

## Pure 13 – Integration: Standard Integrals

Please **complete** this homework by \_\_\_\_\_. Start it early. If you can't do a question you will then have time to ask your teacher for help or go to a drop in session.

### Section 1 – Review of previous topics.

Please complete all questions.

1. Find  $\int \left( \frac{2}{x^3} - 3\sqrt{x} \right) dx$
2. Find  $\int \left( (2x)^2 + \frac{\sqrt{x+5}}{x^2} \right) dx$
3. The curve with equation  $f(x)$  passes through the point  $(-1, 0)$ . Given that  $f'(x) = 9x^2 + 4x - 3$ , find  $f(x)$ .
4. Evaluate  $\int_1^3 \frac{x^3 + 2x^2}{x} dx$
5. Evaluate  $\int_1^4 \frac{2 + \sqrt{x}}{x^2} dx$
6. Given that  $\int_1^k \frac{1}{\sqrt{x}} dx = 3$ , find the value of the constant  $k$ .
7. Find the area enclosed by the curve  $y = 4 - 3x - x^2$  and the  $x$  axis.
8. Find the roots of the curve  $y = x^3 - 5x^2 + 6x$ . Find the total area enclosed by the curve and the  $x$  axis.
9. The curve  $y = x^2 - 3x + 4$  is intersected by the line  $y = x + 1$  at two points. Find the coordinates of the two points and the area enclosed by the curve and the line.
10. The curve  $y = 2 - x - x^2$  has a tangent at the point where it crosses the  $y$  axis. Show that the area enclosed by the curve, the tangent to the curve and the  $x$  axis is  $\frac{5}{6}$ .

## Section 2 – Consolidation of this week's topic.

Please complete all questions.

1) Integrate:

a)  $\int e^x dx$

b)  $\int \frac{1}{x} dx$

c)  $\int \cos x dx$

d)  $\int \sec^2 x dx$

e)  $\int \operatorname{cosec} x \cot x dx$

[5]

2) Integrate:

a)  $\int \left(5 - \frac{3}{x}\right) dx$

b)  $\int \frac{2e^x+1}{5} dx$

c)  $\int \frac{3x+1}{x} dx$

d)  $\int \frac{\sin x}{\cos^2 x} dx$

e)  $\int \frac{\cos x}{\sin^2 x} dx$

[10]

3) Integrate:

a)  $\int (x - 2)^7 dx$

b)  $\int (2x + 5)^3 dx$

c)  $\int \sqrt{4x - 1} dx$

d)  $\int \left(\frac{x}{4} - 2\right)^5 dx$

e)  $\int \frac{5}{(3-2x)^2} dx$

[10]

4) Integrate:

a)  $\int \frac{1}{2x-1} dx$

b)  $\int \frac{2}{3x+5} dx$

c)  $\int \frac{3}{2-7x} dx$

[6]

5) Integrate:

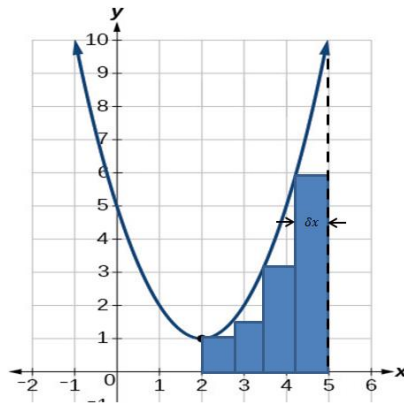
a)  $\int (\cos(5x - 2) + 2e^{x+3}) dx$

b)  $\int (e^{5-2x} + \sec^2 3x) dx$

c)  $\int \left(5 \sin(2x + 3) + \frac{3}{5x-1} + (e^{2x} + 1)^2\right) dx$

[13]

- 6) The area between the curve  $y = x^2 - 4x + 5$ , the x-axis and the lines  $x = 2$  and  $x = 5$  is estimated by finding the areas of four rectangles of equal width,  $\delta x$ , as shown below



- (a) Write down the value of  $\delta x$  (1)
- (b) Calculate the area of the largest rectangle (2)
- (c) If the height of rectangle  $n$  is  $y_n$ , explain what is meant by  $\lim_{\delta x \rightarrow 0} \sum y_n \times \delta x$  (2)
- (d) Calculate the exact value of  $\lim_{\delta x \rightarrow 0} \sum y_n \times \delta x$  (3)

**Total: 52 Marks**