Godalming College

Pure 8 – Differentiation From First Principles and Small Angle Approximations

Please **complete** this homework by _____. Start it early. If you can't do a question you will then have time to ask your teacher for help or go to a drop-in session.

Section 1 – Review of previous topics. Please complete all questions.

1. Find the unit vector in the direction $2\boldsymbol{i} + \boldsymbol{j} - 3\boldsymbol{k}$.

2. The position vector of the point A is $2\boldsymbol{i} + 5\boldsymbol{j} - 4\boldsymbol{k}$ and $\overrightarrow{AB} = 3\boldsymbol{i} - 5\boldsymbol{j} - \boldsymbol{k}$ and the coordinates of point C are $(1, -3, -2)$.

   Find in terms of **i**, **j** and **k**,
   a. the position vectors of B and C
   b. $\overrightarrow{AC}$

   Find the exact value of
   c. The distance between A and C
   d. $\left|\overrightarrow{OC}\right|$

3. Find the angles that the vector $\overrightarrow{AB} = 2\boldsymbol{i} + 3\boldsymbol{j} - 5\boldsymbol{k}$ makes with each of the positive coordinate axes to 1 d.p.

4. Show that the function $f(x) = 4 - x(2x^2 + 3)$ is decreasing for all $x \in \mathbb{R}$.

5. $f(x) = px^3 - 3px^2 + x^2 - 4$. When $x = 2$, $f''(x) = -1$. Find p.

6. Given that $f(x) = x^2$ and $g(x) = 2x + 5$ solve $fg(x) = 9$.

7. Find the inverse function of $f(x) = \frac{1}{x} - 3$, $x \in \mathbb{R}$, $2 < x < 5$.

8. Find the equations of the tangents to the circle $x^2 + y^2 - 10x - 8y + 21 = 0$ at the points where the circle cuts the x axis.

9. Solve the simultaneous equations:
   $$\log(y - x) = 0$$
   $$2\log y = \log(21 + x)$$

10. Solve $2^{2x} - 2^x = 6$.

Section 2 – Consolidation of this week's topic. Please complete all questions.

1. a) When θ is small, show that the expression $\frac{5cos2\theta - sin3\theta - 4}{1 - sin5\theta}$ can be written as 2θ + 1. **(3 marks)**

   b) Hence write down the value of $\frac{5cos2\theta - sin3\theta - 4}{1 - sin5\theta}$ when θ is small. **(1 mark)**

2. For small θ show that $\frac{\sin^2 3\theta}{1 - \cos 2\theta} \cong 4.5$. **(3 marks)**

3. Solve $\frac{\sin^2 5\theta + 2\theta}{\tan \theta} = 3$ for the case when θ can be assumed to be small. **(3 marks)**

4. Differentiate the following from first principles:
   a. $\sin x$ **(5 marks)**
   b. $\cos 3x$ **(7 marks)**
   c. $4\cos x + 3x^2$ **(5 marks)**

Total: 27 marks