predicted that the light travels faster in glass than air. Huygens' predicted that light travels slower in glass than air.

21 Polarisation occurs when the oscillations of the wave are confined to one plane.

Newton's theory used particles, and the particles were seen as solid balls which don't oscillate, so there is no reason why they should be affected by polarisers.

Huygens' theory used longitudinal waves. Longitudinal waves cannot be polarised because the oscillations are parallel to the direction of motion of the waves.

22 Newton was a highly respected scientist and had produced other well thought of research.

There was no way to test which theory was correct, and neither fully explained the behaviour of light.

23 Newton's theory predicted two bright bands.

Young's experiment produced a larger number of light and dark bands.

Particles could not produce dark bands – they could only produce light bands.

24 Improves the accuracy of the experiment because using monochromatic light reduces the spread of the fringes. White light gives an extended spectrum which makes it harder to measure the fringe separation.

25 The laser produces an intense and coherent monochromatic beam of light. This makes the fringes brighter, and easier to observe and measure.

26 Maxwell's theory predicted electromagnetic waves beyond light. Radio waves were another e.m. radiation which had the properties predicted by Maxwell, supporting his theory.

27 The radio waves are reflected from the metal sheet. Constructive and destructive interference occurs and a standing wave is formed.

28 With light of frequency 1.0×10^{15} Hz no photoelectrons are released, but with light of frequency 1.2×10^{15} Hz photoelectrons are released and a photoelectric current may be measured.

29 (a) $c = f\lambda$

$E = hf$

so $E = \dfrac{hc}{\lambda}$

$$E = \frac{6.63 \times 10^{-34}\ \text{J s} \times 3 \times 10^{8}\ \text{m s}^{-1}}{486 \times 10^{-9}\ \text{m}}$$

$E = 4.1 \times 10^{-19}$ J

(b) $KE_{max} = hf - \Phi$

$= 4.1 \times 10^{-19}\ \text{J} - 3.8 \times 10^{-19}\ \text{J}$

$= 3 \times 10^{-20}$ J

30 Energy of incident photons = $hf = \frac{hc}{\lambda}$

$$E = \frac{6.63 \times 10^{-34}\ \text{J s} \times 3 \times 10^{8}\ \text{m s}^{-1}}{460 \times 10^{-9}\ \text{m}} = 4.3\ 10^{-19}\ \text{J}$$

No photoelectrons will be emitted because the work function is greater than the energy of the photons.

31 Any two from:

(i) Threshold frequency below which no photoelectrons are released – electrons must gain enough energy to escape the surface in one go, so photons with too small a value of energy cannot release the electrons.
(ii) Immediate release of photoelectrons when plate irradiated with frequency of light above the threshold frequency – electron can be released as soon as interaction with the photon occurs.
(iii) Photocurrent is proportional to intensity of light – the greater the intensity, the more photons per second reaching the surface and the more electrons which can be released.

32 $p = \frac{h}{\lambda}$

$$v = \frac{h}{m\lambda}$$

$$v = \frac{6.6 \times 10^{-34}}{9.1 \times 10^{-31} \times 2.3 \times 10^{-8}}$$

$v = 3.15 \times 10^{4}\ \text{m s}^{-1}$

33 The de Broglie wavelengths of the proton and electron are different.

$$\lambda \propto \frac{1}{m}$$

So the proton will have a smaller de Broglie wavelength because it has a greater mass.

34 1% of speed of light = $3 \times 10^{6}\ \text{m s}^{-1}$

$$\lambda = \frac{h}{mv}$$

$$\lambda = \frac{6.63 \times 10^{-34}}{9.1 \times 10^{-31} \times 3 \times 10^{6}}$$

$\lambda = 2.4 \times 10^{-10}\ \text{m}$

35 $\lambda = \frac{h}{\sqrt{mev}}$

$$\lambda = \frac{6.63 \times 10^{-34}\ \text{J s}}{\sqrt{2 \times 9.11 \times 10^{-31} \times 1.6 \times 10^{-19} \times 1500}}$$

$\lambda = 3.17 \times 10^{-11}\ \text{m}$

36 $\lambda = \dfrac{h}{\sqrt{2mev}}$

so $V = \dfrac{h^2}{2me\lambda^2}$

$V = \dfrac{(6.6 \times 10^{-34}\ \text{J s})^2}{2 \times 9.1 \times 10^{-31}\ \text{kg} \times 1.6 \times 10^{-19}\ \text{C} \times (2.05 \times 10^{-11}\ \text{m})^2}$

$V = 3559\ \text{V}$

37 Fringes are produced due to constructive and destructive interference of light from the two arms of the apparatus.

38 The fringes were expected to shift when the apparatus was rotated through 90°.

39 The null result showed: that there was no effect on light due to the aether; the absolute movement of the Earth could not be measured; the speed of light appeared to be constant.

40 $t = \dfrac{t_0}{\sqrt{1 - \dfrac{v^2}{c^2}}}$

$t = ?$
$t_0 = 5\ \text{s}$
$v = 0.5c$

$t = \dfrac{5}{\sqrt{1-(0.5c)^2}} \dfrac{c^2 = 5}{\sqrt{1-0.25}}$

$t = 5.8\ \text{s}$

41 As viewed from the space station, the heartbeat is 43 beats per minute = 0.717 beats per second. Each beat therefore lasts 1.395 s.
$t = 1.395\text{s}$
$t_0 = ?$
$v = 0.8c$

$t = \dfrac{t_0}{\sqrt{1 - \dfrac{v^2}{c^2}}}$

$t\sqrt{1 - \dfrac{v^2}{c^2}} = t_0 = 1.395\sqrt{1-0.64}$

$t = 0.837\ \text{s}$
so there are $\dfrac{1}{t}$ heartbeats per second = 1.19 per second and her heartbeat is therefore 72 bpm.

42 $L = L_0\sqrt{1 - \dfrac{v^2}{c^2}}$

$L = ?$
$L_0 = 1000\text{m}$
$v = 0.7c$
$L = 1000\sqrt{1-0.72}$
$L = 714\ \text{m}$

43 $L = L_0\sqrt{\frac{1-v^2}{c^2}}$

$L = ?$

$L_0 = 4$ light years

$v = 0.99\ c$

$L = 4\sqrt{1-0.99^2}$

$L = 0.56$ light years

44 $m = \frac{m_0}{\sqrt{1-\frac{v^2}{c^2}}}$

$m_0 = 1.673 \times 10^{-27}$ kg

$m = ?$

$v = 0.8c$

$m = \frac{m_0}{\sqrt{1-0.8^2}}$

$m = 1.667\ m_0$

$m = 2.788 \times 10^{-27}$ kg

45 (a) $m = \frac{m_0}{\sqrt{1-\frac{v^2}{c^2}}}$

$m = ?$

$m_0 = 722$ kg

$v = 1.7 \times 10^4\ \text{m s}^{-1}$

$c = 3 \times 10^8\ \text{m s}^{-1}$

$$m = \frac{722}{\sqrt{1-\frac{(1.7\times 10^8)^2}{(3\times 10^8)^2}}} = \frac{722}{\sqrt{1-\frac{2.89\times 10^{16}}{9\times 10^{16}}}}$$

$$= \frac{722}{\sqrt{1-0.321}} = \frac{722}{0.8239}$$

$m = 876$ kg

(b) Mass would increase as speed increased. Doubling its speed would mean that it was travelling faster than light. However, as its speed increased, its mass would increase and so it would not be possible to do enough work to continue to accelerate the probe.

46 (a) $E = \frac{m_0c^2}{\sqrt{1-\frac{v^2}{c^2}}} = \frac{(9.11\times 10^{-31}\,\text{kg})(3\times 10^8)^2}{\sqrt{1-0.85^2}}$

$E = 1.56 \times 10^{-13}$ J

$1\text{MeV} = 1.6 \times 10^{-13}$ J

so $E = \frac{1.56 \times 10^{-13}}{1.6 \times 10^{-13}} = 0.975$ MeV

(b) $KE = E - m_0c^2$

Rest energy of an electron = 0.511 MeV

ANSWER

so $KE = 0.975$ MeV – 0.511 MeV

$KE = 0.464$ MeV

47 $KE = E - m_0c^2$

$KE = 2m_0\,c^2 - m_0c^2$

$KE = m_0c^2$

$= 9.1 \times 10^{-31}\ \text{kg} \times (3 \times 10^8\ \text{m s}^{-1})^2$

$= 8.19 \times 10^{-14}$ J

48 (a) If relativistic effects are ignored, then energy $= \frac{1}{2}mv^2 = eV$

$E = 3.2 \times 10^{-15}$ J

(b) Relativistic energy : $E = m_0c^2 + QV$

$E = (9.11 \times 10^{-31}\ \text{kg} \times (3 \times 10^8\ \text{m s}^{-1})^2) + (1.6 \times 10^{-19}\ \text{C} \times 20000\ \text{V})$

$E = 8.199 \times 10^{-14}\ \text{J} + 3.2 \times 10^{-15}$ J

$E = 8.52 \times 10^{-14}$ J

49 Bertozzi measured the kinetic energy for electrons moving at a fraction of the speed of light. His values were consistent with the predictions made by relativity.

Answers to exam style questions

1 (a) Electrons from the wire would be scattered or stopped by the atoms in the gas if the tube wasn't evacuated. The electrons would lose kinetic energy due to collisions.

(b) The 4.5 V supply is used to heat the metal filament. The filament emits electrons when it is heated because the conduction electrons in the metal gain sufficient kinetic energy to leave the surface of the metal.

(c) $KE = eV$

therefore $V = \frac{KE}{e}$

$$V = \frac{8 \times 10^{-16}\ \text{J}}{1.6 \times 10^{-19}\ \text{C}}$$

$V = 5000$ V

(d) The speed of the electrons (cathode rays) will increase because there is a greater force of electrostatic attraction between the electrons and the anode. A larger force means that there is a greater acceleration.

2 (a) The magnetic force on each electron is perpendicular to the direction of motion. No work is done by the force so the speed is constant.

The magnetic force acts as a centripetal force as it is perpendicular to the velocity.

(b) $Bev = \frac{mv^2}{r}$

so

$$\frac{e}{m} = \frac{8.3 \times 10^6\ \text{m s}^{-1}}{0.6 \times 10^{-3}\ \text{T} \times 79 \times 10^{-3}\ \text{m}} = 1.751 \times 10^{11}\ \text{C kg}^{-1}$$

specific charge of the electron = 1.8×10^{11} C kg^{-1}

3 First calculate the speed of the electrons.

$$\frac{eV}{d} = Bev$$

so

$$v = \frac{V}{Bd}$$

$$v = \frac{300}{6.0 \times 10^{-4} \times 0.02\ \text{m}} = 2.5 \times 10^7\ \text{m s}^{-1}$$

then

$$Bev = \frac{mv^2}{r} = \text{or}\ \frac{e}{m} = \frac{v}{Br}$$

$$\frac{e}{m} = \frac{2.5 \times 10^7\ \text{m s}^{-1}}{6.0 \times 10^{-4}\ \text{T} \times 0.25\ \text{m}} = 1.7 \times 10^{11}\ \text{C kg}^{-1}$$

4 First calculate the speed of the electrons using

$$\frac{eV}{d} = Bev$$

$$v = \frac{V}{Bd}$$

$$v = \frac{3000}{2.05 \times 10^{-3} \times 50 \times 10^{-3}\,\text{m}} = 2.93 \times 10^7\ \text{m s}^{-1}$$

We now calculate how long the electrons are in the electric field to work out their acceleration.

$$\text{time} = \frac{\text{length of plates}}{\text{speed of electrons}}$$

$$t = \frac{6.0 \times 10^{-2}\,\text{m}}{2.93 \times 10^7\ \text{m s}^{-1}} = 2.05 \times 10^{-9}\ \text{s}$$

$$\text{acceleration} = t = \frac{2s}{t^2} = \frac{2 \times 22.0 \times 10^{-3}\,\text{m}}{(2.05 \times 10^{-9}\text{s})^2} = 1.05 \times 10^{16}\ \text{m s}^{-2}$$

$$\frac{e}{m} = \frac{ad}{V} = 1.75 \times 10^{11}\ \text{C kg}^{-1}$$

5 (a) (i) At terminal velocity, weight of droplet = drag force.

$$6\pi\eta r v = mg$$

the mass of the droplet,

$$m = \frac{4}{3}\pi r^3 \rho$$

so substituting and rearranging we obtain

$$r = \sqrt{\frac{9\eta v}{2\rho g}}$$

$$r = \sqrt{\frac{9 \times 1.8 \times 10^{-5}\ \text{N s m}^{-1} \times 1.1 \times 10^{-4}\text{m s}^{-1}}{2 \times 880\ \text{kg m}^{-3} \times 9.8\ \text{N kg}^{-1}}} = 1.03 \times 10^{-6}\ \text{m}$$

(ii) $m = \frac{4}{3}\pi r^3 \rho$

$m = 3.7 \times 10^{-15}$ kg

(b) (i) When falling at a steady speed, the drag force is equal to the weight so the forces are balanced and there is no acceleration. When the potential difference is increased, there is an additional upward force on the drop which is proportional to plate p.d.

When the upwards force is equal in magnitude to the downwards force due to the weight, there is no resultant force and the drop remains stationary.

(ii) $\frac{QV}{d} = mg$

rearranging gives

$$Q = \frac{mgd}{V}$$

$$Q = \frac{3.7 \times 10^{-15}\ \text{kg} \times 9.8\ \text{N kg}^{-1} \times 15.0 \times 10^{-3}\text{m}}{1130\ \text{V}}$$

$Q = 4.81 \times 10^{-19}$ C

6 (a) Corpuscles (or particles of light) are attracted towards the glass surface.

The force of attraction means that the velocity perpendicular to the surface increases, but the velocity of the corpuscle parallel to the surface is unchanged.

Resultant velocity vector means that the angle at which the corpuscle is travelling has changed

(b) Evidence: Young's double slits - shows interference – this is a wave property, not shown by particles.

Evidence: measurement of speed of light – speed of light is less than in air – predicted by wave theory.

7 (a) There is a pattern of light and dark fringes on the screen.

Light passing through each slit is diffracted.

Diffracted light from the slits overlaps.

Bright fringes are formed where light is in phase and constructively interferes. The path difference is a whole number of wavelengths.

Dark fringes are formed where light is out of phase and destructively interferes. The path difference is $(n + \frac{1}{2})$ wavelengths.

(b) Corpuscles pass through a slit to form a bright fringe on the screen.

Two slits would produce two bright fringes.

Dark fringes won't be produced because two corpuscles can't cancel each other out.

8 Electric wave and magnetic wave in phase. Electric wave is perpendicular to magnetic wave and both waves are perpendicular to the direction of propagation of the wave.

9 (a) A discrete quanta of energy of e.m. radiation / the smallest quantity of energy of e.m radiation that can be obtained.

(b) $E = hf$

$E = \frac{hc}{\lambda}$

$E = 3.05 \times 10^{-19}$ J

(c) The work function is the energy required for an electron to leave the surface of a metal with zero kinetic energy. The work function is different for different metals due to differences in size of atoms and the force of attraction between the outermost electrons and the nucleus of each metal.

(d) Either convert photon energy to eV, or convert the work functions to J.

1 eV = 1.6×10^{-19} J

Therefore

$$\text{photon energy} = \frac{3.05 \times 10^{-19}}{1.6 \times 10^{-19}} = 1.9 \text{ eV}$$

None of the surfaces will undergo photoelectric emission with the light. The energy of each photon is less than the work function of the metal surfaces so the electrons will not gain sufficient energy to leave the surface.

10 (a) A heated filament produces electrons by thermionic emission.

The electrons are accelerated through a potential difference towards the anode.

The work done on each electron to accelerate it is the same and therefore the kinetic energy of each electron is the same.

A small hole in the anode allows electrons travelling in approximately the same direction to go through which producs a beam.

(b) Force on electron in a magnetic field depends on speed.

Electrons travelling at different speeds would be focused in different places so the image would be blurred.

OR

Electrons at different speeds would have different de Broglie wavelengths so the resolution of the microscope would be reduced.

(c) Increase the potential difference in the electron gun.

11 (a) As the detector moves from X to Y there will be a regular maximum and minimum (zero) in the intensity of radio waves detected.

Explanation: radio waves from the transmitter are reflected back towards the transmitter by the metal sheet. The reflected and incident waves pass through each other. Both waves have the same frequency and amplitude so there is superposition of the waves to form a stationary wave. This leads to equally spaced nodes and antinodes being formed along XY.

(b) The detector signal is zero at the nodes. The distance between nodes is $\frac{\lambda}{2}$. Move detector along XY and measure distance between adjacent nodes. Double this value to give the wavelength.

Use the known value of frequency and $v = f\lambda$ to calculate the speed.

12 (a) Electrons have a wave-like nature. There is a small probability that an electron can cross the gap (transfer across the gap by quantum tunnelling). Transfer is from negative to positive p.d.

(b) Constant height mode:

Tip scanned across at a constant height.

Gap width varies due to bumps and troughs on the specimen surface.

Tunnelling current measured.

Current decreases as gap width increases (and vice versa).

Variation of current with position (time) is used to map surface.

Constant current mode:

Tunnelling current measured.

Feedback is used to keep the current constant by changing the height of the probe tip.

This means that the height of the probe tip about the bumps and troughs is kept constant.

Variation of height of probe tip with position (time) used to map surface.

13 (a) $E_k = \frac{1}{2}mv^2$

$$v = \sqrt{\frac{2E_k}{m}}$$

substitute for *v* in the de Broglie equation

$$\lambda = \frac{h}{mv}$$

hence

$$\lambda = \frac{h}{\sqrt{2mE_k}}$$

(b) Accelerating p.d. will be equal to kinetic energy:

$E_k = eV$

$$\lambda = \frac{h}{\sqrt{2meV}}$$

so

$$V = \frac{h^2}{2me\lambda^2}$$

$$V = \frac{(6.6 \times 10^{-34}\,\text{J s})^2}{2 \times 9.1 \times 10^{-31}\,\text{kg} \times 1.6 \times 10^{-19}\,\text{C} \times (0.4 \times 10^{-9}\text{m})^2}$$

$V = 9.3$ V

14 (a) To prevent the electrons interacting with the air molecules.

(b) To gain full marks for this question, quality of writing should be taken into account.

A series of concentric bright and dark rings will be observed.

As each electron passes through the film it will undergo diffraction.

The diffracted waves will undergo interference.

Where there is constructive interference (path difference = $n\lambda$) there will be bright points, where there is destructive interference (path difference = $(n + \frac{1}{2})\lambda$ there will be dark points.

These combine to form the pattern of rings on the screen.

(c) Higher speed, more momentum. $\lambda = \frac{h}{p}$ so wavelength decreases and therefore diameter of rings also decrease.

15 (a) $hf = \Phi + KE_{max}$

$\Phi = hf - KE_{max}$

KE_{max} = work done due to stopping potential.

$KE_{max} = eV$

$= 1.6 \times 10^{-19}\ \text{C} \times 0.35\ \text{V} = 5.6 \times 10^{-20}\ \text{J}$

$KE_{max} = 5.6 \times 10^{-20}\ \text{J}$

$$hf = \frac{hc}{\lambda} = \frac{(6.6 \times 10^{-34}\ \text{J} \times 3 \times 10^{8}\ \text{m s}^{-1})}{590 \times 10^{-9}} = 3.356 \times 10^{-19}\ \text{J}$$

$\Phi = hf - KE_{max}$

$\Phi = 3.356 \times 10^{-19}\ \text{J} - 5.6 \times 10^{-20}\ \text{J}$

$\Phi = 2.8 \times 10^{-19}\ \text{J}$

(b) This is an example of threshold frequency. The electron gains all the energy from a photon. If the photon doesn't carry enough energy (i.e. if the frequency is too small) the electron cannot escape. Wave theory would allow energy to be built up gradually so there would be no threshold frequency.

16 (a) The laws of physics are the same regardless of the frame of reference. This means that objects will follow physical laws e.g. conservation of momentum regardless of how they are moving relative to an observer.

The speed of light is invariant. This means that the speed of light is always the same regardless of which frame of reference is being used.

(b) The proper time is the time measured in the same frame of reference as the object (i.e. relative velocity is zero).

(c) $$t = \frac{t_0}{\sqrt{1 - \frac{v^2}{c^2}}}$$

$t_0 = 10$ minutes

$t = ?$

$v = 0.93c$

$$t = \frac{10\ \text{min}}{\sqrt{1 - \frac{(0.93c)^2}{c^2}}}$$

$t = 74$ minutes

17 (a) speed of light $= 3 \times 10^{8}\ \text{m s}^{-1}$

speed = distance/time

so distance = speed × time

distance $= 3 \times 10^{8}\ \text{m s}^{-1} \times (10.49 \times 365.25 \times 24 \times 60 \times 60)$

$= 9.5 \times 10^{15}\ \text{m}$

(b) $$L = \frac{L_0}{\sqrt{1 - \frac{v^2}{c^2}}}$$

The astronauts are in the moving frame of reference so they are measuring L. The distance to the exoplanet as measured by the scientists on Earth is the proper length.

$L = ?$

$L_0 = 9.5 \times 10^{15}$ m

$v = 0.9c$

$$L = 9.5 \times 10^{15}\sqrt{1 - \frac{(0.9c)^2}{c^2}}$$

$L = 4.1 \times 10^{15}$ m

(c) In this case, the dimensions measured by the astronaut are the proper lengths. Only lengths which are parallel to the direction of motion of the rocket will undergo length contraction. The diameter will be unchanged.

$$L = L_0\sqrt{1 - \frac{v^2}{c^2}}$$

$L = ?$

$L_0 = 82$ m

$v = 0.9c$

$L = 35.74$ m

As seen from Earth, the rocket will be 36 m long and 21 m in diameter.

Answers to multiple choice questions

1 (b)

2 (c)

3 (d)

4 (a)

5 (d)

6 (b)

7 (d)

8 (b)

9 (a)

10 (d)

Stretch and challenge questions

1 A certain group of bacteria, called mycobacteria, have a generation time of 24 hours. This is the time it takes for the number of bacteria to double. A biologist prepares two flasks of nutrient broth, each containing a single bacterium. One flask is left on Earth, and the other is put on a rocket which travels at a speed of 0.865c relative to Earth. Some time later, the flask on Earth contains 1024 bacteria. How many bacteria will be in the flask on the rocket, as measured by the biologist on Earth?

2 A burglar alarm is designed to make use of the photoelectric effect. A photocell is used as the detector in the alarm. Figure S16.1 shows a simplified circuit diagram for the alarm.

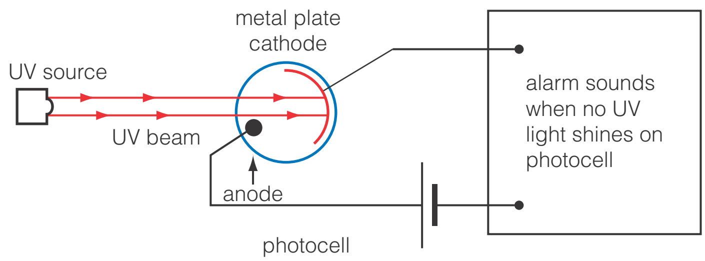


Figure S16.1

Explain how the alarm system works.

3 Newton's law of gravity is a universal law which has solved nearly all problems related to planetary orbits. However, astronomers noticed that there was something not quite right about Mercury's orbit. Over a century, Mercury is about 252 seconds ahead of where Newton's laws predict it should be. With an orbital period of 87.97 days, this amounts to creeping ahead by about 0.6 s per orbit.

Einstein's theory of relativity suggests that, due to the strong gravitational field near the Sun and the planet's great orbital speed, the problem may be solved by adding a small additional term to Newton's theory of gravitation.

An approximate correction to Newton's law is suggested as:

$$F = \frac{GMm}{r^2} + \frac{6GMm}{r^2} \bullet \frac{v^2}{c^2}$$

where G is Newton's gravitational constant, 6.7×10^{-11} N m^2 kg^{-2}

M is the mass of the Sun, 2×10^{30} kg

m is the mass of Mercury, 3×10^{23} kg

r is the orbital distance of Mercury, 5.79×10^{10} m

v is the speed of Mercury's orbit, 4.8×10^5 m s^{-1}

and c is the speed of light, 3×10^8 m s^{-1}

Show that Einstein's correction accounts for a difference of about 0.6 s per orbit of Mercury.

Answers to stretch and challenge questions

1 The proper time is the time measured on Earth. Number of generations, n, it takes for the bacteria to double is given by $2^n = 1024$ so $n = 10$ days.

The Earth observer would observe time dilation for the rocket.

$$t = \frac{t_0}{\sqrt{1 - \frac{v^2}{c^2}}}$$

$$t = \frac{24 \text{ hours}}{\sqrt{1 - 0.865^2}} = 48 \text{ hours}$$

This means that the doubling time for the bacteria on the flask is 2 days as seen from Earth.

In the 10 days that it takes for the Earth flask to contain 1052 bacteria, the space bacteria will have had 5 generations. $2^5 = 32$ bacteria.

2 The UV light causes electrons to be emitted from the cathode. This causes an electric current to flow in the alarm circuit. When the UV beam is broken, the photoelectric current will stop. This 'trips' the circuit and the alarm will sound.

3 Einstein's correction to Newton's laws

$$F = \frac{GMm}{r^2} + \frac{6GMm}{r^2} \bullet \frac{v^2}{c^2}$$

But the gravitational attraction provides the centripetal force so

$$\frac{mv^2}{r} = \frac{GMm}{r^2} + \frac{6GMm}{r^2} \bullet \frac{v^2}{c^2}$$

or

$$v^2 - \frac{6GMm}{r^2} \bullet \frac{v^2}{c^2} = \frac{Gm}{r}$$

and

$$v^2\left(1 - \frac{6gM}{rc^2}\right) = \frac{Gm}{r}$$

but

$$v = \frac{2\pi r}{T}$$

Therefore

$$\frac{4\pi r^2}{T^2}\left(1 - \frac{6GM}{rc^2}\right) = \frac{GM}{r}$$

So

$$T^2 = \frac{4\pi r^3}{GM}\left(1 - \frac{6GM}{rc^2}\right)$$

and

$$T = \left[\frac{4\pi r^3}{GM}\right]^{\frac{1}{2}}\left(1 - \frac{3GM}{rc^2}\right)$$ for $rc^2 >> 6GM$ (using binomial expansion)

For Newton

$$T = \left[\frac{4\pi r^3}{GM}\right]^{\frac{1}{2}} = 7.969 \text{ days}$$

$T = 7.6 \times 10^6$ s

$$\Delta T = \frac{-3GM}{rc^2} \times 7.6\ 10^6\ s$$

$$= \frac{-3 \times 6.7\ 10^{-11} \times 2 \times 10^{30}}{5.79 \times 10^{10} \times 9 \times 10^{16} \times (7.6 \times 10^6)\ s}$$

$= -\ 0.59$ s

ANSWER

Photo credits