Answers to examination-style questions

Answers	Marks	Examiner's tips
1 (a) (i) With the object on the spring: the mean value of $x = 72$ mm, $e = 70$ mm	1	
(ii) 1.4%	1	Each reading was ±0.5 mm. As the extension was the subtraction of two readings the absolute errors are added to give an absolute error of ±1.0 mm and a percentage error of $\frac{1}{70} \times 100 = 1.4\%$.
(b) (i) 0.551s	1	$T_{av} = 11.02$ s
(ii) 0.6%	1	The absolute error can be taken as half the range of the values so the error in T_{av} is $\frac{11.11 - 10.97}{2} = 0.07$s.
(c) (i) $mg = ke$ therefore $e = \frac{mg}{k}$	1	
(ii) Using the above equation gives $\frac{m}{k} = \frac{e}{g}$	1	
Substituting this expression for $\frac{m}{k}$ into the mass–spring time period equation $T = 2\pi\sqrt{\frac{m}{k}}$ gives the required equation	1	
(d) Plot either T^2 against e or T against $\sqrt{e}$ to give a straight line.	4	One mark each for: • correct labels and units • suitable scales • all points plotted correctly • best-fit line
According to the equation, the line should pass through the origin and the gradient is equal to $\frac{4\pi^2}{g}$ for the T^2 against e line or $\frac{2\pi}{\sqrt{g}}$ for the T against $\sqrt{e}$ line.	1	
To determine g, the gradient of the line should be measured.	1	
Gradient given the correct unit ($s^2\ m^{-1}$ or $s^2\ mm^{-1}$).	1	
A large triangle used correctly to determine the gradient.	1	Draw a triangle or use points that cover over half of the line you have drawn. It improves accuracy. Make sure the points used are on your line and not just two of the plotted points.
… and used with the appropriate gradient formula above to find g.	1	For example, the graph for T^2 against e is shown below. Its gradient = 4.074 $s^2\ m^{-1}$. Hence $g = \frac{4\pi^2}{4.074} = 9.69\ m\ s^{-2}$

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(e) The graph should give a best-fit line that passes through the origin (or almost does). Without carrying out detailed error calculations, the percentage errors in the measurement of e and in T suggests an overall percentage error in g of at least 2%, which would give an error in g of ± 0.2 m s^{-2}.	1	Alternatively an attempt can be made to find the error in the gradient of the graph by drawing the "worst" possible line as well as the best fit line and comparing the gradients. However with a very good best fit graph just taking readings for the gradient can introduce a small error.
The accepted value of g is within this range.	1	
An improved method of measuring the extension would give a more accurate value of g. Or the extension contributes the largest percentage error and improving this measurement is important.	1	
For example, a convex lens could be used as a magnifying glass to observe the position of the marker pin on the mm scale.	1	
2 (a) (i) Distance d is twice the amplitude.	1	The amplitude A is the maximum displacement from the equilibrium position. In half an oscillation, each prong of the fork oscillates from one rest position, through equilibrium, to the opposite rest position. This is a distance of $2A$.
(ii) *Graph drawn to include*: • At least one cycle of a sinusoidal graph.	1	Start this by drawing axes: displacement/mm (upwards) and time/ms (across the page). 'Sinusoidal' means 'like the shape of a sine wave' (a cosine wave, a −sine wave or a −cosine wave would be satisfactory). The period $T = \frac{1}{f} = \frac{1}{512} = 1.95 \times 10^{-3}$ s Marking this time as $\frac{1}{512}$ s on the axis would not be acceptable.
• Amplitude correctly shown on displacement axis at ±0.85 mm	1	
• Period correctly shown on time axis at 1.95 (or 2.0) m s	1	
(b) (i) Maximum speed of tip of a prong $V_{max} = 2\pi fa = 2\pi \times 512 \times 0.85 \times 10^{-3}$	1	The maximum speed occurs at the centre of the oscillation, where displacement $x = 0$. In general, $v = \pm 2\pi f\sqrt{A^2 - x^2}$; but when $x = 0$ this reduces to $v_{max} = 2\pi fA$.
$= 2.73$ m s^{-1}	1	
(ii) Maximum acceleration of tip of a prong $A_{max} = (2\pi f)^2 a$ $= (2\pi \times 512)^2 \times 0.85 \times 10^{-3}$	1	The maximum acceleration occurs at the extremity of an oscillation, where displacement $x = \pm A$. Simple harmonic motion always satisfies the equation $a = -(2\pi f)^2 x$, so a_{max} is found by substituting $x = A$ into this equation.
$= 8.8 \times 10^3$ m s^{-2}	1	

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3 (a) Period $T = 2\pi\sqrt{\frac{l}{g}} = 2\pi\sqrt{\frac{0.800}{9.81}}$ $= 1.79$ s	1 1	Part **(a)** is an easy starter, where the answer is found by direct substitution into the equation for the period of a simple pendulum.
(b) When bob falls, E_P lost $= E_K$ gained $\therefore \frac{1}{2}mv^2 = mg\Delta h$ gives $v = \sqrt{2gh} = \sqrt{2 \times 9.81 \times 20 \times 10^{-3}}$ $\therefore v_{max} = 0.626$ m s^{-1} but $v_{max} = 2\pi fA$ $\therefore A = \frac{v_{max}}{2\pi f} = \frac{v_{max}T}{2\pi}$ $= \frac{0.626 \times 1.79}{2\pi} = 0.178$ m	1 1 1 1	Part **(b)** is more demanding. You have to realise that you can find the maximum speed (at the lowest point) from conservation of energy. The amplitude can then be found from the maximum speed by applying $v_{max} = 2\pi fA$, together with $f = \frac{1}{T}$. Alternatively, you could find the approximate value of A by applying Pythagoras: $A^2 = 800^2 - 780^2$ gives $A = 178$ mm. $v_{max} = 2\pi fA$ would then give v_{max}.
(c) At lowest point of swing, centripetal force on mass is $(F - mg)$ where F is the tension in the string. $\therefore F - mg = \frac{mv_{max}^2}{r}$ Tension $F = m\left(g + \frac{v_{max}^2}{r}\right)$ $= 25 \times 10^{-3}\left(9.81 + \frac{0.626^2}{0.800}\right)$ $= 0.257$ N	1 1	Part **(c)** involves some of the work covered in Chapter 2 on circular motion. The mass swings in a circular arc whose radius r is equal to the length of the pendulum. At the lowest point of the circle, the speed is v_{max} and the centripetal force is the **resultant** force towards the centre of the circle. This is (tension – weight of mass).
4 (a) (i) Angular speed of turntable $\omega = \frac{2\pi}{T} = \frac{2\pi}{2.2} = 2.86$ rad s^{-1} Horizontal force on sphere $= m\omega^2 r = 0.050 \times 2.86^2 \times 0.13$ $= 0.053$ N	1 1	This question links simple harmonic motion with a circular model that can generate some of its features. It begins with some useful revision of Chapter 2.
(ii) Towards the centre **of the turntable**.	1	'Towards the centre' would not be sufficient, because it would raise the question '*which* centre?'.
(b) (i) Using $T = 2\pi\sqrt{\frac{l}{g}}$ and rearranging gives $l = \frac{T^2 g}{4\pi^2} = \frac{2.2^2 \times 9.81}{4\pi^2} = 1.20$ m or 1.2 m	1	Direct use of the pendulum equation leads to a straightforward mark. Compare this with Question 3**(a)**, where you had to identify the data with a simple pendulum before you could give an answer.
(ii) Maximum acceleration of bob: $a_{max} = (2\pi f)^2 A = \left(\frac{2\pi}{2.2}\right)^2 \times 0.13$ $= 1.06$ m s^{-2} or 1.1 m s^{-2}	1 1	In this calculation, you have to relate the frequency and period as an intermediate step, by using $f = \frac{1}{T}$.

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(c) *Acceleration–time graph to show*: • $a = 0$ when $t = 0$ • a curve of the correct –sine form, with correct period.	**2**	In shm, acceleration $a \propto -x$, making this graph the mirror image in the time axis of the shape of the displacement graph.
Kinetic energy–time graph to show: • E_K is a maximum when $t = 0$, and E_K is always positive • two cycles for every single cycle of the displacement–time graph, and the correct shape of curve (technically, this is a **(cos)**^{2} graph)	**2**	At $t = 0$, the displacement is zero and so the bob is at its maximum speed; E_K is therefore a maximum at the start. E_K is zero every time the displacement is a maximum (when $t = \frac{T}{4}, \frac{3T}{4}$, etc.), making two cycles for every single cycle of the displacement graph.
5 (a) (i) The minus sign indicates that the acceleration is in the opposite direction to the displacement	**1**	In your answer you must show that you know what the symbols a and × mean. 'a is in the opposite direction to x' would not be worth this mark. Alternatively, you could say that the minus sign means that the restoring force (or acceleration) always acts towards the equilibrium position.
(ii) v–t graph drawn as a sine curve, with $v = 0$ at $t = 0$ and the same period as the a–t graph. (**Phase**: comparing the two sinusoidal graphs, the a–t graph leads the v–t graph by 270° (or $\frac{3\pi}{2}$ rad)	**1**	At $\boldsymbol{t = 0}$, a is at its maximum **positive** value; this shows that × is then at its maximum **negative** value, so $v = 0$. Starting at a point of maximum negative displacement, v then increases positively from zero whilst the motion is towards the equilibrium point. The graph must therefore be a sine curve.
(b) (i) Period $T = 2\pi\sqrt{\frac{m}{k}} = 2\pi\sqrt{\frac{24}{60}} = 3.97$ s	**1**	This calculation requires you to think through a strategy about the steps to be taken. Many similar calculations are set in a structured way, so that you know the steps to take and the order in which to take them. The vertical displacement of the mass from its equilibrium position (0.035m) is the amplitude A of the shm. Alternatively maximum E_K = maximum $E_P = \frac{1}{2} k x^2 = \frac{1}{2} \times 60 \times 0.035^2 = 37$ mJ
Frequency $f = \frac{1}{T} = \frac{1}{3.97} = 0.252$ Hz	**1**	
$v_{max} = 2\pi f A = 2\pi \times 0.252 \times 0.035$ $= 5.54 \times 10^{-2}$ m s^{-1}	**1**	
Maximum $E_K = \frac{1}{2} m\, v_{max}^2$ $= \frac{1}{2} \times 24 \times (5.54 \times 10^{-2})^2$	**1**	
$= 3.7 \times 10^{-2}$ J (i.e. about 40 mJ)	**1**	
(ii) Graph of E_K against t drawn to show: • Maxima of E_K at $t = 0, 2, 4$ s and minima (zero) at $t = 1, 3$ s • Progressive reduction in size of maxima • $E_K = 40$ mJ at $t = 0$ and $E_K = 10$ mJ at $t = 4.0$ s	**3**	Note that the 'end of a complete cycle' is the period, which is 4.0 s, and choose scales to allow the graph to include 40 mJ and 4.0 s. The maximum $E_K \propto v_{max}^2 \propto A^2$, so halving A will reduce the maximum E_K to $\frac{1}{4}$ of its original value. At $t = 1$s and 3s, $v = 0$ and E_K is therefore zero.

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6 **(a)** Forced vibrations/oscillations	**1**	**Resonance** occurs only when the forcing vibrations have exactly the same frequency as a natural frequency of the system that is being vibrated. It is doubtful whether the Millennium Bridge was actually in resonance, but it was certainly subject to forced vibrations of large amplitude.
(b) *Relevant points include*: • A structure has a natural frequency (or frequencies) of vibration • Resonance • This occurs when the frequency of a driving force is equal to a natural frequency of the structure • Large amplitude vibrations are then produced (or there is a large transfer of energy to the structure) • This could damage the structure (or cause a bridge to fail)	**any 4**	The condition is that of resonance, which occurs when the frequency of the applied driving oscillator exactly matches a natural frequency of the driven oscillator. It is well known that simple suspension bridges (as used by an advancing army) are subject to this effect, and that troops are instructed to break step when crossing them. The Millennium Bridge, as originally built, presented a particular problem. The vibrating bridge deck fed oscillations back to the pedestrians on it, almost forcing them to walk in step, thereby enhancing its own vibrations.
(c) *Possible measures* to reduce the effect: • install dampers (shock absorbers) • stiffen (or reinforce) the structure • any other acceptable step e.g. redesign to change natural frequency, increase the mass of the bridge, restrict numbers of pedestrians	**any 2**	The Millennium Bridge problem was solved by installing very large shock absorbers, similar in design to those used on motor vehicles. Changing any physical feature of a structure would change its natural frequency, but a bridge over a river should not be shortened!
7 **(a)** **(i)** Period = 1.8 s Frequency $f = \frac{1}{T} = \frac{1}{1.8} = 0.556$ Hz or 0.56 Hz	**1** **1**	If the time axis is read carefully, it is clear that one cycle takes 1.8 s, two cycles 3.6 s and three cycles 5.4 s.
(ii) Amplitude = 0.076 m (± 0.002 m)	**1**	Some variation has to be allowed in this answer, because the vertical scale is not finely calibrated.
(iii) Damping does not alter the frequency … but it does reduce the amplitude	**1** **1**	It is important to realise that the period of vibration (and therefore the frequency) is unaffected by damping. Damping usually removes a fixed proportion of the existing energy in each cycle, and so the amplitude is progressively reduced.

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(b) *Graph drawn to show*: • Maximum +ve displacement at $t = 0$, 1.8, 3.6 s etc and max –ve displacement at $t = 0.9$, 2.7 s, etc. or vice versa **and** zero displacement at $t = 0.45$, 1.35, 2.25, 3.15 s, etc. • Sinusoidal shape, with constant amplitude and period	**2**	This tests your understanding of **phase**. At $t = 0$, the displacement is zero on the original graph. On the new graph (for a 90° phase difference) the displacement at $t = 0$ must be either $+A$ or $-A$. Your graph should be either a cosine curve or a –(cosine) curve. The new graph may be 90° ahead or behind the original graph.
(c) **(i)** Maximum acceleration of bob $= (2\pi f)^2 A = (2\pi \times 0.556)^2 \times 0.076$ $= 0.93$ m s^{-2}	**1** **1**	The values substituted for f and A are those found from the graph in part **(a)** above.
(ii) Maximum speed of bob $v_{max} = 2\pi f A$ $= 2\pi \times 0.556 \times 0.076 = 0.266$ m s^{-1} Total energy of shm = maximum E_K $= \frac{1}{2} m\, v_{max}{}^2$ $= \frac{1}{2} \times 8.0 \times 10^{-3} \times 0.266^2$ $= 2.8 \times 10^{-4}$ J	**1** **1** **1**	In this part you have to realise that the 'total energy of the oscillations' must be equal to the kinetic energy of the bob when its potential energy can be taken to be zero – at the centre of each swing, when it travels fastest. The first step is therefore to find this maximum speed.
8 **(a)** **(i)** Use of $m\,g = k\,\Delta L$ gives spring constant k $= \frac{mg}{\Delta L} = \frac{0.25 \times 9.81}{40 \times 10^{-3}} = 61.3$ N m^{-1} or 61 N m^{-1}	**1** **1**	This is another question on the mass–spring system that requires you to be familiar with Hooke's law from *AS Physics A* Unit 2.
(ii) Period $T = 2\pi\sqrt{\frac{m}{k}} = 2\pi\sqrt{\frac{0.69}{61.3}} = 0.667$ s or 0.67 s Frequency $f = \frac{1}{T} = \frac{1}{0.667} = 1.50$ Hz	**1** **1**	It is essential to read this part of the question carefully. In **(ii)**, a mass of 0.44 kg has been added to the original 0.25 kg, making the total mass supported by the spring 0.69 kg.

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(b) *Relevant points include*: **(i)** (*at 0.2 Hz*) • Forced vibrations at a frequency of 0.2 Hz are produced • The amplitude is similar to the driver's (≈30mm) (or less than at resonance) • The displacements of the masses are almost in phase with the displacements of the support rod **(ii)** (*at 1.5 Hz*) • Resonance is produced (or vibrations at a frequency of 1.5 Hz) • The amplitude is very large (>30 mm) • The displacements of the masses have a phase lag of 90° on the displacements of the support rod • The motion may appear violent **(iii)** (*at 10 Hz*) • Forced vibrations at a frequency of 10 Hz are produced • The amplitude is small (« 30 mm) • The displacements of the masses have a phase lag of almost 180° on the displacements of the support rod	**any 6**	Part **(b)** is still about the system described in part **(a)(ii)**, which has a natural frequency of 1.5 Hz. Resonance will therefore occur if the driving frequency matches this. Any successful answer must refer to **amplitude**, **frequency**, and **phase** in all of the parts, because the question demands this. Resonant systems always vibrate with a phase lag of 90° on the driver. At 0.2 Hz, the driving frequency is much less than 1.5 Hz; the driven system then practically follows the driving vibrations, with a small phase lag. At 10 Hz the driving frequency is much greater than 1.5 Hz, and so the forced vibrations produced are of small amplitude, almost in antiphase with the driver.