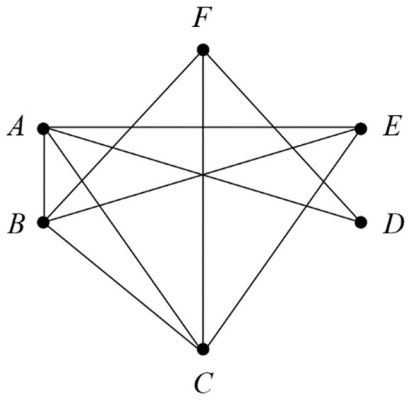
DECISION MATHS 1: UNIT TEST 6 PLANARITY, FLOYD, TRAVELLING SALESPERSON

1 Look at Figure 1.

Figure 1



a Complete a Hamiltonian cycle starting *ADF* for the graph in Figure 1. **(1 mark)**

b Use the planarity algorithm to determine whether the graph shown in Figure 1 is planar.

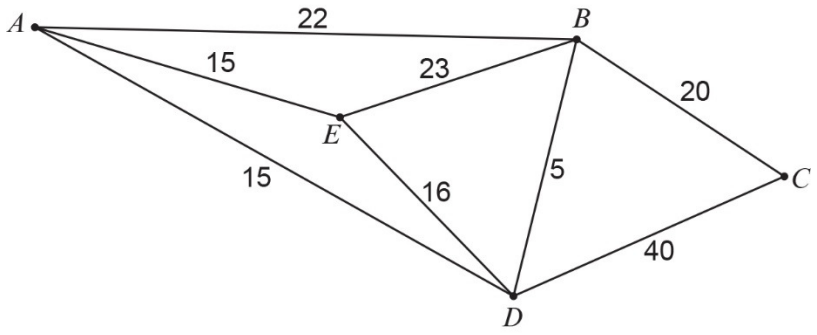
Draw a diagram to justify your answer. **(3 marks)**

Arcs *ED* and *CD* are now added to the graph.

c Explain why the new graph is not planar. **(1 mark)**

2 Figure 2 shows roads connecting five towns. The numbers show distances in kilometres.

Figure 2



These are the distance and route matrices after the third iteration of Floyd's algorithm:

	A	*B*	*C*	*D*	*E*
A	-	22	42	15	15
B	22	-	20	5	23
C	42	20	-	25	43
D	15	5	25	-	16
E	15	23	43	16	-

	A	*B*	*C*	*D*	*E*
A	-	B	B	D	E
B	A	-	C	D	E
C	B	B	-	B	B
D	A	B	B	-	E
E	A	B	B	D	-

a Perform the fourth iteration. (**4 marks**)

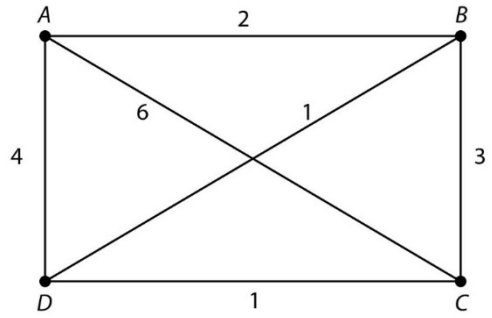
There are no changes on the fifth iteration.

b Explain how to find the shortest distance and route from town *C* to town *A* using your answer from part **a**.

State the route and the distance. (**4 marks**)

3 Figure 6 shows a network with the weights on the arcs representing distances.

Figure 3



a Apply Floyd's algorithm to find the complete network of shortest distances.

You should show both the distance table and the route table after each iteration. **(9 marks)**

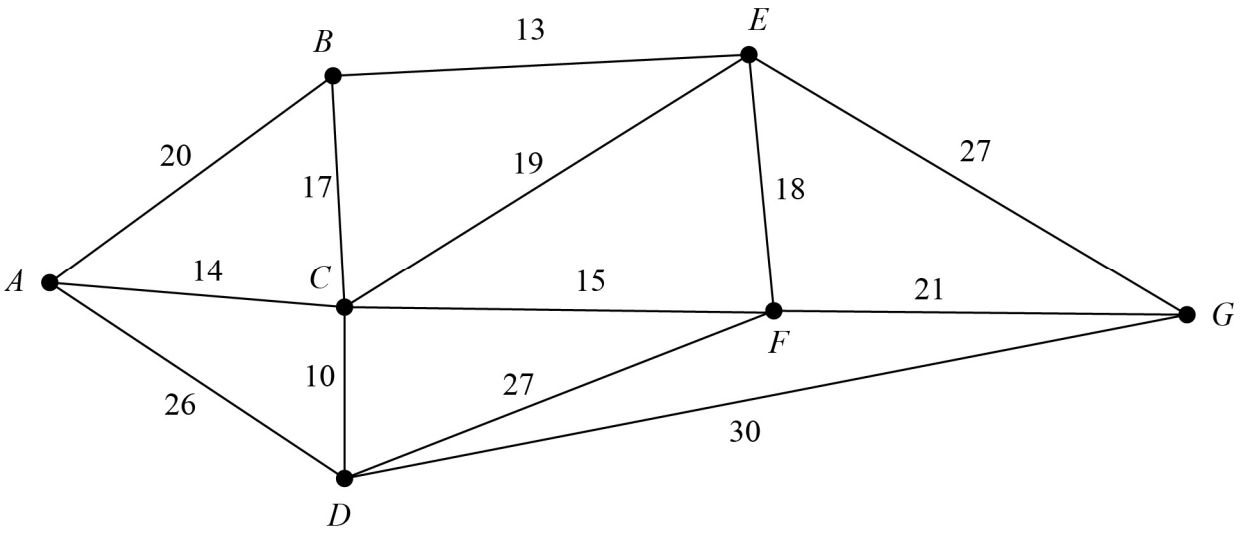
	A	*B*	*C*	*D*
A				
B				
C				
D				

	A	*B*	*C*	*D*
A				
B				
C				
D				

b Explain how to use your final matrix to find the shortest route from vertex *A* to vertex *C*.

State this distance. **(3 marks)**

4 The network in the diagram shows the distances, in km, between seven wind turbines. All the turbines need to be serviced by an engineer.



By deleting *C* a lower bound for the length of the route is found to be 122 km.

a By deleting *F*, find another lower bound of the route. State which is the better lower bound of the two. **(5 marks)**

b By inspection, complete the table of least distances below. **(3 marks)**

	A	***B***	***C***	***D***	***E***	***F***	***G***
A	–	20	14		33	29	
B	20	–	17		13	31	40
C	14	17	–	10	19	15	
D			10	–			30
E	33	13	19		–	18	27
F	29	31	15		18	–	21
G		40		30	27	21	–

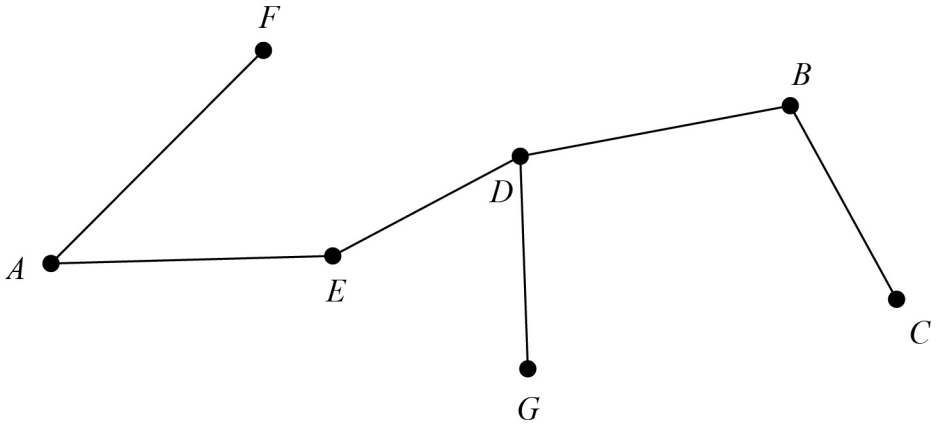
c Starting at *F* use the nearest neighbour algorithm with the completed table to obtain an upper bound for the length of the route. State your route. **(3 marks)**

5

	A	***B***	***C***	***D***	***E***	***F***	***G***
A	–	4	5	3	2	5	6
B	4	–	1	2	4	7	6
C	5	1	–	3	4	6	7
D	3	2	3	–	2	6	4
E	2	4	4	2	–	6	6
F	5	7	6	6	6	–	10
G	6	6	7	4	6	10	–

The table shows the distance, in km, between seven houses.

Prim's algorithm is used on the above table to find a minimum spanning tree. The order in which the arcs are chosen is *AE ED BD BC DG AF* with the associated tree.



Melissa delivers parcels. She has a parcel to deliver to each of the seven houses.

The distance between the parcel depot and each house, in km, is shown in the table below.

House	***A***	***B***	***C***	***D***	***E***	***F***	***G***
Distance from depot P (km)	11	15	16	16	12	17	18

Melissa wants to travel the shortest distance from the depot to each house before returning to the depot.

a Using the given information, calculate a lower bound for the length of Melissa's route. You must show all your working clearly. (**3 marks)**

b Use the nearest neighbour algorithm, starting from the parcel depot (P), to find a
route for Melissa to deliver all her parcels. Hence find an upper bound for the length of Melissa's route. **(3 marks)**