# Worksheet 6 Limits of computation Answers

# Task 1

1. (a) Complete the table below for different values of n.

|  |  |  |  |
| --- | --- | --- | --- |
| **n** | **8** | **16** | **128** |
| log2n | 3 | 4 | 7 |
| n log2n | 24 | 64 | 896 |
| n2 | 64 | 256 | 16,384 |
| n3 | 512 | 4096 | 2,097,152 |
| 2n | 256 | 65536 | 340,282,366,920,938,463,463,374,607,431,768,211,456 (a 39-digit number) |
| n! | 40320 | 20,922,789,888,000 | A very large number! |

 (b) Algorithms for problems A, B and C have time complexities O(n log2n), O(n!) and O(n4).

 Which of these problems are tractable, and which are intractable?

 B is intractable, the others are tractable

2. Choose a password of 8 lowercase alphabetic characters. Write the password below:

 e.g. veryeasy

 What is the big-O notation for a brute force algorithm to crack this password?

 There are 26 x 26 x 26 x 26 x 26 x 26 x 26 x 26 possible combinations of letters. This is O(26n)

 Log on to a site to test the strength of the password. You could try

 <https://howsecureismypassword.net/> or type into Google “How secure is my password” to find a different site.

 How long will it take an average PC to crack the password? 52 seconds

 How long would it take to crack a password with 16 lowercase letters?

 345 thousand years

 If you use a mixture of eight uppercase and lowercase letters, digits and 28 other symbols, what is the big-O notation for the time complexity of the brute force algorithm?

 Total of 26+26+10+28 = 90 different symbols, O(90 n)

 How long will it take an average PC to crack it?

 8 days for h&r+BQ.f

 Is cracking a password an intractable problem? Yes – for a password of anything other than a few characters, it will be impossible to crack within a reasonable time.

 Is cracking a password an insoluble problem? No – it can theoretically be solved, given enough time. There may be another more efficient algorithms which can be used to crack a password.

# Task 2

3. Complete the missing parts in the following paragraph about the Halting problem.:

 It is \_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_ to devise a program H which can show that, given *any* program and its inputs, \_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_ \_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_

 It is, however, often possible to show that given ­­­­­­­­­­­­­\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_, it will halt for any input.

Answer:

 It is not possible to devise a program H which can show that, given any program and its inputs, it will halt or continue for ever.

 It is, however, often possible to show that given a specific algorithm, it will halt for any input.

4. What is the significance of the Halting problem?

 It proves that there are some problems that cannot be solved by computer.

5. Which of the following statements are true about the Travelling Salesman problem?

 (a) The TSP is a non-computable problem

 (b) x The TSP can sometimes be solved using a brute-force algorithm

 (c) Assuming a specific start city, the number of possible routes for 8 cities is double the number of routes for 4 cities

 (d) Assuming a specific start city, there are fewer than 700 possible routes for 7 cities.

6. What is a tractable problem? Include in your answer the time complexities of tractable and intractable problems, using Big-O notation.

A tractable problem has a polynomial-time solution or better, ie O(n), O(n2), O(n3) etc or O(log n), O(n log n)

An intractable problem may be theoretically soluble but cannot be solved within a reasonable time. Algorithms typically have time complexities O(2n) or O(n!).

7. Go to <http://www.hbmeyer.de/backtrack/backtren.htm> , which models the performance of a backtracking algorithm. The aim of the algorithm is to find an arrangement of the 16 cards such that each side touches a “matching” card. There is no quick and efficient algorithm to solve this problem.

**The jewels of Heuro**



This game was written by Dr Michel Wermelinger (Faculty of Mathematics, Computing and Technology) at the Open University.

Try the game by following the link

<http://www.open.edu/openlearn/science-maths-technology/computing-and-ict/computing/the-jewels-heuro>

Write down the best route you found in your attempt to find the shortest tour at the end of the game, and the length of the tour.