Mark Scheme (Results)

January 2021

Pearson Edexcel International Advanced Level
In Further Pure Mathematics F2
(WFM02/01)

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General Marking Guidance

- All candidates must receive the same treatment. Examiners must mark the first candidate in exactly the same way as they mark the last.
- Mark schemes should be applied positively. Candidates must be rewarded for what they have shown they can do rather than penalised for omissions.
- Examiners should mark according to the mark scheme not according to their perception of where the grade boundaries may lie.
- There is no ceiling on achievement. All marks on the mark scheme should be used appropriately.
- All the marks on the mark scheme are designed to be awarded. Examiners should always award full marks if deserved, i.e. if the answer matches the mark scheme. Examiners should also be prepared to award zero marks if the candidate's response is not worthy of credit according to the mark scheme.
- Where some judgement is required, mark schemes will provide the principles by which marks will be awarded and exemplification may be limited.
- When examiners are in doubt regarding the application of the mark scheme to a candidate's response, the team leader must be consulted.
- Crossed out work should be marked UNLESS the candidate has replaced it with an alternative response.

PEARSON EDEXCEL IAL MATHEMATICS

General Instructions for Marking

1. The total number of marks for the paper is 75

2. The Edexcel Mathematics mark schemes use the following types of marks:

- **M** marks: Method marks are awarded for 'knowing a method and attempting to apply it', unless otherwise indicated.
- **A** marks: Accuracy marks can only be awarded if the relevant method (M) marks have been earned.
- **B** marks are unconditional accuracy marks (independent of M marks)
- Marks should not be subdivided.

3. Abbreviations

   These are some of the traditional marking abbreviations that will appear in the mark schemes.

   - bod – benefit of doubt
   - ft – follow through
   - the symbol $\sqrt{}$ will be used for correct ft
   - cao – correct answer only
   - cso - correct solution only. There must be no errors in this part of the question to obtain this mark
   - isw – ignore subsequent working
   - awrt – answers which round to
   - SC: special case
   - o.e. – or equivalent (and appropriate)
   - d… or dep – dependent
   - indep – independent
   - dp decimal places
   - sf significant figures
   - $\ast$ The answer is printed on the paper or ag- answer given
   - $\sqsubset$ or d… The second mark is dependent on gaining the first mark

4. All A marks are ‘correct answer only’ (cao), unless shown, for example, as A1ft to indicate that previous wrong working is to be followed through. After a misread however, the subsequent A marks affected are treated as A ft, but manifestly absurd answers should never be awarded A marks.

5. For misreading which does not alter the character of a question or materially simplify it, deduct two from any A or B marks gained, in that part of the question affected.

6. If a candidate makes more than one attempt at any question:
    - If all but one attempt is crossed out, mark the attempt which is NOT crossed out.
    - If either all attempts are crossed out or none are crossed out, mark all the attempts and score the highest single attempt.

7. Ignore wrong working or incorrect statements following a correct answer.

General Principles for Further Pure Mathematics Marking

(*But note that specific mark schemes may sometimes override these general principles*)

Method mark for solving 3 term quadratic:

1. Factorisation

$(x^2 + bx + c) = (x + p)(x + q), \text{ where} |pq| = |c|$, leading to $x = \ldots$

$(ax^2 + bx + c) = (mx + p)(nx + q), \text{ where} |pq| = |c| \text{ and } |mn| = |a|$, leading to $x = \ldots$

2. Formula

Attempt to use the correct formula (with values for *a*, *b* and *c*).

3. Completing the square

Solving $x^2 + bx + c = 0$: $\left(x \pm \frac{b}{2}\right)^2 \pm q \pm c = 0, \quad q \neq 0$, leading to $x = \ldots$

Method marks for differentiation and integration:

1. Differentiation

Power of at least one term decreased by 1. ($x^n \to x^{n-1}$)

2. Integration

Power of at least one term increased by 1. ($x^n \to x^{n+1}$)

Use of a formula

Where a method involves using a formula that has been learnt, the advice given in recent examiners' reports is that the formula should be quoted first.

Normal marking procedure is as follows:

<u>Method mark</u> for quoting a correct formula and attempting to use it, even if there are small errors in the substitution of values.

Where the formula is <u>not</u> quoted, the method mark can be gained by implication from <u>correct</u> working with values but may be lost if there is any mistake in the working.

Exact answers

Examiners' reports have emphasised that where, for example, an exact answer is asked for, or working with surds is clearly required, marks will normally be lost if the candidate resorts to using rounded decimals.

Question Number	Scheme	Marks
1.	$\mathrm{i}\left(1+\sqrt{3}\right)=\dfrac{\mathrm{i}\left(1+\sqrt{3}\right)+p\mathrm{i}}{\mathrm{i}^2\left(1+\sqrt{3}\right)+3}$	M1
	$-\mathrm{i}\left(1+\sqrt{3}\right)^2+3\mathrm{i}\left(1+\sqrt{3}\right)=\mathrm{i}\left(1+\sqrt{3}\right)+p\mathrm{i}$	
	$-1-2\sqrt{3}-3+3+3\sqrt{3}=1+\sqrt{3}+p$	dM1
	$p=-2$	A1
		[3]

M1	Substitute $\mathrm{i}\left(1+\sqrt{3}\right)$ for w and z
dM1	Solve to $p = ...$
A1	Correct value for p
	Some solve for p first:
M1	Obtain an expression for p in terms of w and/or z
dM1	Substitute $\mathrm{i}\left(1+\sqrt{3}\right)$ for w and z
A1	Correct value for p

Question Number	Scheme	Marks
2 (a)	$\dfrac{r+2}{r(r+1)}-\dfrac{r+3}{(r+1)(r+2)}=\dfrac{(r+2)^2-r(r+3)}{r(r+1)(r+2)}$	M1
	$=\dfrac{r^2+4r+4-r^2-3r}{r(r+1)(r+2)}=\dfrac{r+4}{r(r+1)(r+2)}$ *	A1* (2)
(b)	$r=1 \quad \dfrac{3}{1\times 2}-\dfrac{4}{2\times 3}$ $\quad r=n-1 \quad \dfrac{n+1}{(n-1)n}-\dfrac{n+2}{n(n+1)}$	
	$r=2 \quad \dfrac{4}{2\times 3}-\dfrac{5}{3\times 4}$ $\quad r=n \quad \dfrac{n+2}{n(n+1)}-\dfrac{n+3}{(n+1)(n+2)}$	M1
	$r=3 \quad \dfrac{5}{3\times 4}-\dfrac{6}{4\times 5}$	
	$\sum_{r=1}^{n}\dfrac{r+4}{r(r+1)(r+2)}=\dfrac{3}{2}-\dfrac{n+3}{(n+1)(n+2)}$	A1
	$\sum_{r=1}^{n}\dfrac{r+4}{r(r+1)(r+2)}=\dfrac{3(n+1)(n+2)-2n-6}{2(n+1)(n+2)}=\dfrac{n(3n+7)}{2(n+1)(n+2)}$	dM1 A1cao (4)
		[6]
(a) M1	Attempt a single fraction with the correct denominator (or 2 separate fractions with the correct common denominator)	
A1*	Correct result obtained with no errors in the working. Must include LHS as shown in question or LHS = ...	
(b) M1	Show sufficient terms to demonstrate the cancelling, min 3 at start and 1 at end or 2 at start and 2 at end. Award by implication if the correct 2 remaining terms are seen	
A1	Extract the correct 2 remaining terms	
dM1	Attempt common denominator of the form $k(n+1)(n+2)$	
A1cao	Correct result obtained. No need to show *a*, *b* and *c* explicitly.	

Question Number	Scheme	Marks
3	$x^2+x-2<\frac{1}{2}x+\frac{5}{2}$	
	$2x^2+x-9<0$	M1
	CVs $x=\frac{-1\pm\sqrt{73}}{4}$	A1
	$-x^2-x+2<\frac{1}{2}x+\frac{5}{2}$	M1
	$2x^2+3x+1>0 \quad (2x+1)(x+1)>0$	M1
	CVs $x=-\frac{1}{2},\ -1$	A1
	$\frac{-1-\sqrt{73}}{4}<x<-1, \quad -\frac{1}{2}<x<\frac{-1+\sqrt{73}}{4}$	M1A1
		[7]
	No algebra implies no marks	
NB	The first 5 marks can all be awarded if equations rather than inequalities are shown	
M1	Obtain and solve a 3TQ (any valid method including calculator)	
A1	2 correct CVs Allow decimal equivalents (1.886..., -2.386...), min 3 sf, rounded or truncated	
M1	Multiply either side by -1	
M1	Obtain and solve a 3TQ (any valid method including calculator)	
A1	2 correct CVs	
M1	Form 2 double inequalities with their CVs. No overlap between these inequalities. Correct inequality signs required here or for final mark	
A1	Correct inequalities obtained. Values must be exact, but note that 0.5 is exact. Allow "and" but not "$\cap$". May be written in set language with "$\cup$" and round brackets	

Question Number	Scheme	Marks
4 (a)	$y^2 = z^{-1} \Rightarrow 2y\frac{dy}{dx} = -\frac{1}{z^2}\frac{dz}{dx}$ oe eg $\frac{dy}{dx} = -\frac{1}{2}z^{-\frac{3}{2}}\frac{dz}{dx}$	B1
	$2y\frac{dy}{dx} + 4y^2 = 6xy^4$	
	$-\frac{1}{z^2}\frac{dz}{dx} + \frac{4}{z} = \frac{6x}{z^2}$	M1
	$\frac{dz}{dx} - 4z = -6x$ *	A1 * (3)
(b)	$\text{IF} = e^{\int -4dx} = e^{-4x}$	B1
	$e^{-4x}\left(\frac{dz}{dx} - 4z\right) = e^{-4x} \times -6x$	
	$ze^{-4x} = -6\int xe^{-4x}dx$	M1
	$= -6\left[-\frac{1}{4}xe^{-4x} + \int \frac{1}{4}e^{-4x}dx\right]$	M1
	$= -6\left[-\frac{1}{4}xe^{-4x} - \frac{1}{16}e^{-4x}\right] (+c)$ oe	A1
	$= \frac{3}{2}xe^{-4x} + \frac{3}{8}e^{-4x} (+c)$	
	$z = \frac{3}{2}x + \frac{3}{8} + ce^{4x}$ oe	A1 (5)
ALT	$\frac{dz}{dx} - 4z = -6x$	
	$m - 4 = 0 \Rightarrow m = 4 \Rightarrow$ CF is $z = Ae^{4x}$	B1
	PI: $z = \lambda + \mu x$	M1
	$\frac{dz}{dx} = \mu \Rightarrow \mu - 4(\lambda + \mu x) = -6x$	
	$4\mu = 6 \quad 4\lambda = \mu, \Rightarrow \mu = \frac{3}{2}, \lambda = \frac{3}{8}$	M1,A1
	$z = \frac{3}{2}x + \frac{3}{8} + Ae^{4x}$	A1
(c)	$y^2 = \frac{1}{\frac{3}{2}x + \frac{3}{8} + ce^{4x}} = \frac{8}{(12x + 3 + Ae^{4x})}$ oe	B1ft (1)
		[9]

Question Number	Scheme	Marks
(a)		
B1	Correct derivative seen explicitly or used	
M1	Substitutions made. Only award when an equation in x and z only is reached (if working equation I to II) or an equation in x and y is reached (if working II to I)	
A1 *	Correct result obtained with no errors in working	
(b)		
B1	Correct IF seen explicitly or used	
M1	Multiply through by their IF and integrate the LHS. Accept I for e^{-4x} on LHS only	
M1	Apply parts in the correct direction to RHS to obtain $Axe^{-4x} + B\int e^{-4x}dx$ with $A = \pm\frac{3}{2}$ and $B = \pm\frac{3}{2}$	
A1	Correct integration of RHS, constant not needed	
A1	Include the constant and treat it correctly. Answer in form $z = ...$	
ALT		
B1	Correct CF May not be seen until GS is formed	
M1	For a PI of the correct form	
M1	Differentiate their PI, substitute in the equation and extract 2 equations for the unknowns	
A1	Solve the two equations to obtain correct values for the unknowns	
A1	Correct GS obtained	
(c)		
B1ft	Any equivalent to that shown. (no need to change letter for constant if rearranged) Must start $y^2 = ...$ and must include a constant.	

Question Number	Scheme	Marks
5(a)	$-2x\frac{\mathrm{d}^2y}{\mathrm{d}x^2}+\left(2-x^2\right)\frac{\mathrm{d}^3y}{\mathrm{d}x^3}$	M1
	$+5\left(\frac{\mathrm{d}y}{\mathrm{d}x}\right)^2+5x\times2\frac{\mathrm{d}y}{\mathrm{d}x}\frac{\mathrm{d}^2y}{\mathrm{d}x^2},\ =3\frac{\mathrm{d}y}{\mathrm{d}x}$	M1A1, B1
	$\frac{\mathrm{d}^3y}{\mathrm{d}x^3}\left(2-x^2\right)+\frac{\mathrm{d}^2y}{\mathrm{d}x^2}\left(10x\frac{\mathrm{d}y}{\mathrm{d}x}-2x\right)+5\left(\frac{\mathrm{d}y}{\mathrm{d}x}\right)^2=3\frac{\mathrm{d}y}{\mathrm{d}x}$	
	$\frac{\mathrm{d}^3y}{\mathrm{d}x^3}=\frac{1}{\left(2-x^2\right)}\left(2x\frac{\mathrm{d}^2y}{\mathrm{d}x^2}\left(1-5\frac{\mathrm{d}y}{\mathrm{d}x}\right)-5\left(\frac{\mathrm{d}y}{\mathrm{d}x}\right)^2+3\frac{\mathrm{d}y}{\mathrm{d}x}\right)$ *	A1* (5)
ALT 1	$\frac{\mathrm{d}^2y}{\mathrm{d}x^2}=\frac{3y-5x\left(\frac{\mathrm{d}y}{\mathrm{d}x}\right)^2}{\left(2-x^2\right)}$	
	$\frac{\mathrm{d}^3y}{\mathrm{d}x^3}=\frac{\left[3\frac{\mathrm{d}y}{\mathrm{d}x}-5\left(\frac{\mathrm{d}y}{\mathrm{d}x}\right)^2-5x\times2\frac{\mathrm{d}y}{\mathrm{d}x}\frac{\mathrm{d}^2y}{\mathrm{d}x^2}\right]\left(2-x^2\right)-\left[3y-5x\left(\frac{\mathrm{d}y}{\mathrm{d}x}\right)^2\right](-2x)}{\left(2-x^2\right)^2}$	M1M1A1
	$\frac{\mathrm{d}^3y}{\mathrm{d}x^3}=\frac{\left[3\frac{\mathrm{d}y}{\mathrm{d}x}-5\left(\frac{\mathrm{d}y}{\mathrm{d}x}\right)^2-10x\frac{\mathrm{d}y}{\mathrm{d}x}\frac{\mathrm{d}^2y}{\mathrm{d}x^2}\right]\left(2-x^2\right)+2x\left(2-x^2\right)\frac{\mathrm{d}^2y}{\mathrm{d}x^2}}{\left(2-x^2\right)^2}$	M1 (NB: B1 on ePEN)
	$\frac{\mathrm{d}^3y}{\mathrm{d}x^3}=\frac{1}{\left(2-x^2\right)}\left(2x\frac{\mathrm{d}^2y}{\mathrm{d}x^2}\left(1-5\frac{\mathrm{d}y}{\mathrm{d}x}\right)-5\left(\frac{\mathrm{d}y}{\mathrm{d}x}\right)^2+3\frac{\mathrm{d}y}{\mathrm{d}x}\right)$ *	A1* (5)

Question Number	Scheme	Marks
ALT 2	$\dfrac{d^2y}{dx^2}=\dfrac{3y}{\left(2-x^2\right)}-\dfrac{5x\left(\dfrac{dy}{dx}\right)^2}{\left(2-x^2\right)}$	
	$\dfrac{d^3y}{dx^3}=\dfrac{3\dfrac{dy}{dx}\left(2-x^2\right)-3y(-2x)}{\left(2-x^2\right)^2}$ $-\dfrac{\left[5\left(\dfrac{dy}{dx}\right)^2+5x\times 2\dfrac{dy}{dx}\dfrac{d^2y}{dx^2}\right]\left(2-x^2\right)-5x\left(\dfrac{dy}{dx}\right)^2(-2x)}{\left(2-x^2\right)^2}$	M1M1A1
	$\dfrac{d^3y}{dx^3}=\dfrac{3\dfrac{dy}{dx}\left(2-x^2\right)-\left(\left(2-x^2\right)\dfrac{d^2y}{dx^2}+5x\dfrac{dy}{dx}\right)(-2x)}{\left(2-x^2\right)^2}$ $-\dfrac{\left[5\left(\dfrac{dy}{dx}\right)^2+5x\times 2\dfrac{dy}{dx}\dfrac{d^2y}{dx^2}\right]\left(2-x^2\right)-5x\left(\dfrac{dy}{dx}\right)^2(-2x)}{\left(2-x^2\right)^2}$	M1(B1 on ePEN)
	$\dfrac{d^3y}{dx^3}=\dfrac{1}{\left(2-x^2\right)}\left(2x\dfrac{d^2y}{dx^2}\left(1-5\dfrac{dy}{dx}\right)-5\left(\dfrac{dy}{dx}\right)^2+3\dfrac{dy}{dx}\right)$ *	A1*
(b)	$x=0 \;\Rightarrow\; 2\dfrac{d^2y}{dx^2}=9 \quad \dfrac{d^2y}{dx^2}=\dfrac{9}{2}$	B1
	$\dfrac{d^3y}{dx^3}=\dfrac{1}{2}\left(-5\left(\dfrac{dy}{dx}\right)^2+3\dfrac{dy}{dx}\right)=\dfrac{1}{2}\left(-5\times\dfrac{1}{16}+\dfrac{3}{4}\right)=\dfrac{7}{32}$	M1
	$y=3+\dfrac{1}{4}x+\dfrac{9}{2}\dfrac{x^2}{2!}+\dfrac{7}{32}\dfrac{x^3}{3!}$	M1
	$y=3+\dfrac{1}{4}x+\dfrac{9}{4}x^2+\dfrac{7}{192}x^3$	A1 (4)
		[9]

Question Number	Scheme	Marks
(a)		
M1	Differentiate $\left(2-x^2\right)\dfrac{\mathrm{d}^2y}{\mathrm{d}x^2}$ using product rule	
M1	Differentiate $5x\left(\dfrac{\mathrm{d}y}{\mathrm{d}x}\right)^2$ using product and chain rule	
A1	Correct derivative of $5x\left(\dfrac{\mathrm{d}y}{\mathrm{d}x}\right)^2$	
B1	Correct derivative of $3y$	
A1*	Correct result obtained from fully correct working	
ALT 1	*Rearrange and use quotient rule*	
M1	Use the quotient rule. Denominator must be $\left(2-x^2\right)^2$ and numerator to be the difference of 2 terms	
M1	Differentiate $\left[3y-5x\left(\dfrac{\mathrm{d}y}{\mathrm{d}x}\right)^2\right]$ using product and chain rule	
A1	Fully correct differentiation	
M1	NB: B1 on ePEN Replace $3y$ with $\left(2-x^2\right)\dfrac{\mathrm{d}^2y}{\mathrm{d}x^2}+5x\dfrac{\mathrm{d}y}{\mathrm{d}x}$	
A1*	Correct result obtained from fully correct working	
ALT 2	*Rearrange, separate into 2 fractions and then use quotient* rule	
M1	Use the quotient rule on both fractions. Denominators must be $\left(2-x^2\right)^2$ and numerator of each to be the difference of 2 terms	
M1	Differentiate $3y$ using the chain rule **and** differentiate $5x\left(\dfrac{\mathrm{d}y}{\mathrm{d}x}\right)^2$ using product and chain rule	
A1	Fully correct differentiation	
M1	NB: B1 on ePEN Replace $3y$ with $\left(2-x^2\right)\dfrac{\mathrm{d}^2y}{\mathrm{d}x^2}+5x\dfrac{\mathrm{d}y}{\mathrm{d}x}$	
A1*	Correct result obtained from fully correct working	
(b)		
B1	Correct value of $\dfrac{\mathrm{d}^2y}{\mathrm{d}x^2}$	
M1	Use the given result from (a) to obtain a value for $\dfrac{\mathrm{d}^3y}{\mathrm{d}x^3}$	
M1	Taylor's series formed using their values for the derivatives (accept 2! or 2 and 3! or 6)	
A1	Correct series, must start (or end) $y = ...$ but accept f(x) provided y = f(x) defined somewhere	

Question Number	Scheme	Marks
6(a)	$m^2+2m+5=0 \Rightarrow m=-1\pm 2i$	M1
	C F: $y=\mathrm{e}^{-x}\left(A\cos 2x+B\sin 2x\right)$ OR $y=\mathrm{e}^{-x}\left(P\mathrm{e}^{\mathrm{i}2x}+Q\mathrm{e}^{-\mathrm{i}2x}\right)$ or $y=P\mathrm{e}^{(-1+2\mathrm{i})x}+Q\mathrm{e}^{(-1-2\mathrm{i})x}$	A1
	PI: $y=a\cos x+b\sin x$	B1
	$y'=-a\sin x+b\cos x$ $y''=-a\cos x-b\sin x$	
	$-a\cos x-b\sin x-2a\sin x+2b\cos x+5a\cos x+5b\sin x=6\cos x$	M1
	$-b-2a+5b=0$ $-a+2b+5a=6$	M1
	$a=\frac{6}{5}$ $b=\frac{3}{5}$	A1
	GS: y = their CF $+\frac{6}{5}\cos x+\frac{3}{5}\sin x$	A1ft (7)
(b)	$x=0, y=0$ $0=A+\frac{6}{5} \Rightarrow A=-\frac{6}{5}$	M1
	$y'=-\mathrm{e}^{-x}\left(A\cos 2x+B\sin 2x\right)+\mathrm{e}^{-x}\left(-2A\sin 2x+2B\cos 2x\right)$ $-\frac{6}{5}\sin x+\frac{3}{5}\cos x$	M1A1ft
	$x=0$ $\frac{\mathrm{d}y}{\mathrm{d}x}=0 \Rightarrow 0=+\frac{6}{5}+2B+\frac{3}{5} \Rightarrow B=-\frac{9}{10}$	dM1
	PS: $y=\mathrm{e}^{-x}\left(-\frac{6}{5}\cos 2x-\frac{9}{10}\sin 2x\right)+\frac{6}{5}\cos x+\frac{3}{5}\sin x$	A1 (5)
		[12]
ALT	$y=\mathrm{e}^{-x}\left(P\mathrm{e}^{\mathrm{i}2x}+Q\mathrm{e}^{-\mathrm{i}2x}\right)+\frac{6}{5}\cos x+\frac{3}{5}\sin x$	
	$x=0$ $y=0$ $0=P+Q+\frac{6}{5}$	M1
	$\frac{\mathrm{d}y}{\mathrm{d}x}=\mathrm{e}^{-x}\left(2\mathrm{i}P\mathrm{e}^{\mathrm{i}2x}-2\mathrm{i}Q\mathrm{e}^{-\mathrm{i}2x}\right)-\mathrm{e}^{-x}\left(P\mathrm{e}^{\mathrm{i}2x}+Q\mathrm{e}^{-\mathrm{i}2x}\right)-\frac{6}{5}\sin x+\frac{3}{5}\cos x$	M1A1ft
	$0=2\mathrm{i}P-2\mathrm{i}Q+\frac{9}{5}$	
	$P+Q=-\frac{6}{5}$ $P-Q=\frac{9}{10}\mathrm{i}$	
	$P=\frac{1}{2}\left(-\frac{6}{5}+\frac{9}{10}\mathrm{i}\right)$ $Q=\frac{1}{2}\left(-\frac{6}{5}-\frac{9}{10}\mathrm{i}\right)$	dM1
	PS: $y=\frac{1}{2}\mathrm{e}^{-x}\left(-\frac{6}{5}+\frac{9}{10}\mathrm{i}\right)\mathrm{e}^{2\mathrm{i}x}+\frac{1}{2}\mathrm{e}^{-x}\left(-\frac{6}{5}-\frac{9}{10}\mathrm{i}\right)\mathrm{e}^{-2\mathrm{i}x}+\frac{6}{5}\cos x+\frac{3}{5}\sin x$	A1 (5)

Question Number	Scheme	Marks
(a)		
M1	Form and solve the auxiliary equation	
A1	Correct CF, either form (Often not seen until GS stated)	
B1	Correct form for the PI	
M1	Differentiate twice and sub in the original equation	
M1	Obtain a pair of simultaneous equations and attempt to solve	
A1	Correct values for both unknowns	
A1ft	Form the GS. Must start y = ... Follow through their CF (writing CF scores A0) Must have scored a minimum of 2 of the M marks	
(b)		
	For CF $y = e^{-x}\left(A\cos 2x + B\sin 2x\right)$	
M1	Sub $x=0,\ y=0$ in their GS and obtain a value for A	
M1	Differentiate their GS Product rule must be used	
A1ft	Correct differentiation of their GS provided this has 4 terms	
dM1	Sub $x=0,\ \frac{dy}{dx}=0$ and their A and obtain a value for B Depends on both previous M marks	
A1	Fully correct PS. Must start y =...	
ALT(b)		
	For CF $y = e^{-x}\left(Pe^{i2x} + Qe^{-i2x}\right)$ or $y = Pe^{(-1+2i)x} + Qe^{(-1-2i)x}$	
M1	Sub $x=0,\ y=0$ in their GS and obtain an equation in P and Q	
M1	Differentiate their GS Product rule must be used if $y = e^{-x}\left(Pe^{i2x} + Qe^{-i2x}\right)$ used	
A1ft	Correct differentiation of their GS	
dM1	Sub $x=0,\ \frac{dy}{dx}=0$ to obtain a second equation and solve the pair of equations The solution must allow for P and Q to be complex	
A1	Fully correct PS. Must start y =...	

Question Number	Scheme	Marks
7 **(a)**	$x = r\cos\theta = 3\sin 2\theta\cos\theta$	B1
	$\dfrac{dx}{d\theta} = 6\cos 2\theta\cos\theta - 3\sin 2\theta\sin\theta = 0$	M1
	$2\cos\theta\left(\cos^2\theta - 2\sin^2\theta\right) = 0$	M1
ALT	For the 2 M marks:	
	$x = 6\sin\theta\cos^2\theta \Rightarrow \dfrac{dx}{d\theta} = 6\cos^3\theta - 12\sin^2\theta\cos\theta = 0$	
	$\tan\phi = \dfrac{1}{\sqrt{2}}$ *	A1* (4)
(b)	$\tan\phi = \dfrac{1}{\sqrt{2}} \Rightarrow \sin\phi = \dfrac{1}{\sqrt{3}},\ \cos\phi = \dfrac{\sqrt{2}}{\sqrt{3}}$	M1
	$R = 3\times 2\times\dfrac{1}{\sqrt{3}}\times\dfrac{\sqrt{2}}{\sqrt{3}} = 2\sqrt{2}$	A1 (2)
(c)	Area of sector $= \dfrac{1}{2}\int r^2 d\theta = \dfrac{9}{2}\int \sin^2 2\theta\, d\theta$	M1
	$= \dfrac{9}{2}\int_0^{\arctan\left(\frac{1}{\sqrt{2}}\right)} \dfrac{1}{2}\left(1-\cos 4\theta\right) d\theta$	M1
	$= \dfrac{9}{2}\left[\dfrac{1}{2}\left(\theta - \dfrac{1}{4}\sin 4\theta\right)\right]_0^{\arctan\frac{1}{\sqrt{2}}}$	M1A1
	$= \dfrac{9}{4}\left[\arctan\dfrac{1}{\sqrt{2}} - \dfrac{1}{4}\sin 4\left(\arctan\dfrac{1}{\sqrt{2}}\right) - 0\right]$	dM1
	$\sin 4\phi = 2\sin 2\phi\cos 2\phi = 4\sin\phi\cos\phi\left(2\cos^2\phi - 1\right)$ $= 4\times\dfrac{1}{\sqrt{3}}\times\dfrac{\sqrt{2}}{\sqrt{3}}\left(2\times\dfrac{2}{3} - 1\right) = \dfrac{4\sqrt{2}}{9}$	M1
	Area of sector $= \dfrac{9}{4}\left(\arctan\dfrac{1}{\sqrt{2}} - \dfrac{1}{4}\times\dfrac{4\sqrt{2}}{9}\right) = \dfrac{9}{4}\arctan\dfrac{1}{\sqrt{2}} - \dfrac{\sqrt{2}}{4}$	A1 (7)
		[13]

Question Number	Scheme	Marks
(a)		
B1	State $x=\left(r\cos\theta\right)=3\sin 2\theta\cos 2\theta$ May be given by implication	
M1	Attempt to differentiate $x=r\cos\theta$ or $x=r\sin\theta$ Product rule must be used	
M1	Use a correct double angle formula **and** equate the derivative of $r\cos\theta$ to 0	
ALT	**M1** Attempt the differentiation of $x=r\cos\theta$ or $x=r\sin\theta$ using the product rule (after using a double angle formula) **M1** Use a correct double angle formula **and** equate the derivative of $r\cos\theta$ to 0	
A1*	Complete to the given answer and no extras with no errors in the working. Accept θ or ϕ All values seen must be exact	
(b)		
M1	Attempt exact values for $\sin\theta$ and $\cos\theta$ and use these to obtain a value for *R*. Values for $\sin\theta$ and/or $\cos\theta$ may have been seen in (a)	
A1	A correct, exact value for *R*, as shown or any equivalent. Award M1A1 for a correct exact answer	
(c)		
M1	Use of Area $=\frac{1}{2}\int r^2\,\mathrm{d}\theta$ Limits not needed (ignore any shown)	
M1	Use the double angle formula to obtain $k\int \frac{1}{2}\left(1\pm\cos 4\theta\right)\mathrm{d}\theta$ Ignore any limits given This is NOT dependent NB: There are other, lengthy, methods of reaching this point	
M1	Attempt the integration $\cos 4\theta \to \pm\frac{1}{4}\sin 4\theta$ (Not dependent)	
A1	Correct integration of $1-\cos 4\theta$	
dM1	Correct use of correct limits. Depends on second and third M marks 0 at lower limit need not be shown	
M1	Attempt an exact numerical value for $\sin 4\left(\arctan\frac{1}{\sqrt{2}}\right)$	
A1	Correct final answer. Award M1A1 for a correct exact final answer	

Question Number	Scheme	Marks
8(a)	$z^n = e^{in\theta} = \cos n\theta + i\sin n\theta$	
	$\frac{1}{z^n} = e^{-in\theta} = \cos(-n\theta) + i\sin(-n\theta) = \cos n\theta - i\sin n\theta$	
	$z^n + \frac{1}{z^n} = \cos n\theta + i\sin n\theta + \cos n\theta - i\sin n\theta = 2\cos n\theta$ *	M1A1cso (2)
(b)	$\left(z+\frac{1}{z}\right)^6 = z^6 + 6z^5\times\frac{1}{z} + \frac{6\times5}{2!}z^4\times\frac{1}{z^2} + \frac{6\times5\times4}{3!}z^3\times\frac{1}{z^3}$ $+\frac{6\times5\times4\times3}{4!}z^2\times\frac{1}{z^4} + \frac{6\times5\times4\times3\times2}{5!}z\times\frac{1}{z^5} + \frac{1}{z^6}$	M1A1
	$(2\cos\theta)^6 = z^6 + 6z^4 + 15z^2 + 20 + 15\times\frac{1}{z^2} + 6\times\frac{1}{z^4} + \frac{1}{z^6}$	
	$64\cos^6\theta = z^6 + \frac{1}{z^6} + 6\left(z^4 + \frac{1}{z^4}\right) + 15\left(z^2 + \frac{1}{z^2}\right) + 20$	M1
	$64\cos^6\theta = 2\cos 6\theta + 6\times2\cos 4\theta + 15\times2\cos 2\theta + 20$	M1
	$\cos^6\theta = \frac{1}{32}(\cos 6\theta + 6\cos 4\theta + 15\cos 2\theta + 10)$ *	A1* (5)
(c)	$\cos 6\theta + 6\cos 4\theta + 15\cos 2\theta + 10 = 10$	
	$32\cos^6\theta = 10$	M1A1
	$\cos\theta = \pm\sqrt[6]{\frac{5}{16}}$	
	$\theta = 0.6027...,\ 2.5388... \quad \theta = 0.603,\ 2.54$	M1A1 (4)
(d)	$\int_0^{\frac{\pi}{3}}(32\cos^6\theta - 4\cos^2\theta)\,d\theta$	
	$= \int_0^{\frac{\pi}{3}}(\cos 6\theta + 6\cos 4\theta + 15\cos 2\theta + 10 - 4\cos^2\theta)\,d\theta$	
	$= \int_0^{\frac{\pi}{3}}(\cos 6\theta + 6\cos 4\theta + 15\cos 2\theta + 10 - 2 - 2\cos 2\theta)\,d\theta$	M1
	$= \left[\frac{1}{6}\sin 6\theta + \frac{3}{2}\sin 4\theta + \frac{13}{2}\sin 2\theta + 8\theta\right]_0^{\frac{\pi}{3}}$	M1A1
	$= (0) + \frac{3}{2}\left(-\frac{\sqrt{3}}{2}\right) + \frac{13}{2}\times\frac{\sqrt{3}}{2} + \frac{8\pi}{3} \quad (-0)$	dM1
	$= \frac{5\sqrt{3}}{2} + \frac{8\pi}{3}$ oe	A1 (5)
		[16]

Question Number	Scheme	Marks
(a)		
M1	Attempt to obtain $z^n + \frac{1}{z^n}$	
A1cso	Reach the given result with clear working and no errors Must see $\cos(-n\theta)+\mathrm{i}\sin(-n\theta)$ changed to $\cos n\theta - \mathrm{i}\sin n\theta$ (ie both included)	
(b)	*The first 3 marks apply to the binomial expansion only*	
M1	Apply the binomial expansion to $\left(z+\frac{1}{z}\right)^6$ Coefficients must be numerical (ie ${}^nC_r$ is not acceptable). The expansion must have 7 terms with at least 4 correct	
A1	Correct expansion, terms need not be simplified	
M1	Simplify the coefficients and pair the appropriate terms on RHS (At least 2 pairs must be correct)	
M1	Use the result from (a) throughout. Must include 2^6 or 64 now	
A1*	Obtain the given result with no errors in the working	
(c)		
M1	Use the result from (b) to simplify the given equation	
A1	Reach $32\cos^6\theta = 10$ oe	
M1	Solve to obtain at least one correct value for θ, in radians and in the given range, 3 sf or better	
A1	2 correct values, and no extras, in radians and in the given range. Must be 3 sf here Ignore extras outside the range	
(d)		
M1	Use the result in (b) to change $\cos^6\theta$ to a sum of multiple angles ready for integration and use $\cos^2\theta = \pm\frac{1}{2}(\cos 2\theta \pm 1)$ on $\cos^2\theta$ Limits not needed, ignore any shown	
M1	Integrate their expression to obtain an expression containing terms in $\sin 6\theta, \sin 4\theta, \sin 2\theta$ and θ Limits not needed	
A1	Correct integration Limits not needed	
dM1	Substitute limit pi/3. Depends second M mark	
A1	Correct, exact, answer (any equivalent to that shown). AwardM1A1 for a correct final answer following fully correct working.	
	There are other ways to integrate the function in (d), eg parts on one or both of the powers of $\cos\theta$, using $\cos^6\theta = \left(\cos^2\theta\right)^3 = \frac{1}{8}(1+\cos 2\theta)^3 = ...$	
	If in doubt about the marking of alternative methods which are not completely correct, send to review	

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