Godalming College

Pure 2 – Functions: domain, range, composites and inverse

Please **complete** this homework by _____. Start it early. If you can't do a question you will then have time to ask your teacher for help or go to a drop in session.

Section 1 – Review of previous topics. Please complete all questions.

1. Express $\frac{4x}{x^2 - 9} - \frac{2}{x+3}$ as a single fraction in its simplest form.

2. Given that

$$\frac{3x^4 - 2x^3 - 5x^2 - 4}{x^2 - 4} \equiv ax^2 + bx + c + \frac{dx + e}{x^2 - 4}, \qquad x \neq \pm 2$$

find the values of the constants a, b, c, d and e.

3. Given that

$$f(x) = \ln x, \qquad x > 0$$

sketch on separate axes the graphs of

(i) $y = f(x)$,

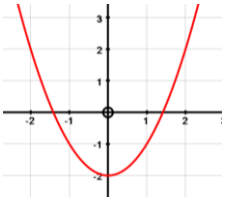
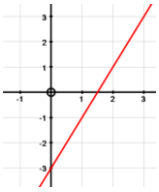
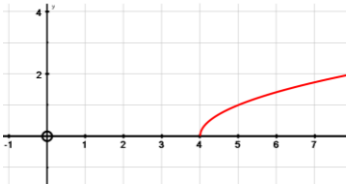
(ii) $y = |f(x)|$,

(iii) $y = -f(x - 4)$.

Show, on each diagram, the point where the graph meets or crosses the x-axis.
In each case, state the equation of the asymptote.

Section 2 – Consolidation of this week's topic. Please complete all 10 questions.

1. State the largest possible domain and range for each function shown:

(a)  (b)  (c) 

f(x) = $x^2 - 2$ f(x) = 2x -3 f(x) = $\sqrt{x-4}$ (6)

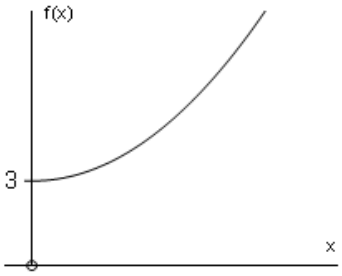
2. The function f is defined as f (x) = 4x – 5, x ε R. Find

a. f (3) b. f (-2) c. f (¼) d. the inverse function $f^{-1}(x)$ (5)

3. Given f(x) = x^3 , x ε R and g(x) = 4x -1, x ε R.
Find the following:

a. fg (x) b. gf (x) c. gg(x) (4)

4. The graph of f(x) = x^2 + 3, x ≥ 0, x ε R is shown.



a. State the range of f(x).

b. Find the inverse function f^{-1}, stating its domain.

c. Sketch the graph of f(x) and f^{-1} (x) on the same diagram.

d. State the transformation which maps y = f(x) onto y = f^{-1} (x). (6)

5. Find f^{-1}, the inverse of function f, where f: x → $\frac{x}{x+5}$ – 3, x ≠ -5, stating its domain. (4)

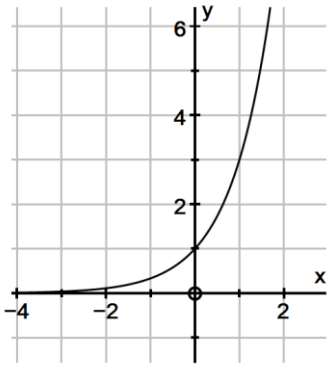
6. Two functions f and g are defined by f(x) = 5x + 6 and g(x) = $\frac{2}{x}$,(x ≠ 0)

a. find rules for fg(x) and gg(x)

b. find a rule for $(fg)^{-1}(x)$

(5)

7. The graph of $f(x) = 3^x$, $x \in R$ is shown.



i. State the range of f(x)

ii. Find the **exact** value of f(-2).

iii. State whether f(x) is a one-to-one function or a many-to-one function

iv Find an expression for $f^{-1}(x)$, stating its domain.

(5)

8. $f(x) = \sqrt{(x + 4)}$, , $x \geq -4$, $x \in R$

a. Sketch the graph of y = f(x)

b. State the range of f(x)

c. Find the inverse function $f^{-1}(x)$ and state its domain. (5)

9. The functions f and g are defined by

$$f : x \to 7x - 1, \qquad x \in \mathbb{R},$$

$$g : x \to \frac{4}{x-2}, \qquad x \neq 2, x \in \mathbb{R},$$

(*a*) Solve the equation $fg(x) = x$.

(4)

(*b*) Hence, or otherwise, find the largest value of a such that $g(a) = f^{-1}(a)$.

(1)

(Total 5 marks)

10.

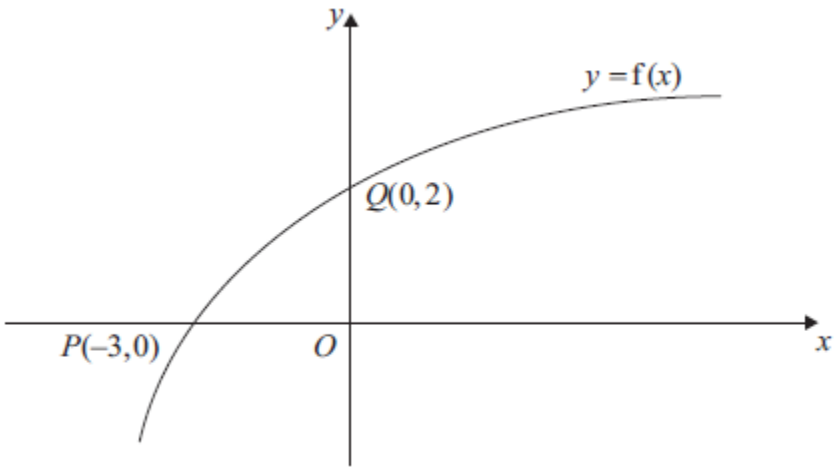


Figure 3

Figure 3 shows part of the curve with equation $y = \mathrm{f}(x)$, $x \in \mathbb{R}$.

The curve passes through the points $Q(0, 2)$ and $P(-3, 0)$ as shown.

(*a*) Find the value of ff (-3).

(2)

On separate diagrams, sketch the curve with equation

(*b*) $y = \mathrm{f}^{-1}(x)$,

(2)

(*c*) $y = \mathrm{f}(|x|) - 2$,

(2)

(*d*) $y = 2\mathrm{f}\left(\frac{1}{2}x\right)$.

(3)

Indicate clearly on each sketch the coordinates of the points at which the curve crosses or meets the axes.

(Total 9 marks)

Total: 54 marks