# Pure 6 – Compound Angles

Please **complete** this homework by ___________. Start it early. If you can't do a question you will then have time to ask your teacher for help or go to a drop in session.

## Section 1 – Review of previous topics. Please complete all questions.

1. $f(x) = x^2 - 8x + 19$

   (a) Express f(x) in the form $(x + a)^2 + b$, where $a$ and $b$ are constants.

   The curve $C$ with equation $y = f(x)$ crosses the $y$-axis at the point $P$ and has a minimum point at the point $Q$.

   (b) Sketch the graph of $C$ showing the coordinates of point $P$ and the coordinates of point $Q$.

   (c) Find the distance $PQ$, writing your answer as a simplified surd.

2. A circle $C$ has centre $(-1, 7)$ and passes through the point $(0, 0)$. Find an equation for $C$.

3.

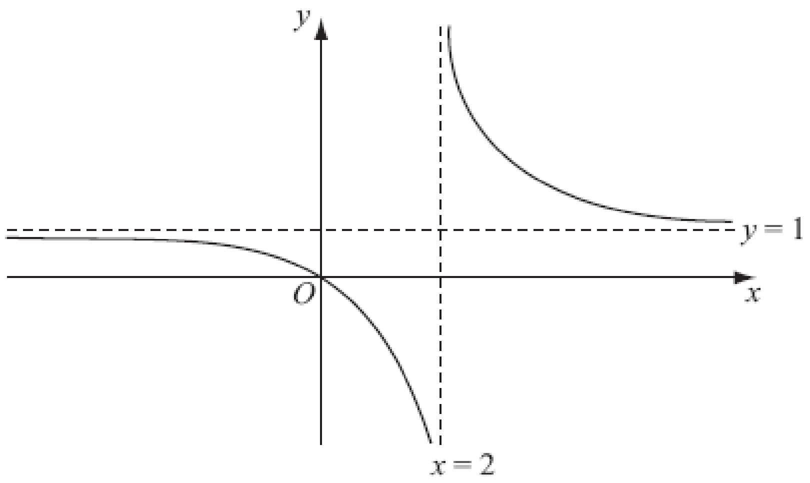


**Figure 1**

Figure 1 shows a sketch of the curve with equation $y = f(x)$ where

$$f(x) = \frac{x}{x-2}, \qquad x \neq 2.$$

The curve passes through the origin and has two asymptotes, with equations $y = 1$ and $x = 2$, as shown in Figure 1.

(a) Sketch the curve with equation $y = f(x - 1)$ and state the equations of the asymptotes of this curve.

(b) Find the coordinates of the points where the curve with equation $y = f(x - 1)$ crosses the coordinate axes.

4. Solve for x in the interval $0 \le x \le 2\pi$ the following equation, giving your answer in terms of $\pi$

$$cosec^2x + cot^2x = 3$$

5. Prove the following identities
   a) $sec^2x - sin^2x \equiv tan^2x + cos^2x$
   b) $(sinx - secx)^2 \equiv sin^2x + (tanx - 1)^2$

## Section 2 – Consolidation of this week's topic. Please <u>complete</u> all questions. Total 51

1) Express in the form sin$\alpha$, where $\alpha$ is acute.
   a) $sin10°cos30° + cos10°sin30°$
   b) $cos14°cos39° - sin14°sin39°$ **(3)**

2) Express as a single trigonometric ratio
   $$\frac{tan2A+tan5A}{1-tan2Atan5A}$$ **(1)**

3) Find the maximum value that each expression can take and the smallest positive value of x, in degrees for which this maximum occurs
   a) $cosxcos30° + sinxsin30°$

   b) $3sinxcos45° + 3cosxsin45°$ **(6)**

4) Find the minimum value that each expression can take and the smallest positive value of x, in radians in terms of π, for which this minimum occurs.
   a) $sinxcos\frac{\pi}{3} - cosxsin\frac{\pi}{3}$
   b) $2cosxcos\frac{\pi}{6} - 2sinxsin\frac{\pi}{6}$ **(6)**

5) Solve each equation for θ in the interval 0 ≤ θ ≤ 360.
   Give your answers to 1 decimal place where appropriate.
   a) $sin\theta cos15 + cos\theta sin15 = 0.4$ **(3)**
   b) $\frac{tan2\theta - tan60}{1+tan2\theta tan60} = 1$ **(5)**
   c) $\cos(\theta - 60) = sin\theta$ **(6)**

6) Given that
   $$2\cos(x + 50)° = \sin(x + 40)°.$$
   *(a)* Show that
   $$\tan x° = \frac{1}{3}\tan 40°.$$ **(4)**

   (*b*) Hence solve, for $0 \le \vartheta < 360$,
   $$2\cos(2\vartheta + 50)° = \sin(2\vartheta + 40)°,$$
   giving your answers to 1 decimal place. **(4)**

7) (*a*) Starting from the formulae for sin (*A* + *B*) and cos (*A* + *B*), prove that

$$\tan(A + B) = \frac{\tan A + \tan B}{1 - \tan A \tan B}.$$

**(4)**

(*b*) Deduce that

$$\tan\left(\theta + \frac{\pi}{6}\right) = \frac{1 + \sqrt{3}\tan\theta}{\sqrt{3} - \tan\theta}.$$

**(3)**

(*c*) Hence, or otherwise, solve, for $0 \le \vartheta \le \pi$,

$$1 + \sqrt{3}\tan\vartheta = (\sqrt{3} - \tan\vartheta)\tan(\pi - \vartheta).$$

Give your answers as multiples of $\pi$.

**(6)**