1.3 Conservation of momentum

Learning objectives:

- Is momentum ever lost in a collision?
- What do we mean by *conservation of momentum?*
- What condition must be satisfied if the momentum of a system is conserved?

Specification reference: 3.4.1

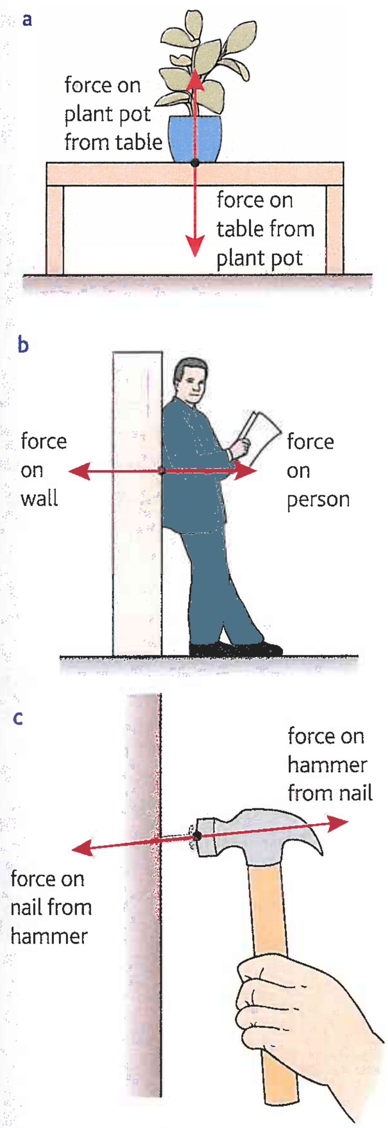

Figure 1 *Examples of Newton's third law*

Newton's third law of motion

When two objects interact, they exert equal and opposite forces on each other.

In other words, if object A exerts a force on object B, there must be an equal and opposite force acting on object A due to object B.

For example,

- an object resting on a table exerts a force on the table which exerts an equal and opposite force on the object;
- a person leaning against a wall exerts a force on the wall which exerts an equal and opposite force on the person;
- a hammer hitting a nail exerts a force on the nail which exerts an equal and opposite force on the hammer;
- the Earth exerts a force due to gravity on an object which exerts an equal and opposite force on the Earth;
- a jet engine exerts a force on hot gas in the engine to expel the gas; the gas being expelled exerts an equal and opposite force on the engine.

The Principle of Conservation of Momentum

When an object is acted on by a resultant force, its momentum changes. If there is no change of its momentum, there can be no resultant force on the object. Now consider several objects which interact with each other. If no external resultant force acts on the objects, the total momentum does not change. However, interactions between the objects can transfer momentum between them. But the total momentum does not change.

The Principle of Conservation of Momentum states that for a system of interacting objects, the total momentum remains constant, provided no external resultant force acts on the system.

Consider two objects that collide with each other then separate. As a result, the momentum of each object changes. They exert equal and opposite forces on each other when they are in contact. So the change of momentum of one object is equal and opposite to the change of momentum of the other object. In other words, if one object gains momentum, the other object loses an equal amount of momentum. So the total amount of momentum is unchanged.

Let's look in detail at the example of two snooker balls A and B in collision, as shown in Figure 2 overleaf.

The impact force F_1 on ball A due to ball B changes the velocity of A from u_A to v_A in time t

Therefore, $F_1 = \frac{m_A v_A - m_A u_A}{t}$, where t = the time of contact between A and B, and m_A = the mass of ball A.

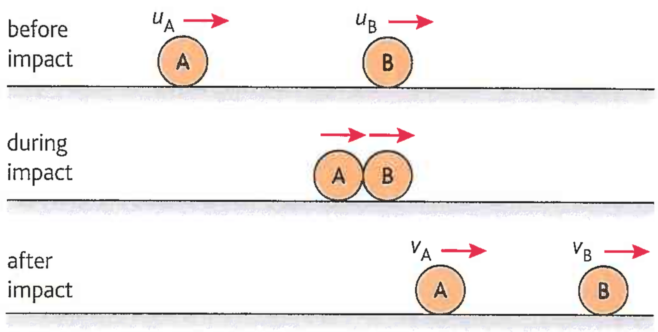

Figure 2 *Conservation of momentum*

The impact force F_2 on ball B due to ball A changes the velocity of B from u_B to v_B in time t.

Therefore, $F_2 = \dfrac{m_B v_B - m_B u_B}{t}$, where t = the time of contact between A and B, and m_B = the mass of ball B.

Because the two forces are equal and opposite to each other, $F_2 = -F_1$

Therefore, $\dfrac{m_B v_B - m_B u_B}{t} = -\dfrac{(m_A v_A - m_A u_A)}{t}$

Cancelling t on both sides gives

$$m_B v_B - m_B u_B = -(m_A v_A - m_A u_A)$$

Rearranging this equation gives

$$m_B v_B + m_A v_A = m_A u_A + m_B u_B$$

Therefore,

the total final momentum = the total initial momentum

Hence the total momentum is unchanged by this collision, i.e. it is conserved.

Note:

If the colliding objects stick together as a result of the collision, they have the same final velocity. The above equation with V as the final velocity may therefore be written

$$(m_B + m_A)V = m_A u_A + m_B u_B$$

Testing conservation of momentum

Figure 3 shows an arrangement that can be used to test **conservation of momentum** using a motion sensor linked to a computer. The mass of each trolley is measured before the test. With trolley B at rest, trolley A is given a push so it moves towards trolley B at constant velocity. The two trolleys stick together on impact. The computer records and displays the velocity of trolley A throughout this time.

The computer display shows that the velocity of trolley A dropped suddenly when the impact took place. The velocity of trolley A immediately before the collision, u_A, and after the collision, V, can be measured. The measurements should show that the total momentum of both trolleys after the collision is equal to the momentum of trolley A before the collision. In other words,

$$(m_B + m_A)V = m_A u_A$$

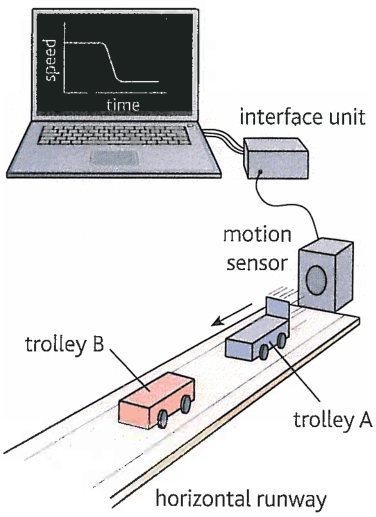

Figure 3 *Testing conservation of momentum*

Worked example:

A rail wagon of mass 4500 kg moving along a level track at a speed of 3.0 m s^{-1} collides with and couples to a second rail wagon of mass 3000 kg which is initially stationary. Calculate the speed of the two wagons immediately after the collision.

Solution

Total initial momentum = initial momentum of A + initial momentum of B

$$= (4500 \times 3.0) + (3000 \times 0) = 13\,500\ \text{kg m s}^{-1}$$

Total final momentum = total mass of A and B × velocity V after the collision

$$= (4500 + 3000)V = 7500V$$

Using the Principle of Conservation of Momentum,

$$7500V = 13\,500$$

$$V = \frac{13\,500}{7500} = 1.8\ \text{m s}^{-1}$$

Head-on collisions

Consider two objects moving in opposite directions that collide with each other. Depending on the masses and initial velocities of the two objects, the collision could cause them both to stop. The momentum of the two objects after the collision would then be zero. This could only happen if the initial momentum of one object was exactly equal and opposite to that of the other object. In general, if two objects move in opposite directions before a collision, then the vector nature of momentum needs to be taken into account by assigning numerical values of velocity + or − according to the direction.

For example, if a car of mass 600 kg travelling at a velocity of 25 m s^{-1} collides head-on with a lorry of mass 2400 kg travelling at a velocity of 10 m s^{-1} in the opposite direction, the total momentum before the collision is 9000 kg m s^{-1} in the direction the lorry was moving. As momentum is conserved in a collision, the total momentum after the collision is the same as the total momentum before the collision. Prove for yourself that if the two vehicles were to stick together after the collision, they must have had a velocity of 3.0 m s^{-1} in the direction the lorry was moving immediately after the impact.

Application and How science works

Crash barriers

Crash barriers on motorways are designed to stop out-of-control vehicles from entering the opposite carriageway. There were no crash barriers on the first major UK motorway, the M1, when it was opened in 1959. There was no speed limit either. EC regulations require that the strongest barriers are designed to withstand the impact of a 38 tonne heavy goods vehicle hitting the barrier at an angle of 20° at a speed of 18 m s^{-1} (40 mph). Motorcycles are very vulnerable to crash barriers, particularly wire rope barriers. In 2007, the UK government rejected an e-petition sent by thousands of people to replace these barriers with stronger conventional barriers.

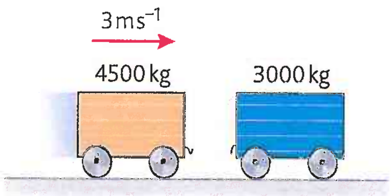

Figure 4

Summary questions

1 A rail wagon of mass 3000 kg moving at a velocity of 1.2 m s^{-1} collides with a stationary wagon of mass 2000 kg. After the collision, the two wagons couple together. Calculate their speed immediately after the collision.

2 A rail wagon of mass 5000 kg moving at a velocity of 1.6 m s^{-1} collides with a stationary wagon of mass 3000 kg. After the collision, the 3000 kg wagon moves away at a velocity of 1.5 m s^{-1}. Calculate the speed and direction of the 5000 kg wagon after the collision.

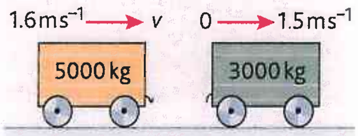

Figure 5

3 In a laboratory experiment, a trolley of mass 0.50 kg moving at a speed of 0.25 m s^{-1} collided with a trolley of mass 1.0 kg moving in the opposite direction at a speed of 0.20 m s^{-1}. The two trolleys couple together on collision. Calculate their speed and direction immediately after the collision.

4 A ball of mass 0.80 kg moving at a speed of 2.5 m s^{-1} along a straight line collided with a ball of mass 2.5 kg which was initially stationary. As a result of the collision, the 2.5 kg ball was given a velocity of 1.0 m s^{-1} along the same line. Calculate the speed and direction of the 0.80 kg ball immediately after the collision.