**1.**

****

(*a*) Show that the matrix **M** is non-singular.

**(2)**

The transformation *T* of the plane is represented by the matrix **M**.

The triangle *R* is transformed to the triangle *S* by the transformation *T*.

Given that the area of *S* is 63 square units,

(*b*) find the area of *R*.

**(2)**

(*c*) Show that the line *y* = 2*x* is invariant under the transformation *T*.

**(2)**

**(Total for Question 1 is 6 marks)**

**2.** The cubic equation

2*x*3 + 6*x*2 − 3*x* + 12 = 0

has roots *α*, *β* and *γ*.

Without solving the equation, find the cubic equation whose roots are (*α* +3), (*β* +3) and (*γ* + 3), giving your answer in the form *pw*3 + *qw*2 + *rw* + *s* = 0, where *p*, *q*, *r* and *s* are integers to be found.

**(5)**

**(Total for Question 2 is 5 marks)**

**3.** Prove by mathematical induction that, for *n* ∈ ℕ



**(6)**

**(Total for Question 3 is 6 marks)**

**4.** The line *l* has equation



The plane *Π* has equation

**r**.(**i** − 2**j** + **k**) =−7

Determine whether the line *l* intersects *Π* at a single point, or lies in *Π*, or is parallel to *Π*
without intersecting it.

**(5)**

**(Total for Question 4 is 5 marks)**

**5.**

The complex numbers *z*1 = −2, z2 = −1 + 2i and *z*3 =1 + i are plotted in Figure 1, on an Argand diagram for the complex plane with *z* = *x* + i*y*

(*a*) Explain why *z*1, *z*2 and *z*3 cannot all be roots of a quartic polynomial equation with real coefficients.

**(2)**

(*b*) Show that 

**(3)**

(*c*) Hence show that arctan (2) − arctan

**(2)**

(*d*) Copy Figure 1 and shade the set of points of the complex plane that satisfy the inequality



**(2)**

**(Total for Question 5 is 9 marks)**

**6.** An art display consists of an arrangement of *n* marbles.

When arranged in ascending order of mass, the mass of the first marble is 10 grams.
The mass of each subsequent marble is 3 grams more than the mass of the previous one,
so that the *r*th marble has mass (7 + 3*r*) grams.

(*a*) Show that the mean mass, in grams, of the marbles in the display is given by

(3*n* + 17)

**(3)**

Given that there are 85 marbles in the display,

(*b*) use the standard summation formulae to find the standard deviation of the mass of the marbles in the display, giving your answer, in grams, to one decimal place.

**(6)**

**(Total for Question 6 is 9 marks)**

**7.** f(*z*) = *z*3 *−* 8*z*2 + *pz −* 24

where *p* is a real constant.

Given that the equation f(*z*) = 0 has distinct roots



(a) solve completely the equation f(*z*) = 0

**(6)**

(b) Hence find the value of *p*.

**(2)**

**(Total for Question 7 is 8 marks)**

**8.** A gas company maintains a straight pipeline that passes under a mountain.

The pipeline is modelled as a straight line and one side of the mountain is modelled as a plane.

There are accessways from a control centre to two access points on the pipeline.

Modelling the control centre as the origin *O*, the two access points on the pipeline have coordinates *P*(−300, 400, −150) and *Q*(300, 300, −50), where the units are metres.

(*a*) Find a vector equation for the line *PQ,* giving your answer in the form **r** = **a** + λ**b**,
where λ is a scalar parameter.

**(2)**

The equation of the plane modelling the side of the mountain is 2*x* + 3*y* − 5*z* = 300

The company wants to create a new accessway from this side of the mountain to the pipeline.

The accessway will consist of a tunnel of shortest possible length between the pipeline and the point (100, *k*, 100) on this side of the mountain, where *k* is a constant.

(*b*) Using the model, find

 (i) the coordinates of the point at which this tunnel will meet the pipeline,

 (ii) the length of this tunnel.

**(7)**

It is only practical to construct the new accessway if it will be significantly shorter than both of the existing accessways, *OP* and *OQ*.

(*c*) Determine whether the company should build the new accessway.

**(2)**

(*d*) Suggest one limitation of the model.

**(1)**

**(Total for Question 8 is 12 marks)**

**9.** f(*x*) =  + , *x* > 0

The finite region bounded by the curve *y* = f(*x*), the line *x* = , the *x*-a*x*is and the line

*x* = 8 is rotated through *θ* radians about the *x*-a*x*is to form a solid of revolution.

Given that the volume of the solid formed is  units cubed, use algebraic integration to

find the angle *θ* through which the region is rotated.

**(8)**

**(Total for Question 9 is 8 marks)**

**10.** The population of chimpanzees in a particular country consists of juveniles and adults.

Juvenile chimpanzees do not reproduce.

In a study, the numbers of juvenile and adult chimpanzees were estimated at the start of each year. A model for the population satisfies the matrix system

 *n* = 0, 1, 2, …

where *a* is a constant, and *Jn* and *An*, are the respective numbers of juvenile and adult chimpanzees *n* years after the start of the study.

(*a*) Interpret the meaning of the constant a in the context of the model.

**(1)**

At the start of the study, the total number of chimpanzees in the country was
estimated to be 64 000

According to the model, after one year the number of juvenile chimpanzees is 15 360 and the number of adult chimpanzees is 43 008

(*a*) (i) Find, in terms of *a*



**(3)**

 (ii) Hence, or otherwise, find the value of *a*.

**(3)**

 (iii) Calculate the change in the number of juvenile chimpanzees in the first year of the study, according to this model.

**(2)**

Given that the number of juvenile chimpanzees is known to be in decline in the country,

(c) comment on the short-term suitability of this model.

**(1)**

A study of the population revealed that adult chimpanzees stop reproducing at the age of 40 years.

(d) Refine the matrix system for the model to reflect this information, giving a reason for your answer.

(*There is no need to estimate any unknown values for the refined model, but any known values should be made clear*.)

**(2)**

**(Total for Question 10 is 12 marks)**

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**TOTAL FOR CORE PURE MATHEMATICS IS 80 MARKS**