****

**1.** The transformation *T* from the *z*-plane, where *z* = *x* + i*y* , to the *w*-plane,

where *w* = *u* + i*v* , is given by

**

The point representing  is invariant under *T*.

Determine the value of *p*.

**(3)**

**(Total 3 marks)**

**\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_**

**2.** (*a*)Show that, for *r* > 0



**(2)**

(*b*)Hence show that

**

where *a*, *b* and *c* are integers to be determined.

**(4)**

**(Total 6 marks)**

**\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_**

**3.** Use algebra to obtain the set of values of *x* for which

**

**(7)**

**(Total 7 marks)**

**\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_**

**4.** (*a*)Show that the substitution transforms the differential equation



into the differential equation



**(3)**

(*b*)Obtain the general solution of differential equation (II).

**(5)**

(*c*)Hence obtain the general solution of differential equation (I), giving your answer in

the form *y*2 = f (*x*)

**(1)**

**(Total 9 marks)**

**\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_**

**5.** Given that



(*a*)show that



**(5)**

Given also that *y* = 3 and  at *x* = 0

(*b*)obtain a series solution for *y* in ascending powers of *x* with simplified coefficients, up

to and including the term in *x*3

**(4)**

**(Total 9 marks)**

**\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_**

**6.** (*a*)Determine the general solution of the differential equation



**(7)**

(*b*)Find the particular solution for which *y* = 0 and  at *x* = 0

**(5)**

**(Total 12 marks)**

**\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_**

**7.**

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Figure 1 shows a sketch of curve *C* with polar equation

*r* = 3 sin 2*θ *

The point *P* on *C* has polar coordinates (*R*, *ϕ*). The tangent to *C* at *P* is perpendicular to

the initial line.

(*a*)Show that tan *ϕ* =

**(4)**

(*b*)Determine the exact value of *R*.

**(2)**

The region *S*, shown shaded in Figure 1, is bounded by *C* and the line *OP*, where *O* is

the pole.

(*c*)Use calculus to show that the exact area of *S* is

**

where *p* and *q* are constants to be determined.

**Solutions relying entirely on calculator technology are not acceptable.**

**(7)**

**(Total 13 marks)**

**\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_**

**8.** Given that *z* = ei*θ*

(*a*)show that **

where *n* is a positive integer.

**(2)**

(*b*)Show that

cos6 *θ* = (cos 6*θ* + 6 cos 4*θ* + 15 cos 2*θ* + 10)

**(5)**

(*c*)Hence solve the equation

cos 6*θ* + 6 cos 4*θ* + 15 cos 2*θ* = 0 0 ≤ *θ* ≤ *π*

Give your answers to 3 significant figures.

**(4)**

(*d*)Use calculus to determine the exact value of



**Solutions relying entirely on calculator technology are not acceptable.**

**(5)**

**(Total 16 marks)**

**TOTAL FOR PAPER: 75 MARKS**