**9MA0/02: Pure Mathematics Paper 2 Mark scheme**

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| **Question** | | **Scheme** | **Marks** | **AOs** |
| **1** | |  | M1 | 1.1a |
|  | A1 | 1.1b |
| length of minor arc | dM1 | 3.1a |
|  | A1 | 1.1b |
|  | **(4)** |  |
| **1**  **Alt** | |  | M1 | 1.1a |
|  | A1 | 1.1b |
| length of major arc |  |  |
| length of minor arc | dM1 | 3.1a |
|  | A1 | 1.1b |
|  | **(4)** |  |
| **(4 marks)** | | | | |
| **Question 1 Notes:** | | | | |
| **M1:** | Applies formula for the area of a sector with  i.e.  with  **Note:** Allow M1 for considering ratios. E.g. | | | |
| **A1:** | Uses a correct equation  to obtain a radius of 7.5 | | | |
| **dM1:** | Depends on the previous M mark.  A complete process for finding the length of the minor arc *AB*, by either | | | |
| **A1:** | Correct exact answer in its simplest form, e.g. | | | |

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| **Question** | | **Scheme** | **Marks** | **AOs** |
| **2(a)** | | Attempts to substitute into either | M1 | 1.1b |
|  |  |  |
|  | M1 | 1.1b |
|  |  |  |
| \* | A1\* | 2.1 |
|  | |  | **(3)** |  |
| **(b)(i)** | | E.g.   * Adele is working in degrees and not radians * Adele should substitute  and not  into the approximation | B1 | 2.3 |
| **(b)(ii)** | | , so  gives a good approximation. | B1 | 2.4 |
|  | **(2)** |  |
| **(5 marks)** | | | | |
| **Question 2 Notes:** | | | | |
| **(a)(i)** |  | | | |
| **M1:** | See scheme | | | |
| **M1:** | Substitutes  into  and attempts to apply  **Note:** It is not a requirement for this mark to write or refer to the term in | | | |
| **A1\*:** | Correct proof with no errors seen in working.  **Note:** It is not a requirement for this mark to write or refer to the term in | | | |
| **(a)(ii)** |  | | | |
| **B1:** | See scheme | | | |
| **(b)(i)** |  | | | |
| **B1:** | See scheme | | | |
| **(b)(ii)** |  | | | |
| **B1:** | Substitutes  into  to give awrt 7.962 ***and*** an appropriate conclusion. | | | |

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| **Question** | | **Scheme** | **Marks** | **AOs** |
| **3 (a)** | |  | B1 | 3.3 |
|  | **(1)** |  |
| **(b)** | |  | M1 | 3.4 |
| minutes (1 dp) | A1 | 1.1b |
|  | **(2)** |  |
| **(c)** | | A valid evaluation of the model, which relates to the large values of *t*.  E.g.   * As  then the model is not true for large values of *t* * does not have any solutions and so the model predicts that tea in the room will never be  So the model does not work for large values of *t* * *t* = 120 *θ* = 25 + 50*e* −0.03(120) = 26.36… which is not approximately equal to 20.3, so the model is not true for large values of *t* | B1 | 3.5a |
|  | **(1)** |  |
| **(4 marks)** | | | | |
| **Question 3 Notes:** | | | | |
| **(a)** |  | | | |
| **B1:** | Applies  to give the complete model | | | |
| **(b)** |  | | | |
| **M1:** | Applies  and their value of *A* to the model and rearranges to make  the subject. | | | |
|  | **Note:** Later working can imply this mark. | | | |
| **A1** | Obtains 11.9 (minutes) with no errors in manipulation seen. | | | |
| **(c)** |  | | | |
| **B1** | See scheme | | | |

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| **Question** | **Scheme** | | **Marks** | **AOs** |
| **4(a)** | 5  *x*  *y*  *O* | Correct graph in  quadrant 1 and quadrant 2  with V on the *x*-axis | B1 | 1.1b |
| States  and  **or** marked in the correct position on the *x‑*axis  **and** 5 marked in the correct position on the *y*-axis | B1 | 1.1b |
|  | | **(2)** |  |
| **(b)** |  | |  |  |
| **and** | | M1 | 1.1b |
| {critical values are } | | A1 | 1.1b |
|  | | **(2)** |  |
| **(c)** |  | |  |  |
| E.g.   * Solves  to give   and solves  to also give   * Sketches graphs of  and .   Indicates that these graphs meet at the point | | M1 | 3.1a |
| Hence using set notation, e.g. | | A1 | 2.5 |
|  |  | | **(2)** |  |
| **(6 marks)** | | | | |

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| **Question 4 Notes:** | |
| **(a)** |  |
| **B1:** | See scheme |
| **B1:** | See scheme |
| **(b)** |  |
| **M1:** | See scheme |
| **A1:** | Correct answer, e.g. |
| **(c)** |  |
| **M1:** | A complete process of finding that  and  meet at ***only*** one point.  This can be achieved either algebraically or graphically. |
| **A1:** | See scheme.  **Note:** Final answer must be expressed using set notation. |

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| **Question** | **Scheme** | **Marks** | **AOs** |
| **5** | intersects  at two distinct points |  |  |
| Eliminate *y* and forms quadratic equation = 0 or quadratic expression | M1 | 3.1a |
|  | A1 | 1.1b |
|  | dM1 | 2.1 |
|  |  |  |
| Critical value obtained of | B1 | 1.1b |
| o.e. | A1 | 1.1b |
|  | **(5)** |  |
| **5**  **Alt 1** | Eliminate *y* and forms quadratic equation = 0 or quadratic expression | M1 | 3.1a |
|  |  |  |
|  | A1 | 1.1b |
|  | dM1 | 2.1 |
|  |  |  |
| Critical value obtained of | B1 | 1.1b |
| o.e. | A1 | 1.1b |
|  | **(5)** |  |
| **5**  **Alt 2** | . So | M1 | 3.1a |
| A1 | 1.1b |
|  | dM1 | 2.1 |
| Critical value obtained of | B1 | 1.1b |
| o.e. | A1 | 1.1b |
|  | **(5)** |  |
| **(5 marks)** | | | |

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| **Question 5 Notes:** | |
| **M1:** | Complete strategy of eliminating *x* or *y* and manipulating the resulting equation to form a quadratic equation = 0 or a quadratic expression |
| **A1:** | Correct algebra leading to either   * or   or a one-sided quadratic of either  or      or a one-sided quadratic of e.g. |
| **dM1:** | Depends on the previous M mark.  Interprets  intersecting  at two distinct points by applying  to their quadratic equation or one-sided quadratic. |
| **B1:** | See scheme |
| **A1:** | Correct answer, e.g. |
| **Alt 2** |  |
| **M1:** | Complete strategy of using differentiation to find the values of *x* and *y* where  is a tangent to |
| **A1:** | Correct algebra leading to |
| **dM1:** | Depends on the previous M mark.  Full method of substituting their  into *l* and attempting to find the value for *k*. |
| **B1:** | See scheme |
| **A1:** | Deduces correct answer, e.g. |

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| **Question** | **Scheme** | **Marks** | **AOs** |
| **6(a)** |  |  |  |
| Crosses *x*-axis |  |  |
| *x* coordinates are 1 and 8 | B1 | 1.1b |
|  | **(1)** |  |
| **(b)** | Complete strategy of setting  and rearranges to make | M1 | 3.1a |
|  |  |  |
|  | M1 | 1.1b |
| A1 | 1.1b |
| \* | A1\* | 2.1 |
|  | **(4)** |  |
| **(c)** | Evaluates both  and | M1 | 1.1b |
| and  Sign change and as  is continuous, the *x* coordinate of *Q* lies between  and | A1 | 2.4 |
|  | **(2)** |  |
| **(d)(i)** |  | B1 | 1.1b |
| **(d)(ii)** | (2 dp) | B1 | 2.2a |
|  | **(2)** |  |
| **(9 marks)** | | | |

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| **Question 6 Notes:** | |
| **(a)** |  |
| **B1:** | Either   * 1 and 8 * on Figure 2, marks 1 next to *A* and 8 next to *B* |
| **(b)** |  |
| **M1:** | Recognises that *Q* is a stationary point (and not a root) and applies a complete strategy of setting  and rearranges to make |
| **M1:** | Applies , where  **Note:** This mark can be recovered for work in part (c) |
| **A1:** | or equivalent  **Note:** This mark can be recovered for work in part (c) |
| **A1\*:** | Correct proof with no errors seen in working. |
| **(c)** |  |
| **M1:** | Evaluates both  and |
| **A1:** | and  or  (truncated)  **and** a correct conclusion |
| **(d)(i)** |  |
| **B1:** | See scheme |
| **(d)(ii)** |  |
| **B1:** | Deduces (e.g. by the use of further iterations) that the *x* coordinate of *Q* is 3.54 accurate to 2 dp |
|  | **Note:** |

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| **Question** | **Scheme** | **Marks** | **AOs** |
| **7(a)** |  | B1 | 3.3 |
|  | M1 | 1.1b |
|  | A1 | 1.1b |
| \* | A1 \* | 2.1 |
|  | **(4)** |  |
| **(b)** | and evidence of understanding that either    * vertical intercept  or | M1 | 2.1 |
|  | A1 | 1.1b |
| vertical intercept | A1 | 1.1b |
|  | **(3)** |  |
| **(c)** | e.g. | B1 | 2.2a |
| which can be implied by | B1 | 1.1b |
|  | **(2)** |  |
| **(d)(i)** | Initial area (i.e. ) of bacterial culture that was first placed onto the circular dish. | B1 | 3.4 |
| **(d)(ii)** | E.g.   * Rate of increase per hour of the area of bacterial culture * The area of bacterial culture increases by “15%” each hour | B1 | 3.4 |
|  |  | **(2)** |  |
| **(e)** | The model predicts that the area of the bacteria culture will increase indefinitely, but the size of the circular dish will be a constraint on this area. | B1 | 3.5b |
|  | **(1)** |  |
| **(12 marks)** | | | |

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| **Question 7 Notes:** | |
| **(a)** |  |
| **B1:** | Translates the scientist’s statement regarding proportionality into a differential equation, which involves a constant of proportionality. e.g. |
| **M1:** | Correct method of separating the variables *p* and *t* in their differential equation |
| **A1:** | with or without a constant of integration |
| **A1\*:** | Correct proof with no errors seen in working. |
| **(b)** |  |
| **M1:** | See scheme |
| **A1:** | Correctly finds |
| **A1:** | Correctly finds |
| **(c)** |  |
| **B1:** | Uses algebra to correctly deduce either   * from * from |
| **B1:** | See scheme |
| **(d)(i)** |  |
| **B1:** | See scheme |
| **(d)(ii)** |  |
| **B1:** | See scheme |
| **(e)** |  |
| **B1:** | Gives a correct long-term limitation of the model for *p*. (See scheme). |

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| **Question** | | **Scheme** | **Marks** | **AOs** |
| **8(a)** | | , |  |  |
|  | M1 | 1.1b |
| A1 | 1.1b |
|  | M1 | 3.1a |
| When | dM1 | 3.4 |
|  | A1 | 1.1b |
|  | **(5)** |  |
| **(b)** | |  | M1 | 3.4 |
|  | A1 | 1.1b |
|  | **(2)** |  |
| **(7 marks)** | | | | |
| **Question 8 Notes:** | | | | |
| **(a)** |  | | | |
| **M1:** | Differentiates *V* with respect to *h* to give | | | |
| **A1:** |  | | | |
| **M1:** | Attempts to solve the problem by applying a complete method of | | | |
| **M1:** | Depends on the previous M mark. | | | |
|  | Substitutes  into their model for  which is in the form | | | |
| **A1:** | Obtains the correct answer 0.4 | | | |
| **(b)** |  | | | |
| **M1:** | Realises that rate for of for  has no effect when the rate is increased to for  and so substitutes  into their model for  which is in the form | | | |
| **A1:** | Obtains the correct answer 0.5 | | | |

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| **Question** | **Scheme** | **Marks** | **AOs** |
| **9(a)** | E.g. midpoint *PQ* | M1 | 1.1b |
| which is the centre point *A*,  so *PQ* is the diameter of the circle. | A1 | 2.1 |
|  | **(2)** |  |
| **(a)**  **Alt 1** |  | M1 | 1.1b |
| So  so *PQ* is the diameter of the circle. | A1 | 2.1 |
|  | **(2)** |  |
| **(a)**  **Alt 2** | **and either** | M1 | 1.1b |
| e.g. as , then *PQ* is the diameter of the circle. | A1 | 2.1 |
|  | **(2)** |  |
| **(b)** | Uses Pythagoras in a correct method to find  either the radius or diameter of the circle. | M1 | 1.1b |
|  | M1 | 1.1b |
| A1 | 1.1b |
|  | **(3)** |  |
| **(c)** | Distance  or | M1 | 3.1a |
|  | A1 | 1.1b |
|  | **(2)** |  |
| **(d)** | or | M1 | 3.1a |
| (to 0.1 of a degree) | A1 | 1.1b |
|  | **(2)** |  |
| **(9 marks)** | | | |

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| **Question 9 Notes:** | |
| **(a)** |  |
| **M1:** | Uses a correct method to find the midpoint of the line segment *PQ* |
| **A1:** | Completes proof by obtaining  and gives a correct conclusion. |
| **(a)** |  |
| **Alt 1** |  |
| **M1:** | Full attempt to find the equation of the line *PQ* |
| **A1:** | Completes proof by showing that  lies on *PQ* and gives a correct conclusion. |
| **(a)** |  |
| **Alt 2** |  |
| **M1:** | Attempts to find distance *PQ* and either one of distance *AP* or distance *AQ* |
| **A1:** | Correctly shows either   * , supported by  and gives a correct conclusion * , supported by  and gives a correct conclusion |
| **(b)** |  |
| **M1:** | **Either**   * uses Pythagoras correctly in order to find the **radius**. Must clearly be identified as the **radius**. E.g.  or  or  or   **or**   * uses Pythagoras correctly in order to find the **diameter**. Must clearly be identified as the **diameter**. E.g.  or |
|  | **Note:** This mark can be implied by just 30 clearly seen as the **diameter** or 15 clearly seen as the **radius** (may be seen or implied in their circle equation) |
| **M1:** | Writes down a circle equation in the form |
| **A1:** | or  or |
| **(c)** |  |
| **M1:** | Attempts to solve the problem by using the circle property “the perpendicular from the centre to a chord bisects the chord” and so applies Pythagoras to write down an expression of the form . |
| **A1:** | by correct solution only |
| **(d)** |  |
| **M1:** | Attempts to solve the problem by e.g. using the circle property “the angle in a semi-circle is a right angle” and writes down either  or  **Note:** Also allow |
| **A1:** | 41.8 by correct solution only |

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| **Question** | **Scheme** | **Marks** | **AOs** |
| **10 (a)** |  | B1 | 2.2a |
|  | **(1)** |  |
| **(b)** | Attempts to apply | M1 | 3.1a |
|  | A1 | 1.1b |
|  | M1 | 3.1a |
|  | A1 | 1.1b |
|  | M1 | 1.1b |
| A1 | 1.1b |
|  | M1 | 2.2a |
|  |  |  |
| \* | A1\* | 2.1 |
|  | **(8)** |  |
| **(b)**  **Alt 1** | Attempts to apply ,  with a substitution of | M1 | 3.1a |
|  | A1 | 1.1b |
|  | M1 | 3.1a |
|  | A1 | 1.1b |
|  | M1 | 1.1b |
| A1 | 1.1b |
|  | M1 | 2.2a |
|  |  |  |
| \* | A1 \* | 2.1 |
|  | **(8)** |  |
| **(9 marks)** | | | |

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| **Question** | **Scheme** | **Marks** | **AOs** |
| **10 (b)**  **Alt 2** | Attempts to apply ,  with a substitution of | M1 | 3.1a |
|  | A1 | 1.1b |
|  | M1 | 3.1a |
|  | A1 | 1.1b |
|  | M1 | 1.1b |
| A1 | 1.1b |
|  | M1 | 2.2a |
|  |  |  |
| \* | A1 \* | 2.1 |
|  | **(8)** |  |

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| **Question 10 Notes:** | |
| **(a)** |  |
| **B1:** | Uses  with  to deduce the correct domain, |
| **(b)** |  |
| **M1:** | Attempts to solve the problem by either   * a parametric process or * a Cartesian process with a substitution of either  or |
| **A1:** | Obtains   * from a parametric approach * from a Cartesian approach with * from a Cartesian approach with |
| **M1:** | Applies a strategy of attempting to express either ,  or  as partial fractions |
| **A1:** | Correct partial fractions for their method |
| **M1:** | Integrates to give either    * where * where |
| **A1:** | Correct integration for their method |
| **M1:** | Either   * Parametric approach: Deduces and applies limits of 2 and 0 in *t* and subtracts the correct way round * Cartesian approach: Deduces and applies limits of 3 and 1 in *u*, where  and subtracts the correct way round * Cartesian approach: Deduces and applies limits of 4 and 2 in *v*, where  and subtracts the correct way round |
| **A1\*:** | Correctly shows that the area of *R* is , with no errors seen in their working |

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| **Question** | | **Scheme** | **Marks** | **AOs** |
| **11** | | Arithmetic sequence, |  |  |
|  | M1 | 2.1 |
|  | A1 | 1.1b |
| . So | M1 | 2.2a |
|  | M1 | 1.1b |
| which is a square number | A1 | 2.1 |
|  | **(5)** |  |
| **(5 marks)** | | | | |
| **Question 11 Notes:** | | | | |
| **M1:** | Complete method to find the value of *k* | | | |
| **A1:** | Uses a correct method to find | | | |
| **M1:** | Uses their value of *k* to deduce the common difference and the first term  of the arithmetic series. | | | |
| **M1:** | Applies  with their  and their *d*. | | | |
| **A1:** | Correctly shows that the sum of the series is  and makes an appropriate conclusion. | | | |

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| **Question** | | **Scheme** | **Marks** | **AOs** |
| **12** | | Complete process to find at least one set of coordinates for *P*.  The process must include evidence of   * differentiating * setting  to find * substituting  into  to find | M1 | 3.1a |
|  | B1 | 1.1b |
| Applies | M1 | 2.2a |
| giving at least one of either  or | A1 | 1.1b |
|  | M1 | 1.1b |
| So in specified range, , by cso | A1 | 1.1b |
| has no solutions,  and so there are exactly 2 possible points *P*. | B1 | 2.1 |
|  | **(7)** |  |
| **(7 marks)** | | | | |
| **Question 12 Notes:** | | | | |
| **M1:** | See scheme | | | |
| **B1:** | Correct differentiated equation. E.g. | | | |
| **M1:** | Uses the information “the tangent to *C* at the point *P* is parallel to the *x*-axis”  to deduce and apply  and finds | | | |
| **A1:** | See scheme | | | |
| **M1:** | For substituting one of their values from  into  and so finds | | | |
| **A1:** | Selects coordinates for *P* on *C* satisfying  and | | | |
|  | i.e. finds  and no other points by correct solution only | | | |
| **B1:** | Complete argument to show that there are exactly 2 possible points *P*. | | | |

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| **Question** | **Scheme** | **Marks** | **AOs** |
| **13(a)** |  |  |  |
|  | M1 | 1.2 |
|  | M1 | 1.1b |
|  | M1 | 2.1 |
| A1 | 1.1b |
|  | A1\* | 2.1 |
|  | **(5)** |  |
| **(b)** | ; |  |  |
|  | M1 | 2.2a |
|  | M1 | 1.1b |
| A1 | 1.1b |
|  | M1 | 2.1 |
|  | A1 | 1.1b |
|  | **(5)** |  |
| **(10 marks)** | | | |

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| **Question 13 Notes:** | |
| **(a)** |  |
| **M1:** | Writes  and |
| **M1:** | Combines into a single fraction with a common denominator |
| **M1:** | Applies  to the denominator **and** applies either |
|  | * and * and   to the numerator and manipulates to give a one term numerator expression |
| **A1:** | Correct algebra leading to  or equivalent. |
| **A1\*:** | Correct proof with correct notation and no errors seen in working |
| **(b)** |  |
| **M1:** | Uses the result in part (a) in an attempt to deduce either  or  and uses  to write down or imply |
| **M1:** | Applies  or |
|  | and attempts to solve  to give |
| **A1:** | Uses a correct method to obtain |
| **M1:** | Uses  in a complete method to find the second solution, |
| **A1:** | Uses a correct method to obtain , with no extra solutions given either inside or outside the required range |

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| **Question** | **Scheme** | **Marks** | **AOs** |
| **14 (i)** | For an explanation or statement to show when the claim  fails  This could be e.g.   * when   or  is not greater than or equal to * when   or  is not greater than or equal to | M1 | 2.3 |
| followed by an explanation or statement to show when the claim  is true. This could be e.g.   * or 9 is greater than or equal to 4 * when   and a correct conclusion. E.g.   * so the claim  is sometimes true | A1 | 2.4 |
|  | **(2)** |  |
| **(ii)** | Assume that  is a rational number  So , where *p* and *q* integers,  and the HCF of *p* and *q* is 1 | M1 | 2.1 |
| is divisible by 3 and so *p* is divisible by 3 | M1 | 1.1b |
| A1 | 2.2a |
| So where *c* is an integer  From earlier, | M1 | 2.1 |
| is divisible by 3 and so *q* is divisible by 3 | A1 | 1.1b |
| As both *p* and *q* are both divisible by 3 then the HCF of *p* and *q* is not 1  This contradiction implies that  is an irrational number | A1 | 2.4 |
|  | **(6)** |  |
| **(8 marks)** | | | |

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| **Question 14 Notes:** | |
| **(i)** |  |
| **M1:** | See scheme |
| **A1:** | See scheme |
| **(ii)** |  |
| **M1:** | Uses a method of proof by contradiction by initially assuming that  is rational and expresses |
|  | in the form , where *p* and *q* are correctly defined. |
| **M1:** | Writes  and rearranges to make  the subject |
| **A1:** | Uses a logical argument to prove that *p* is divisible by 3 |
| **M1:** | Uses the result that *p* is divisible by 3, (to construct the initial stage of proving that  is also divisible by 3), by substituting  into their expression for |
| **A1:** | Hence uses a correct argument, in the same way as before, to deduce that *q* is also divisible by 3 |
| **A1:** | Completes the argument (as detailed on the scheme) that  is irrational. |
|  | **Note:** All the previous 5 marks need to be scored in order to obtain the final A mark. |