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**1.** Relative to a fixed origin *O*, the points *A*, *B*, *C* and *D* have coordinates (0, 4, 1), (4, 0, 0),

(3, 5, 2) and (2, 2, *k*) respectively, where *k* is a constant.

(*a*)Determine the exact area of triangle *ABC*.

**(3)**

(*b*)Determine in terms of *k*, the volume of the tetrahedron *ABCD*, simplifying your answer.

**(3)**

**(Total 6 marks)**

**\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_**

**2.** *y* = ln (tanh 2*x*) *x* > 0

(*a*)Show that



where *p* is a constant to be determined.

**(4)**

(*b*)Hence determine, in simplest form, the exact value of *x* for which 

**(2)**

**(Total 6 marks)**

**\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_**

**3.**

 where *k* is a constant

(*a*)Determine the values of *k* for which **A** is singular.

**(2)**

Given that **A** is non-singular,

(*b*)find **A**−1 , giving your answer in terms of *k*.

**(4)**

**(Total 6 marks)**

**\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_**

**4.** Using the substitution *x* = 4 cosh *θ* show that



where *a* is a constant to be determined and *c* is an arbitrary constant.

**(6)**

**(Total 6 marks)**

**\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_**

**5.**



Given that 8 is an eigenvalue of **M**

(*a*)determine an eigenvector corresponding to the eigenvalue 8

**(2)**

(*b*)Determine the other two eigenvalues of **M**.

**(3)**

(*c*)Hence find an orthogonal matrix **P** and a diagonal matrix **D** such that **P**T**MP** = **D**

**(4)**

**(Total 9 marks)**

**\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_**

**6.**

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(*a*)Show that



**(6)**

(*b*)Hence show that



where *p* and *q* are integers to be determined and *k* is an arbitrary constant.

**(4)**

**(Total 10 marks)**

**\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_**

**7.** The point *P* has coordinates (1, 2, 1)

The line *l* has Cartesian equation

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The plane *Π*1 contains the point *P* and the line *l*.

(*a*)Show that a Cartesian equation for *Π*1 is

6*x* − 2*y* + 3*z* = 5

**(5)**

The point *Q* has coordinates (2, *k*, −7), where *k* is a constant.

(*b*)Show that the shortest distance between *Π*1 and *Q* is



**(2)**

The plane *Π*2 has Cartesian equation 8*x* − 4*y* + *z* = −3

Given that the shortest distance between *Π*1 and *Q* is the same as the shortest distance

between *Π*2 and *Q*,

(*c*)determine the possible values of *k*.

**(4)**

**(Total 11 marks)**

**\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_**

**8.** The curve *C* has equation

*y* = 2 + ln (1 − *x*2) 

(*a*)Show that the length of the curve *C* is given by

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**(4)**

(*b*)Hence, using algebraic integration, show that the length of the curve *C* is *p* + ln *q*

where *p* and *q* are rational numbers to be determined.

**(5)**

**(Total 9 marks)**

**\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_**

**9.** The ellipse *E* has equation



The point *P* lies on the ellipse and has coordinates (5 cos *θ*, 4 sin *θ*) where 0 < *θ* <

The line *l* is the normal to the ellipse at the point *P*.

(*a*)Show that an equation for *l* is

5*x* sin *θ* − 4*y* cos *θ* = 9 sin *θ* cos *θ*

**(5)**

The point *F* is the focus of *E* that lies on the positive *x*-axis.

(*b*)Determine the coordinates of *F*.

**(2)**

The line *l* crosses the *x*-axis at the point *Q*.

(*c*)Show that

**

where *e* is the eccentricity of *E*.

**(5)**

**(Total 12 marks)**

**TOTAL FOR PAPER: 75 MARKS**