Pure 7 – Double Angle and Rsin(x+α)

Please **complete** this homework by __________. Start it early. If you can't do a question you will then have time to ask your teacher for help or go to a drop in session.

Section 1 – Review of previous topics. Please complete all questions.

1. Given $y = \sqrt{x} + \dfrac{4}{\sqrt{x}} + 4, \quad x > 0$

find the value of $\dfrac{dy}{dx}$ when $x = 8$, writing your answer in the form $a\sqrt{2}$, where a is a rational number.

2. Joan brings a cup of hot tea into a room and places the cup on a table. At time t minutes after Joan places the cup on the table, the temperature, θ °C, of the tea is modelled by the equation

$$\theta = 20 + Ae^{-kt},$$

where A and k are positive constants.
Given that the initial temperature of the tea was 90 °C,
(a) find the value of A.
The tea takes 5 minutes to decrease in temperature from 90 °C to 55 °C.
(b) Show that $k = \dfrac{1}{5}\ln 2$.

3. The curve C_1 has equation

$$y = x^2(x + 2).$$

(a) Find $\dfrac{dy}{dx}$.
(b) Sketch C_1, showing the coordinates of the points where C_1 meets the x-axis.
(c) Find the gradient of C_1 at each point where C_1 meets the x-axis.
The curve C_2 has equation

$$y = (x - k)^2(x - k + 2),$$

where k is a constant and $k > 2$.
(d) Sketch C_2, showing the coordinates of the points where C_2 meets the x and y axes.

4. Find, giving your answer to 3 significant figures where appropriate, the value of x for which
(a) $5^x = 10$,
(b) $\log_3(x - 2) = -1$.

5. Given that $2\log_2(x + 15) - \log_2 x = 6$,
(a) show that $x^2 - 34x + 225 = 0$.
(b) Hence, or otherwise, solve the equation $2\log_2(x + 15) - \log_2 x = 6$.

6. Find

$$\int\left(2x^5 - \frac{1}{4x^3} - 5\right)dx$$

giving each term in its simplest form.

Section 2 – Consolidation of this week's topic. Please complete all questions. (Total 60 marks)

1. a) Express 3cosθ + 4sinθ in the form R cos(θ – α). (3)
   b) Solve 3cosθ + 4sinθ = 1 for 0º < θ < 360º. (3)
   c) Find the minimum value of 3cosθ + 4sinθ. (1)

2. a) Given that tan $x \neq 1$, show that $\dfrac{\cos 2x}{\cos x - \sin x} \equiv \cos x + \sin x$ (3)

   b) By expressing cos x + sin x in the form Rsin(x + α), solve, for 0º ≤ x ≤ 360º,

$$\frac{\cos 2x}{\cos x - \sin x} = \frac{1}{2}$$ (6)

3. Prove that $\dfrac{2\tan x}{1+\tan^2 x} \equiv \sin 2x$ (4)

4. (*a*) Express sin x + √3 cos x in the form R sin (x + α), where R > 0 and 0 < α < 90°. (3)

   (*b*) Show that the equation sec x + √3 cosec x = 4 can be written in the form

   sin x + √3 cos x = 2 sin 2x. (3)

   (*c*) Deduce from parts (*a*) and (*b*) that sec x + √3 cosec x = 4 can be written in the form

   sin 2x – sin (x + 60°) = 0 (1)

5. Find all the solutions of

   2 cos 2θ = 1 – 2 sin θ

   in the interval 0 ≤ θ < 360°. (6)

6. (*a*) Express 6 cos θ + 8 sin θ in the form R cos (θ – α), where R > 0 and $0 < \alpha < \dfrac{\pi}{2}$. Give the value of α to 3 decimal places. (4)

(*b*) $p(\theta) = \dfrac{4}{12 + 6\cos\theta + 8\sin\theta}$, $0 \le \theta \le 2\pi$.

Calculate

(i) the maximum value of $p(\theta)$,

(ii) the value of θ at which the maximum occurs. (4)

7. (i) Without using a calculator, find the exact value of

$$(\sin 22.5° + \cos 22.5°)^2.$$

You must show each stage of your working. (5)

(ii) (*a*) Show that $\cos 2\theta + \sin\theta = 1$ may be written in the form

$$k\sin^2\theta - \sin\theta = 0,$$ stating the value of k. (2)

(*b*) Hence solve, for $0 \le \theta < 360°$, the equation

$$\cos 2\theta + \sin\theta = 1.$$ (4)

8.

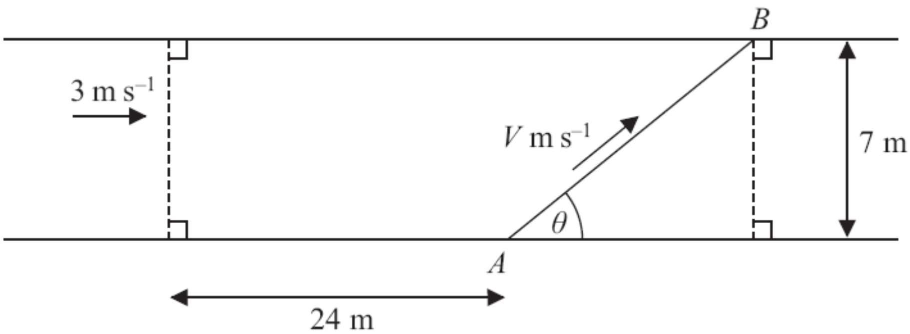

Figure 2

Kate crosses a road, of constant width 7 m, in order to take a photograph of a marathon runner, John, approaching at 3 m s^{-1}.

Kate is 24 m ahead of John when she starts to cross the road from the fixed point *A*.

John passes her as she reaches the other side of the road at a variable point *B*, as shown in Figure 2.

Kate's speed is V m s^{-1} and she moves in a straight line, which makes an angle θ, $0 < \theta < 150°$, with the edge of the road, as shown in Figure 2.

You may assume that V is given by the formula

$$V = \frac{21}{24\sin\theta + 7\cos\theta}, \qquad 0 < \theta < 150°$$

(*a*) Express $24\sin\theta + 7\cos\theta$ in the form $R\cos(\theta - \alpha)$, where R and α are constants and where $R > 0$ and $0 < \alpha < 90°$, giving the value of α to 2 decimal places. (3)

Given that θ varies,

(*b*) find the minimum value of V. (2)

Given that Kate's speed has the value found in part (*b*),

(*c*) find the distance *AB*. (3)