Godalming College

Pure 19– Parametrics

Please **complete** this homework by _____. Start it early. If you can't do a question you will then have time to ask your teacher for help or go to a drop in session.

Section 1 – Review of previous topics. Please complete all questions.

1. Given that the binomial expansion of $(1 + kx)^{-4}$, $|kx| < 1$, is

$$1 - 6x + Ax^2 + \ldots$$

(*a*) find the value of the constant k,

(*b*) find the value of the constant A, giving your answer in its simplest form.

2. (*a*) Express $2\cos\theta - \sin\theta$ in the form $R\cos(\theta + \alpha)$, where R and α are constants, $R > 0$ and $0 < \alpha < 90°$ Give the exact value of R and give the value of α to 2 decimal places.

(*b*) Hence solve, for $0 \leq \theta < 360°$,

$$\frac{2}{2\cos\theta - \sin\theta - 1} = 15.$$

Give your answers to one decimal place.

(*c*) Use your solutions to parts (*a*) and (*b*) to deduce the smallest positive value of θ for which

$$\frac{2}{2\cos\theta + \sin\theta - 1} = 15.$$

Give your answer to one decimal place.

3. (*a*) Express $4\,\text{cosec}^2 2\theta - \text{cosec}^2\theta$ in terms of $\sin\theta$ and $\cos\theta$.

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(b) Hence show that

$$4\,\text{cosec}^2 2\theta - \text{cosec}^2\theta = \sec^2\theta .$$

(c) Hence or otherwise solve, for $0 < \theta < \pi$,

$$4\,\text{cosec}^2 2\theta - \text{cosec}^2\theta = 4$$

giving your answers in terms of π.

Section 2

Exercise

1. Find the Cartesian equations of the curves given by
   (i) $x = 1 - t,\ y = t^2 - 4$
   (ii) $x = 2t^2,\ y = \dfrac{1}{t}$
   (iii) $x = 2\cos\theta + \sin\theta,\ y = \cos\theta - 2\sin\theta$ [9]

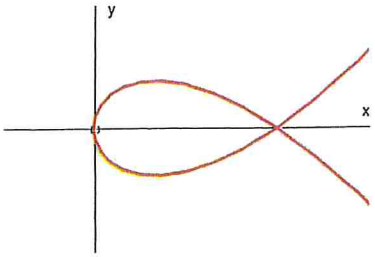
2. A curve has parametric equations $x = t^2, y = t^3$.
   (i) Calculate values for x and y for values of t between –3 and +3.
   (ii) Sketch the curve.
   (iii) Find the Cartesian equation of the curve. [10]

3. A curve has parametric equations $x = 3\cos\theta,\ y = 2\sin\theta$.
   (i) Calculate values for x and y for values of θ from 0 to π, at intervals of $\dfrac{\pi}{12}$.
   (ii) Using what you know about angles greater than π calculate values for x and y for θ from π to 2π.
   (iii) Sketch the curve.
   (iv) Find the Cartesian equation of the curve. [16]

4. A curve has parametric equations $x = t + t^2,\ y = t - t^2$.
   (i) Find the values of t for which the curve meets the x-axis. [3]

5. Given the parametric equations $x = t - \dfrac{1}{t},\ y = 2\left(t + \dfrac{1}{t}\right)$,
   (i) For what value of t is the curve undefined?
   (ii) Find the coordinates of any points where the curve meets the coordinate axes.
   (iii) Find the Cartesian equation. [11]

6. The curve shown below has parametric equations $x = 4t^2$, $y = 2t(1-t^2)$.



(i) By eliminating t, find the Cartesian equation of the curve. (3)

7. A ball is struck at ground level and projected with a speed of 16 ms^{-1} at an angle θ to the horizontal. The parametric equations of the path of the ball are given by $x = 16t\cos\theta$, $y = 16t\sin\theta - 5t^2$.

(i) By eliminating t show that the Cartesian equation of the path can be written as a quadratic in $\tan\theta$.

Given that $\theta = 30°$,

(ii) How far does the ball travel horizontally before bouncing? (9)

TOTAL: 61