**Simplifying a Boolean Expression**

So now we are able to derive a Boolean Expression for a logic system either from a truth table or from a logic diagram. The expression we end up with is often very long and complex and can result in very complicated logic systems being built. What is needed is a method of simplifying these logic expressions using Boolean algebra.

The introduction of the word algebra may remind many of you of the algebra you used in GCSE Mathematics to solve questions like:

simplify 2x + 6y.

Giving an answer of 2(x+3y).

Or

abc + bcd = bc(a+d)

Or

2xy – 4yz = 2y(x+2z) etc.

The solution involves looking for common elements that can be removed from each term to simplify the expression by using brackets. The original expression can be regenerated by multiplying everything inside the bracket by the term outside. All we have done is to find an alternative way of writing down the expression. Boolean algebra is very similar, we try to remove common elements to simplify the expression. However there are a few things that we can do to enable the expression to become much simpler if we can remember some basic combinations that help to reduce terms significantly. These special terms are called **identities**, and you will have to remember them as they are not reproduced on the information sheet of the examination paper.

We will now have a look at these special identities.

1. Consider the following Logic Circuit.

A

1

Q

The output is **Q = A.1** in Boolean Algebra.

If **A** is a Logic 0, then **Q** will also be Logic 0, since 0.1 = 0,

If **A** is a Logic 1, then **Q** will also be a Logic 1, since 1.1 = 1.

The output **Q** will therefore be the same Logic Level as **A**.

So wherever the term **A.1** appears this can be replaced with **A** so our first identity is

**A.1 = A**

2. Consider the following Logic Circuit.

A

0

Q

The output is **Q = A.0** in Boolean Algebra.

If **A** is a Logic 0, then **Q** will also be Logic 0, since 0.0 = 0,

If **A** is a Logic 1, then **Q** will be a Logic 0, since 1.0 = 0.

The output **Q** will therefore always be Logic 0

So wherever the term **A.0** appears this can be replaced with **0** so our second identity is

**A.0 = 0**

3. Consider the following Logic Circuit.

A

A

Q

The output is **** in Boolean Algebra.

If **A** is a Logic 0, then  will be Logic 1, so **Q** will be Logic 0,

since 0.1 = 0,

If **A** is a Logic 1, then  will be Logic 0, so **Q** will be Logic 0,

since 1.0 = 0,

The output **Q** will therefore always be Logic 0

So wherever the term ****appears this can be replaced with **0** so our third identity is

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4. Consider the following Logic Circuit.

A

1

Q

The output is **Q = A+1** in Boolean Algebra.

If **A** is a Logic 0, then **Q** will also be Logic 1, since 0+1 = 1,

If **A** is a Logic 1, then **Q** will also be Logic 1, since 1+1 = 1.

The output **Q** will therefore always be Logic 1

So wherever the term **A+1** appears this can be replaced with **1** so our fourth identity is

**A+1 = 1**

5. Consider the following Logic Circuit.

A

0

Q

The output is **Q = A+0** in Boolean Algebra.

If **A** is a Logic 0, then **Q** will also be Logic 0, since 0+0 = 0,

If **A** is a Logic 1, then **Q** will also be Logic 1, since 0+1 = 1.

The output **Q** will therefore be the same Logic Level as **A**.

So wherever the term **A+0** appears this can be replaced with **A** so our fifth identity is

**A+0 = A**

6. Consider the following Logic Circuit.

A

A

Q

The output is in Boolean Algebra.

If **A** is a Logic 0, then  will be Logic 1, so **Q** will be Logic 1,

since 0+1 = 1,

If **A** is a Logic 1, then  will be Logic 0, so **Q** will be Logic 1,

since 1+0 = 1,

The output **Q** will therefore always be Logic 1.

So wherever the term appears this can be replaced with **1** so our sixth identity is



In all of these examples the variable A has been used, but the rules apply for any variable. i.e.  etc.

Now let’s use these to simplify some Boolean expressions.

Examples :

1. Simplify the following expression.



Simplification can begin by looking for common terms, in this case A is common to all terms so we can remove this outside a bracket as shown below:



Now we can see that B is common to the first two terms inside the bracket, but it is not common to the last term so we cannot include this in the simplification, which now becomes:



Using our sixth identity the term  so the expression now becomes:



Using the first identity **B.1 = B** so the expression becomes



Using our sixth identity again the term  so the expression now becomes:



Using the first identity **A.1 = A** so the expression finally becomes



Using this mathematical approach the solution where common factors were identified in all terms actually made the expression a little bit more complex with the insertion of all the brackets, before we were able to start simplification. If we group pairs of terms instead of the whole, with intention of obtaining identities such as or  inside the brackets we often reach the simplest solution, without having to insert multiple brackets as shown below. We will solve the same example by this different technique.

Simplify the following expression.



Simplification can begin by combining the first two terms only as follows:



Using our sixth identity the term  so the expression now becomes:



Using the first identity **A.B.1 = A.B** so the expression becomes



Now we can remove the common term again to leave:



Using our sixth identity the term  so the expression now becomes:



Using the first identity **A.1 = A** so the expression finally becomes



I’m sure you’ll agree this is a lot simpler than .

Hopefully you will see the advantage of simplifying these expressions whenever we have the opportunity. It is normal practice in ***electronics*** to take the smaller groups and remove common factors to leave a useful combination behind like , as this means the combination can be removed. Let’s try another one!

2. Simplify the following expression:



Start by looking for possible common factors which will leave behind one of our two key identities inside the bracket, in this case **A.B** in terms 2 and 3. This gives the following simplification.



Using our fourth identity the term  so the expression now becomes:



Using the first identity **A.B.1 = A.B** so the expression becomes



Again we look for common terms, this time between the first two terms to give the following:



Using our sixth identity the term  so the expression now becomes:



Using the first identity **B.1 = B** so the expression becomes



And so we arrive at the simplest solution once again. You would not be expected to write out all of the steps as has been shown here in your own simplifications. These have only been included to show you how each step has been arrived at.

In the next two examples these intervening descriptions will be removed, to show you what typical solutions look like.

3. Simplify the following expression.



Solution:



4. Simplify the following expression.



Solution:



In this last example, notice that the original expression first needed to be expanded before the expression could be simplified. This is sometimes necessary to be able to group terms together in different ways.

The process may at first seem complicated and you may find it hard in the beginning to identify the terms to group together, but this comes from experience and the old phrase ‘practice makes perfect’ is very true here. So why don’t you try the next few examples.

Exercise 4 : Simplify the following Boolean expressions.

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