# Homework 3: Big-O notation Answers

1. (a) What is meant by the **time complexity** of an algorithm? [2]

It is a measure of the execution time of an algorithm for a problem of size n.

(b) The time complexity of an algorithm can be expressed in Big-O notation. Why is this measure of interest to computer scientists? [2]

Many problems involve vast amounts of data. (1) If the time complexity is exponential, it cannot be solved in a reasonable time. (1) Big-O is a way of comparing algorithms so that the most efficient one can be chosen. (1)

(c) Time complexity can be roughly calculated by counting the number of steps involved in processing n items.

If the number of steps in an algorithm is computed as 3n3 + n2 + 10n +6, what is the Big-0 notation that expresses the time complexity of the algorithm? Explain how you arrive at your answer. [3]

O(n3) (1 mark)

Only the most significant term is considered. (1) The coefficient of this term does not alter the order of magnitude so is ignored (1)

The constant term is ignored since as n increases, this does not significantly affect the running time.(1)

2. (a) Arrange the following in ascending order of time complexity:

O(n2) O(log n) O(n!) O(n) O(2n) [3]

O(log n) O(n) O(n2) O(2n)

(b) For each one, state whether it is constant time, exponential time,   
polymonial time or linear time. [5]

(i) O(n2) polynomial

(ii) O(log n) logarithmic

(iii) O(n!) exponential

(iv) O(n) linear

(v) O(2n) exponential

3. The algorithm for a bubble sort of n items is given below.

FOR i 🡨 1 to n-1

FOR j 🡨 1 to(n-i)

IF numbers [j] > numbers[j+1]

# Swap the names in the array

temp 🡨 numbers[j]

numbers[j] 🡨 numbers[j+1]

numbers[j+1] 🡨 temp

ENDIF

ENDFOR

ENDFOR

(a) Calculate the number of steps needed to sort a list of 5 items. Count the IF statement in the inner loop as one statement. [2]

The outer loop is performed 4 times. The inner loop is performed 4, 3, 2, 1 times. Within the loop, there is one IF statement.

This gives a total of 10 steps for the inner loop.

Total number of steps = 10

(b) Calculate the number of steps to sort a list of n items, given that

1 + 2 +… + (n-1) = n(n - 1)/2 [2]

Number of steps = (n2 - n)/2

(c) What is the time complexity of the algorithm expressed in Big-O notation? [1]

The significant term is n2/2.

The coefficient can be ignored, so the answer is O(n2)

[Total 20 marks]