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| **Pearson Edexcel Level 3** |
| **GCE Mathematics** **Advanced** **Paper 2: Pure Mathematics** |
| **Mock paper Spring 2018** **Time: 2 hours** | **Paper Reference(s)** |
| **9MA0/02** |
| **You must have:** **Mathematical Formulae and Statistical Tables, calculator** |

**Candidates may use any calculator permitted by Pearson regulations. Calculators must not have the facility for algebraic manipulation, differentiation and integration, or have retrievable mathematical formulae stored in them.**

**Instructions**

• Use black ink or ball-point pen.

• If pencil is used for diagrams/sketches/graphs it must be dark (HB or B).

• Answer **all** questions and ensure that your answers to parts of questions are clearly labelled.

• Answer the questions in the spaces provided – *there may be more space than you need*.

• You should show sufficient working to make your methods clear. Answers without working may not gain full credit.

• Inexact answers should be given to three significant figures unless otherwise stated.

**Information**

• A booklet ‘Mathematical Formulae and Statistical Tables’ is provided.

• There are 14 questions in this paper. The total mark is 100.

• The marks for each question are shown in brackets – *use this as a guide as to how much time to spend on each question.*

**Advice**

• Read each question carefully before you start to answer it.

• Try to answer every question.

• Check your answers if you have time at the end.

 • If you change your mind about an answer, cross it out and put your new answer and any working underneath.

**Answer ALL questions.**

**1.**

**Figure 1**

Figure 1 shows a circle with centre *O*. The points *A* and *B* lie on the circumference of the circle.

The area of the major sector, shown shaded in Figure 1, is 135 cm2. The reflex angle *AOB* is 4.8 radians.

Find the exact length, in cm, of the minor arc *AB*, giving your answer in the form *aπ* + *b*, where *a* and *b* are integers to be found.

 **(Total for Question 1 is 4 marks)**

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**2.** (a) Given that *θ* is small, use the small angle approximation for cos *θ* to show that

1 + 4 cos *θ* + 3 cos2 *θ* ≈ 8 – 5*θ*2.

**(3)**

Adele uses *θ* = 5° to test the approximation in part (a).

Adele’s working is shown below.

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| Using my calculator, 1 + 4 cos (5°) + 3 cos2 (5°) = 7.962, to 3 decimal places.Using the approximation 8 – 5*θ*2 gives 8 – 5(5)2 = –117Therefore, 1 + 4 cos *θ* + 3 cos2 *θ* ≈ 8 – 5*θ*2 is not true for *θ* = 5°. |

(b) (i) Identify the mistake made by Adele in her working.

 (ii) Show that 8 – 5*θ* 2 can be used to give a good approximation to 1 + 4 cos *θ* + 3 cos2*θ* for an angle of size 5°.

**(2)**

 **(Total for Question 2 is 5 marks)**

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**3.** A cup of hot tea was placed on a table. At time *t* minutes after the cup was placed on the table, the temperature of the tea in the cup, *θ* °C, is modelled by the equation

*θ* = 25 + *A*e–0.03*t*

where *A* is a constant.

The temperature of the tea was 75 °C when the cup was placed on the table.

(*a*) Find a complete equation for the model.

**(1)**

(*b*) Use the model to find the time taken for the tea to cool from 75 °C to 60 °C, giving your answer in minutes to one decimal place.

**(2)**

Two hours after the cup was placed on the table, the temperature of the tea was measured as 20.3 °C.

Using this information,

(*c*) evaluate the model, explaining your reasoning.

**(1)**

 **(Total for Question 3 is 4 marks)**

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**4.** (*a*) Sketch the graph with equation

*y* = |2*x* – 5|,

 stating the coordinates of any points where the graph cuts or meets the coordinate axes.

**(2)**

(*b*) Find the values of *x* which satisfy

|2*x* – 5| > 7.

**(2)**

(*c*) Find the values of *x* which satisfy

|2*x* – 5| > *x* – *.*

 Write your answer in set notation.

**(2)**

 **(Total for Question 4 is 6 marks)**

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**5.** The line *l* has equation 3*x* – 2*y* = *k*, where *k* is a real constant.

Given that the line *l* intersects the curve with equation *y* = 2*x*2 – 5 at two distinct points, find the range of possible values for *k*.

**(Total for Question 5 is 5 marks)**

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**6.**

**Figure 2**

Figure 2 shows a sketch of the curve with equation *y* = f(*x*), where f(*x*) = (8 – *x*) ln *x*, *x* > 0.

The curve cuts the *x*-axis at the points *A* and *B* and has a maximum turning point at *Q*, as shown in Figure 2.

(*a*) Find the *x* coordinate of *A* and the *x* coordinate of *B*.

**(1)**

(*b*) Show that the *x-*coordinate of *Q* satisfies *x* = .

**(4)**

(*c*) Show that the *x-*coordinate of *Q* lies between 3.5 and 3.6

**(2)**

(*d*) Use the iterative formula *xn* + 1 =   with *x*1 = 3.5 to find

 (i) the value of *x*5 to 4 decimal places,

 (ii) the *x-*coordinate of *Q* accurate to 2 decimal places.

**(2)**

 **(Total for Question 6 is 9 marks)**

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**7.** A bacterial culture has area *p* mm2 at time *t* hours after the culture was placed onto a circular dish.

A scientist states that at time *t* hours, the rate of increase of the area of the culture can be modelled as being proportional to the area of the culture.

(*a*) Show that the scientist’s model for *p* leads to the equation *p* = *a*e*kt*, where *a* and *k* are constants.

**(4)**

The scientist measures the values for *p* at regular intervals during the first 24 hours after the culture was placed onto the dish. She plots a graph of ln *p* against *t* and finds that the points on the graph lie close to a straight line with gradient 0.14 and vertical intercept 3.95.

(*b*) Estimate, to 2 significant figures, the value of *a* and the value of *k*.

**(3)**

(*c*) Hence show that the model for *p* can be rewritten as *p* = *abt*,stating, to 3 significant figures, the value of the constant *b*.

**(2)**

With reference to this model,

(*d*) (i) interpret the value of the constant *a*,

 (ii) interpret the value of the constant *b*.

**(2)**

(*e*) State a long term limitation of the model for *p*.

**(1)**

 **(Total for Question 7 is 12 marks)**

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**8.**

**Figure 3**

A bowl is modelled as a hemispherical shell as shown in Figure 3.

Initially the bowl is empty and water begins to flow into the bowl.

When the depth of the water is *h* cm, the volume of water, *V* cm3, according to the model is given by

*V* = *πh*2(75 – *h*), 0 ≤ *h* ≤ 24.

The flow of water into the bowl is at a constant rate of 160*π* cm3 s–1 for 0 ≤ *h* ≤ 12.

(*a*) Find the rate of change of the depth of the water, in cm s–1, when *h* = 10.

**(5)**

Given that the flow of water into the bowl is increased to a constant rate of 300π cm3s–1 for 12 < *h* ≤ 24,

(*b*) find the rate of change of the depth of the water, in cm s–1, when *h* = 20

**(2)**

 **(Total for Question 8 is 7 marks)**

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**9.** A circle with centre *A* (3, –1) passes through the point *P* (–9, 8) and the point *Q* (15, –10).

(*a*) Show that *PQ* is a diameter of the circle.

**(2)**

(*b*) Find an equation for the circle.

**(3)**

A point *R* also lies on the circle.

Given that the length of the chord *PR* is 20 units,

(*c*) find the length of the shortest distance from *A* to the chord *PR*, giving your answer as a surd in its simplest form.

**(2)**

(*d*) Find the size of angle *ARQ*, giving your answer to the nearest 0.1 of a degree.

**(2)**

 **(Total for Question 9 is 9 marks)**

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**10.**

**Figure 4**

Figure 4 shows a sketch of the curve *C* with parametric equations

*x* = ln (*t* + 2), *y* = , *t* > –.

(*a*) State the domain of values of *x* for the curve *C*.

**(1)**

The finite region *R*, shown shaded in Figure 4, is bounded by the curve *C*, the line with equation *x* = ln 2, the *x*-axis and the line with equation *x* = ln 4

(*b*) Use calculus to show that the area of *R* is ln .

 **(8)**

 **(Total for Question 10 is 9 marks)**

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**11.** The second, third and fourth terms of an arithmetic sequence are 2*k*, 5*k* – 10 and 7*k* – 14 respectively, where *k* is a constant.

Show that the sum of the first *n* terms of the sequence is a square number.

**(Total for Question 11 is 5 marks)**

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**12.** A curve *C* is given by the equation

sin *x* + cos *y* = 0.5,  ≤ *x* < , –*π* < *y* < *π*.

A point *P* lies on *C*. The tangent to *C* at the point *P* is parallel to the *x*-axis.

Find the exact coordinates of all possible points *P*, justifying your answer.

(*Solutions based entirely on graphical or numerical methods are not acceptable*.)

**(Total for Question 12 is 7 marks)**

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**13.** (*a*) Show that

cosec 2*x* + cot 2*x* ≡ cot *x*, *x* ≠ 90*n*°, *n* ∈ ℤ.

**(5)**

(*b*) Hence, or otherwise, solve, for 0 ≤ *θ* < 180°,

cosec(4*θ +* 10°) + cot(4*θ +* 10°) = √3.

 You must show your working.

(*Solutions based entirely on graphical or numerical methods are not acceptable*.)

**(5)**

 **(Total for Question 13 is 10 marks)**

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**14.** Kayden claims that 3*x* ≥ 2*x*.

(i) Determine whether Kayden’s claim is always true, sometimes true or never true, justifying your answer.

**(2)**

(ii) Prove that √3 is an irrational number.

**(6)**

 **(Total for Question 14 is 8 marks)**

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**TOTAL FOR PAPER IS 100 MARKS**

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