Pure 12 – Differentiation: Implicit, Second Derivative & Rates of Change

Please **complete** this homework by _____. Start it early. If you can't do a question you will then have time to ask your teacher for help or go to a drop in session.

Section 1 – Review of previous topics. Please complete all questions.

1. The surface area of an expanding sphere of radius r is given by $4\pi r^2$. Find the rate of change of the area with respect to the radius when r = 6cm.

2. Differentiate $2x^3 + \sqrt{x} + \frac{x^2+2x}{x^2}$.

3. The point (1, -3) lies on the circle $(x-3)^2 + (y+4)^2 = r^2$. Find the value of r.

4. The line with equation $y = 4x - 1$ does not intersect the circle with equation $x^2 + 2x + y^2 = k$. Find the range of possible values of k.

5. The points (-2,8), (7,7), (-3,-1) lie on a circle. Find the equation of the circle.

6. Use the factor theorem to show whether $(x-2)$ is a factor of $x^3 + x^2 - 4x - 4$.

7. Given that $(x-1)$ is a factor of $5x^3 - 9x^2 + 2x + a$, find the value of a.

8. Use algebraic division to find the cubic polynomial that arises from $(3x^4 + 8x^3 - 11x^2 + 2x + 8) \div (3x + 2)$.

9. It is asserted that $a + \frac{1}{a} \geq 2$. Prove that the inequality is true only if $a > 0$.

10. Prove that, for any distinct positive numbers p and q, $p + q > \sqrt{4pq}$.

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Section 2 – Consolidation of this week's topic.
Please complete all questions.

1) Differentiate these implicitly:
a) $x^2 + y^2 = 3$ b) $2x - y + y^2 = 5$ c) $\sin x + \cos y = 1$
d) $2e^x - 3e^{2y} = y$ e) $\ln 2x + 3\ln y^2 = x$ **[10]**

2) Find the equation of the tangent to $4\sin y - \sec x = 0$ at $(\frac{\pi}{3}, \frac{\pi}{6})$. **[5]**

3) Differentiate these implicitly:
a) $x^3 y = 5$ b) $xe^{2y} - \ln xy = 2$ c) $x\sin y + y^2 \operatorname{cosec} x = 0$
d) $xy - \sin x = e^y$ e) $\ln(x+2) = \ln(2y+1)$ **[16]**

4) A curve has the equation $3^x + y^2 = (x+3)y$. Find the equation of the normal to the curve at the point $(1,1)$. **[5]**

5) Find the points of inflection of these functions and determine the intervals over which they are concave or convex.
a) $y = x^4 - 54x^2$ b) $y = xe^{-x}$ **[11]**

6) Given that $y = e^x \sin x$, show that $\frac{d^2y}{dx^2} - 2\frac{dy}{dx} + 2y = 0$. **[3]**

7) The volume of water in a vase is given by $V = 10\pi(e^{0.2h} - 1)$ where h = depth of water in the vase. The depth of water is increasing at a rate of 0.6 $cm\ s^{-1}$. What is the rate of increase of the volume when the depth is 5cm? **[4]**

8) An inverted cone is being filled with sand at a rate of 10 $cm^3 s^{-1}$. After 5 seconds the depth of the sand is 20cm. What is the rate of increase of the depth of sand when the depth of sand is 10cm? (Hint: $V = \frac{1}{3}\pi r^2 h$ where both r and h are variables but we can put $\tan\theta = \frac{r}{h}$ and, since θ is not variable, we can substitute for r and reformulate the expression for V with only h as a variable). **[6]**

9) A biological culture is growing exponentially such that the number of bacteria present N at time t minutes is given by $N = 500 \times 1.05^{0.4t}$. Find the rate at which the bacteria are growing when N = 2000. **[3]**

Total: 63 Marks