# Pure 17 – Integration: Partial Fractions and Trapezium Rule

Please **complete** this homework by ___________. Start it early. If you can't do a question you will then have time to ask your teacher for help or go to a drop in session.

## Section 1 – Review of previous topics.
## Please complete all questions.

**1)** The equation of a circle is $x^2 + y^2 - 10x + 2y - 23 = 0$
   a) Showing your working clearly, work out
      i) Its centre ii) its radius
   b) The line $y = x + 2$ meets the circle at the points $P$ and $Q$. Work out, in exact form, the coordinates of $P$ and $Q$.

**2)** Given vectors $\mathbf{p} = \begin{pmatrix} 6 \\ -1 \end{pmatrix}$ and $\mathbf{q} = \begin{pmatrix} -3 \\ 4 \end{pmatrix}$
   a) Evaluate $3\mathbf{p} + 5\mathbf{q}$
   b) Write down the unit vector, $\hat{\mathbf{q}}$ in the direction of $\mathbf{q}$

**3)** Write the Cartesian equation of the curve that is given parametrically by

$$x = \frac{1}{2t+1}, y = \frac{2}{3-t}, t > 3$$

**4)** a) Show that $\sec^4 x - \tan^4 x \equiv \sec^2 x + \tan^2 x$
   b) Find the values in the range $-\pi \le x \le \pi$ that satisfy $\sec^4 x - \tan^4 x \equiv 5 + \tan^2 x$. Show your working.

**5)** Find $\frac{dy}{dx}$ given that $5xy - y^3 = 7$

**6)** Use implicit differentiation to prove that the derivative of $a^x$ is $a^x \ln a$

## Section 2 – Consolidation of this week's topic.
## Please complete all questions.

**1)** a) Express $\frac{3x+5}{(x+1)(x+3)}$ in partial fractions

b) Hence, find $\int \frac{3x+5}{(x+1)(x+3)}\,dx$ **[8]**

**2)** Show that $\int \frac{3}{(t-2)(t+1)}\,dt = \ln\left|\frac{t-2}{t+1}\right| + c$ **[7]**

**3)** Integrate with respect to $x$:

a) $\frac{14-x}{x^2+2x-8}$ b) $\frac{3x^2-5}{x^2-1}$ c) $\frac{x(4x+13)}{(2+x)^2(3-x)}$ **[23]**

**4)** Find the **exact** value of:

a) $\int_1^3 \frac{x+3}{x(x+1)}\,dx$ b) $\int_0^1 \frac{5x+7}{(x+1)^2(x+3)}\,dx$ **[15]**

**5)**

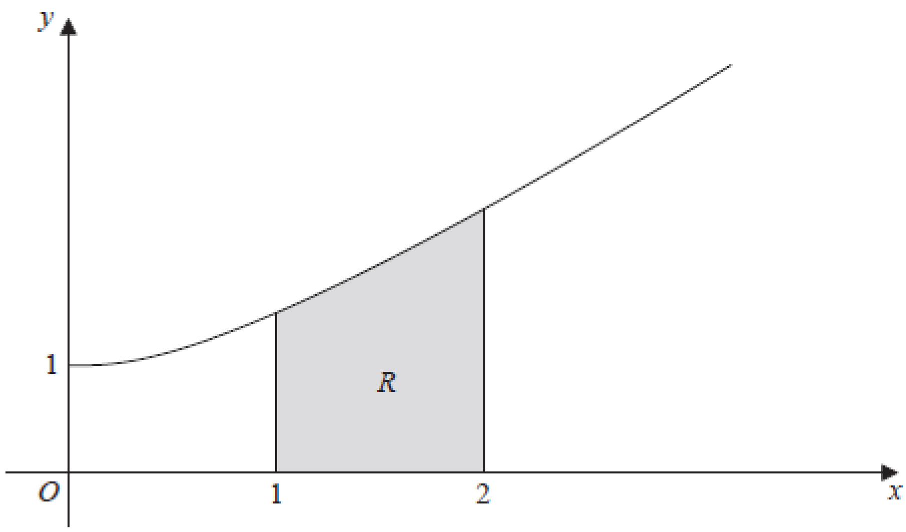


**Figure 1**

Figure 1 shows a sketch of part of the curve with equation $y = \sqrt{(x^2+1)}, x \geq 0$.

The finite region $R$, shown shaded in Figure 1, is bounded by the curve, the $x$-axis and the lines $x = 1$ and $x = 2$.

The table below shows corresponding values for $x$ and $y$ for $y = \sqrt{(x^2+1)}$.

| $x$ | 1 | 1.25 | 1.5 | 1.75 | 2 |
|---|---|---|---|---|---|
| $y$ | 1.414 | | 1.803 | 2.016 | 2.236 |

a) Complete the table above, giving the missing value of $y$ to 3 decimal places.

**[1]**

b) Use the trapezium rule, with all the values of $y$ in the completed table, to find an approximate value for the area of $R$, giving your answer to 2 decimal places.

**[4]**

**6)** The curve *C* has equation

$$y = 8 - 2^{x-1}, \qquad 0 \leq x \leq 4.$$

a) Complete the table below with the value of *y* corresponding to $x = 1$

| $x$ | 0 | 1 | 2 | 3 | 4 |
|---|---|---|---|---|---|
| $y$ | 7.5 | | 6 | 4 | 0 |

**(1)**

b) Use the trapezium rule, with all the values of *y* in the completed table, to find an approximate value for $\int_0^4 (8 - 2^{x-1})\ \mathrm{d}x$ .

**(3)**

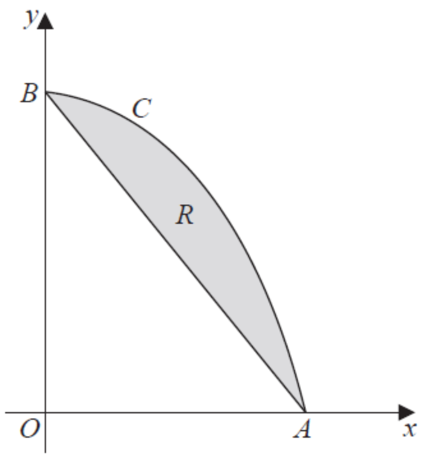


**Figure 2**

Figure 2 shows a sketch of the curve $C$ with equation $= 8 - 2^{x-1}$, $0 \leq x \leq 4$.

The curve $C$ meets the *x*-axis at the point $A$ and meets the *y*-axis at the point $B$.

The region $R$, shown shaded in Figure 2, is bounded by the curve $C$ and the straight line through $A$ and $B$.

c) Use your answer to part (b) to find an approximate value for the area of $R$.

**(2)**

**7)**

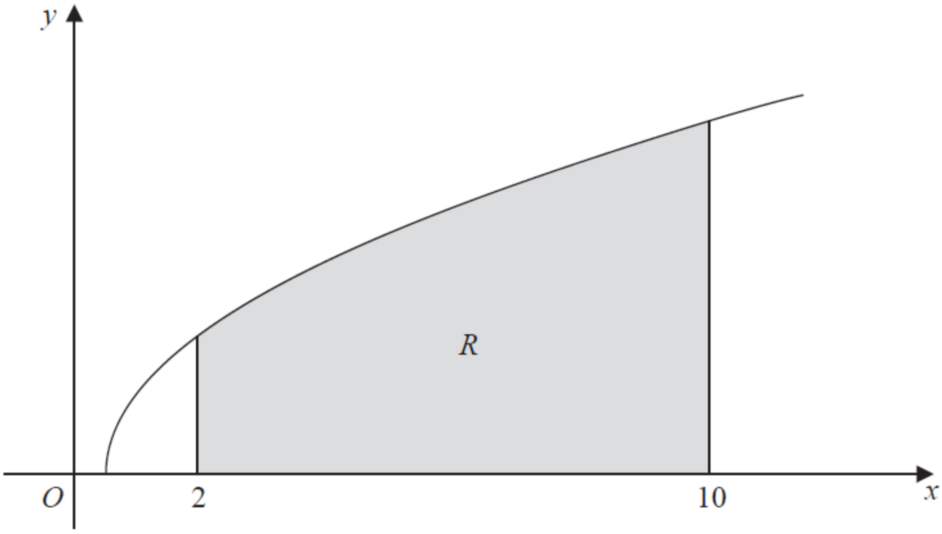


**Figure 3**

Figure 3 shows a sketch of part of the curve with equation $y = \sqrt{(2x - 1)}$, $x \geq 0.5$.

The finite region $R$, shown shaded in Figure 1, is bounded by the curve, the *x*-axis and the lines with equations $x = 2$ and $x = 10$.

The table below shows corresponding values of $x$ and $y$ for $y = \sqrt{(2x - 1)}$.

| $x$ | 2 | 4 | 6 | 8 | 10 |
|---|---|---|---|---|---|
| $y$ | $\sqrt{3}$ | | $\sqrt{11}$ | | $\sqrt{19}$ |

a) Complete the table with the values of $y$ corresponding to $x = 4$ and $x = 8$.

**[1]**

b) Use the trapezium rule, with all the values of $y$ in the completed table, to find an approximate value for the area of $R$, giving your answer to 2 decimal places.

**[3]**

c) State, giving a reason, whether your approximate value in part (b) is an overestimate or an underestimate for the area of $R$.

**[2]**

**Total: 69 Marks**