Summer Work: Part 1 – Sequences and Series, Proof and Algebraic Fractions

1. Given that $u_1 = 2, u_2 = 5$, find the next four terms in the sequence defined by the recurrence relation $u_{n+2} = 4u_{n+1} - 5u_n$ **(2)**

2. An arithmetic sequence has a third term of 20 and a seventh term of 52. Find the first term and the sum of the first 10 terms. **(4)**

3. An arithmetic sequence has $u_1 = 3, d = 2$. A second arithmetic sequence is such that $u_1 = 136, d = -5$. Find the value of n such that u_n is the same in both sequences. **(3)**

4. By using the formula for the first n terms of an arithmetic series, find the sum of all the integers from 1 to 300 which are **NOT** divisible by 5. **(4)**

5. On Martin's 20th birthday he received the first of an annual birthday gift of money from his uncle. This first gift was £40 and on each subsequent birthday the gift was £25 more than the year before. The amounts of these gifts form an arithmetic sequence.
   (a) Write down the amount that Martin received on his 22nd birthday. **(1)**
   (b) Find the total amount received up to and including is 25th birthday. **(2)**
   (c) On which birthday does the sum of Martin's from his uncle gifts exceed £2000? **(4)**

6. A geometric series has first term a and common ratio $r = -\frac{2}{3}$
   The sum of the first 4 terms of this series is 520.
   (a) Show through algebra that a = 1080. **(2)**
   (b) Find the sum to infinity of the series. **(2)**
   (c) Find the difference between the 9th and 10th terms of the series. Give your answer to 2 decimal places. **(3)**

7. All the terms of a geometric series are positive. The sum of the first two terms is 34 and the sum to infinity is 162. Find the common ratio and the first term. **(5)**

8. A geometric series has a first term of 49 and a common ratio of $\frac{6}{7}$. Find the smallest value of n for which the sum of the first n terms exceeds 342. **(5)**

9. The first three terms of a geometric series are $8p$, $(3p - 3)$ and $(2p - 10)$ respectively, where p is a **positive** constant.
   (a) Show that $7p^2 - 62p - 9 = 0$. **(4)**
   (b) Hence show that $p = 9$ **(2)**
   (c) Find the common ratio. **(2)**
   (d) Find the difference, to 3 significant figures, between the sum to infinity of this series and the sum of the first ten terms. **(3)**

10. Prove by contradiction that there are no integers p and q for which $12p + 21q = 5$ **(3)**

11. Prove by contradiction that $\sqrt{3}$ is irrational. **(4)**

12. Write the following as single fractions in their simplest form

(a) $\frac{3}{x-1}+\frac{2}{x+3}$ (b) $\frac{6}{x^2-9}-\frac{5}{x^2-x-6}$ (c) $\frac{3}{x+4}+\frac{2}{x^2+9x+2}$ **(9)**

13. Write each of the following as partial fractions

(a) $\frac{12}{(x-1)(x+3)}$ (b) $\frac{3}{x^2+5x+6}$ (c) $\frac{3x+1}{x^2(x-1)}$ (d) $\frac{2x-5}{(x+2)^2(x-4)}$ **(2,2,4,4)**

14. Using algebraic division, write the fraction $\frac{4x^3+2x^2-3x+5}{x^2-4}$ in the form $Ax+B+\frac{C}{x+2}+\frac{D}{x-2}$ **(6)**

15. Write the fraction $\frac{2x+6}{(x+1)(x+2)}$ in partial fractions, and hence write down the binomial expansion of $\frac{2x+6}{(x+1)(x+2)}$ up to and including the term in x^3 **(6)**

16. Given that $f(x)=(5-3x)^{-3}$

(a) Find the binomial expansion of $f(x)$, in ascending powers of x, up to and including the term in x^3. Give each coefficient as a simplified fraction. **(5)**

(b) State the range of values of x for which the expansion is valid. **(1)**

17. The coordinates of A and B are $(-2,4,8)$ and $(0,k,7)$ respectively. Given that the distance from A to B is 3 units, find the possible values of k. **(3)**

18. Given that $\overrightarrow{AB}=4\boldsymbol{i}+\boldsymbol{j}+3\boldsymbol{k}$ and $\overrightarrow{AC}=5\boldsymbol{i}-6\boldsymbol{j}-\boldsymbol{k}$, find the unit vector in the direction $\overrightarrow{BC}$. **(3)**

19. Given that $(2q-3p)\boldsymbol{i}+5\boldsymbol{j}+r\boldsymbol{k}=13\boldsymbol{i}+(4p+q)\boldsymbol{j}+(2q-p)\boldsymbol{k}$ find the values of p, q and r. **(3)**

20. ABCD is a quadrilateral and A, B, C are the points $(6,-8,4)$, $(-3,-4,2)$ and $(-7,9,-3)$ respectively. Find the coordinates of D such that ABCD is a parallelogram. **(3)**

Pure Total: 107 marks

Summer Work: Part 2 – Correlation, the Normal Distribution and Probability

1. The number of bacteria on a mould was recorded at hourly intervals in the table below:

Time (t) hours	1	2	3	4	5	6
Number of bacteria (n)	700	1280	1920	3160	5100	8000
log n						

(i) Complete the table, showing values of log n to 3 decimal places **(2)**

(ii) The data is coded using $x = t$ and $y = \log n$. By calculating the PMCC for x and y, explain why a model of $n = ab^t$ is appropriate for these data **(3)**

(iii) Given that the regression line of y on x is $y = 2.66 + 0.209x$, find the values of a and b for this model. **(3)**

2. The marks achieved by ten students in a Maths test and a Physics test are recorded in the table:

Student	A	B	C	D	E	F	G	H	I	J
Maths Score (x)	60	84	56	76	92	68	62	30	42	34
Physics score (y)	55	81	85	73	75	69	61	59	63	55

(i) Calculate, to 3 decimal places, the product moment correlation coefficient x and y **(1)**

(ii) It is suggested that there is a positive correlation between the Maths score and the Physics score. Test this suggestion at the 5% level, stating your hypotheses clearly. **(4)**

3. A student wishes to test at the 1% level for correlation between the English test scores and Maths test scores for her class. From a sample of twelve students she calculates a product moment correlation of -0.65. Given that her hypotheses were $H_0: \rho = 0$ and $H_1: \rho \neq 0$, write down a suitable conclusion to her test.

(3)

4. The random variable $X \sim N(30, 5^2)$. Find, to 3 decimal places;
(i) $P(X > 36)$ **(ii)** $P(X \leq 20)$ **(iii)** $P(25 < X < 32)$ **(iv)** $P(|X - 30| < 2)$ **(1,1,1,3)**

5. The length of drive, in metres, struck by golfers on a particular hole is modelled by a normal distribution with mean 250 metres and standard deviation 20 metres.

(a) Find the proportion of golfers drive more than 280 metres on this hole. **(2)**

(b) A golfer sets herself the target of being in the top 20% of the longest drives. Using the above model estimate the shortest drive that she can hit and achieve her aim. **(2)**

6. Bag A contains 7 black counters and 9 white counters. Bag B contains 5 black counters and 6 white counters. A counter is chosen from random from bag A and its colour is recorded. The counter is then added to bag B. A counter is then chosen at random from bag B and its colour is recorded.

(a) Draw a tree diagram to represent this information. **(3)**

Find the probability of choosing:

(b) A black counter from bag B **(2)**

(c) A white counter from bag A, given that a white counter is chosen from bag B. **(2)**

7. The events A and B are independent and such that P(A)=3P(B) and P(AUB) = $\frac{7}{12}$.

(a) Show that P(B) = $\frac{1}{6}$ **(5)**

(b) Find P(A∩B) **(3)**

(c) Find P(B|A') **(2)**

Applied Total: 43 Marks

Your Benchmark 5 will be in the week commencing 27th September. It will include questions on the following topics:

Pure Year 2: Chapters 1, 3, 4, 12
Applied Year 2: Chapters 1, 2, 3 (only 3.1 – 3.3)

In addition to the work above, you should prepare yourself thoroughly for this test. Use the 'questions by topic' powerpoints on www.westiesworkshop.com for extra exam practice as well as the practice exam papers on GO.