Pure 15 – Integration: Trig and Reverse Chain Rule

Please **complete** this homework by __________. Start it early. If you can't do a question you will then have time to ask your teacher for help or go to a drop in session.

Section 1 – Review of previous topics.
Please complete all questions.

1) a) Express $x^2 + 6x + 13$ in the form $(x + a)^2 + b$

b) Hench sketch the curve $y = x^2 + 6x + 13$ and label the vertex, and the point where the curve cuts the y-axis.

2) A radioactive isotope has mass, M grams, at time t days given by the equation $M = 50e^{-0.3t}$

a) What is the initial mass of the isotope?

b) What is the half-life of the isotope?

3) Functions $\mathrm{f}(x)$ and $\mathrm{g}(x)$ are defined by:

$\mathrm{f}(x) = \frac{x}{x-3}, x \in \mathbb{R}, x \neq 3$ and $\mathrm{g}(x) = \frac{5x-2}{x}, x \in \mathbb{R}, x \neq 0$

a) Work out an expression for $\mathrm{f}^{-1}(x)$

b) Work out an expression for $\mathrm{gf}(x)$

c) Solve the equation $\mathrm{f}^{-1(x)} = \mathrm{gf}(x)$

4) A sequence of terms is defined by the recurrence relation $u_{n+1} = 4 - ku_n$, where k is a constant. Given that $u_1 = 3$.

a) Work out an expression in terms of k for u_2

b) Work out an expression in terms of k for u_3

Given also that $u_1 + u_2 + u_3 = 9$

c) Calculate the possible values of k

5) a) i) Prove that $\frac{\cos x}{\sin x} - \frac{\sin x}{1-\cos x} = -\operatorname{cosec} x$

ii) For what values of x is this identity valid?

b) Solve the equation $\frac{\cos x}{\sin x} - \frac{\sin x}{1-\cos x} = 3$ for $0 \leq x \leq 2\pi$

6) a) Differentiate these expressions with respect to x:

i) $\frac{x}{x+2}$ ii) $\frac{3x^2}{\cos x}$ iii) $(3x^3 + 5)\mathrm{e}^x$

b) Show that the derivative of $\frac{x^2+3x}{x-5}$ can be written as $\frac{ax^2+bx+c}{(x-5)^2}$ where a, b, and c are constants to be found.

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Section 2 – Consolidation of this week's topic.
Please complete all questions.

1) Integrate with respect to x:

b) $2\cos x$ b) $\sin 4x$ c) $3\sin(\frac{\pi}{3} - x)$ d) $\sec x \tan x$

e) $\operatorname{cosec}^2 x$ f) $\operatorname{cosec}\frac{1}{4}x \cot\frac{1}{4}x$ **[6]**

2) Evaluate:

a) $\int_0^{\frac{\pi}{2}} \cos(2x - \frac{\pi}{3})dx$ b) $\int_{\frac{\pi}{4}}^{\frac{\pi}{3}} \sec^2 3x\, dx$ c) $\int_{\frac{\pi}{2}}^{\frac{2\pi}{3}} \operatorname{cosec} x \cot x\, dx$ **[9]**

3) a) Express $\tan^2\theta$ in terms of $\sec\theta$

b) Show that $\int \tan^2 x\, dx = \tan x - x + c$ **[4]**

4) Find:

a) $\int \sin x \cos x \; dx$ b) $\int 4\cos^2 3x\, dx$ c) $\int \operatorname{cosec} 2x \cot x \; dx$ **[9]**

5) Integrate with respect to x:

a) $3x^2(x^3 - 2)^3$ b) $e^{\sin x} \cos x$ c) $\frac{x}{x^2+1}$

d) $\cot^3 x \operatorname{cosec}^2 x$ e) $\frac{e^x}{1+e^x}$ f) $\frac{x^3}{(x^4-2)^2}$

g) $\frac{(\ln x)^3}{x}$ h) $x^{\frac{1}{2}}\left(1 + x^{\frac{3}{2}}\right)^2$ **[16]**

6) Evaluate:

a) $\int_0^{\frac{\pi}{2}} \sin x\,(1 + \cos x)^2 \; dx$ b) $\int_{-1}^{0} \frac{e^{2x}}{2-e^{2x}}$

c) $\int_{\frac{\pi}{6}}^{\frac{\pi}{4}} \cot x \operatorname{cosec}^4 x \; dx$ d) $\int_2^4 \frac{x+1}{x^2+2x+8} \; dx$ **[16]**

Total: 60 Marks