Pure 20 – Parametric Calculus

Please **complete** this homework by ___________. Start it early. If you can't do a question you will then have time to ask your teacher for help or go to a drop-in session.

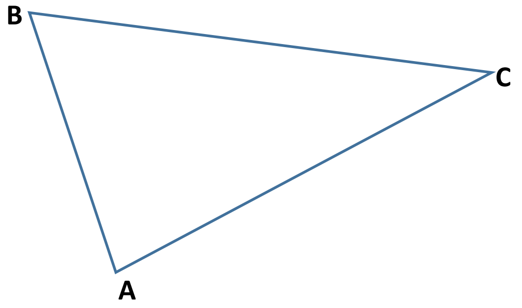
Section 1 – Review of previous topics. Please complete all questions.

1. Harry hits a ball from point A on a horizontal surface. The motion of the ball is modelled as that of a partical travelling with constatnt velocity $(3\boldsymbol{i} + 11\boldsymbol{j})ms^{-1}$

a) Find the speed of the ball

b) Find the distance of the ball from A after 7 seconds

c) Comment on the validity of this model for large values of t.



2. In trangle ABC, $\overrightarrow{AB} = -2\boldsymbol{i} + 6\boldsymbol{j}$ and $\overrightarrow{AC} = 6\boldsymbol{i} + 3\boldsymbol{j}$, find:

a) $\overrightarrow{BC}$

b) $B\hat{A}C$

c) the area of the triangle

3. Two forces $\boldsymbol{F_1}$ and $\boldsymbol{F_2}$, are given by the vectors $F_1 = (3\boldsymbol{i} - 5\boldsymbol{j})N$ and $F_2 = (p\boldsymbol{i} + q\boldsymbol{j})N$. The resultatnt force, $\boldsymbol{R} = \boldsymbol{F_1} + \boldsymbol{F_2}$ acts in a direction which is parallel to the vector $(5\boldsymbol{i} - \boldsymbol{j})$

a) Find the angle between **R** and the vector $\boldsymbol{i}$

b) Show that $5q + p = 22$

c) Given that $p = 7$, find the magnitude of **R.**

4. Find the unit vector in the direction $2\boldsymbol{i} + \boldsymbol{j} - 3\boldsymbol{k}$

5. Vectors **a** and **b** are defined by $\boldsymbol{a} = \begin{pmatrix} 3 \\ -1 \\ 5 \end{pmatrix}$ and $\boldsymbol{b} = \begin{pmatrix} 4 \\ 0 \\ -3 \end{pmatrix}$

   a) Find $\boldsymbol{a} - \boldsymbol{b}$ b) Find $2\boldsymbol{a} - 3\boldsymbol{b}$

   c) State with a reason whether any of the vectors from a) or b) are parallel to $3\boldsymbol{i} + 3\boldsymbol{j} - 24\boldsymbol{k}$

6. The position vector of the point A is $-3\boldsymbol{i} + 6\boldsymbol{j} + 4\boldsymbol{k}$ and $\overrightarrow{AB} = 7\boldsymbol{i} - 8\boldsymbol{j} - \boldsymbol{k}$ and the coordinates of point C are (2, -2, -1)

Find in terms of **i**, **j** and **k**,

a) the position vectors of B and C

b) $\overrightarrow{AC}$

Find the exact value of

c) The distance between A and C

d) $|\overrightarrow{OC}|$

7. The vectors a and b are defined by $\boldsymbol{a} = \begin{pmatrix} 2 \\ -5 \\ 6 \end{pmatrix}$ and $\boldsymbol{b} = \begin{pmatrix} p \\ q \\ r \end{pmatrix}$

   Given that 2**a** – **b** = $\begin{pmatrix} 3 \\ -6 \\ 10 \end{pmatrix}$ find the values of *p*, *q* and *r*.

8. Given that $\boldsymbol{a} = 3t\boldsymbol{i} - 12t\boldsymbol{j} + 4t\boldsymbol{k}$ and that $|\boldsymbol{a}| = 39$ find the possible values of t

9. Find the angles that the vector $\overrightarrow{AB} = -2\boldsymbol{i} + 5\boldsymbol{j} - 3\boldsymbol{k}$ Makes with each of the positive coordinate axes to 1 d.p.

Section 2 – Consolidation of this week's topic. Please complete all questions.

1. Find $\frac{\mathrm{dy}}{\mathrm{dx}}$ in terms of the parameter t:

   a) $\mathrm{x} = \mathrm{t}^3$ $\quad$ $\mathrm{y} = \mathrm{t}$ $\quad$ b) $x = 3t - 1$ $\quad$ $\mathrm{y} = 2 - \frac{1}{\mathrm{t}}$

   c) $\mathrm{x} = \cos 2\mathrm{t}$ $\quad$ $\mathrm{y} = \sin \mathrm{t}$ $\quad$ d) $\mathrm{x} = \mathrm{e}^{\mathrm{t}+1}$ $\quad$ $\mathrm{y} = \mathrm{e}^{2\mathrm{t}-1}$ **[7]**

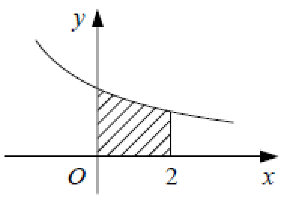
2. A curve is given by the parametric equations $\mathrm{x} = \mathrm{t} + \frac{1}{\mathrm{t}}, y = \mathrm{t} - \frac{1}{\mathrm{t}}$ $\quad$ $(t \neq 0)$.

   a) Find an equation for the tangent to the curve at the point P where $t = 3$.

   b) Show that the Cartesian equation of the curve is $x^2 - y^2 = k$ where k is a constant to be found.

   c) Show that the tangent to the curve at P does not meet the curve again. **[10]**

3.



The diagram shows part of the curve with parametric equations $x = 2t - 4, y = \frac{1}{t}$.

The shaded region is bounded by the curve, the coordinate axes and the line $x = 2$

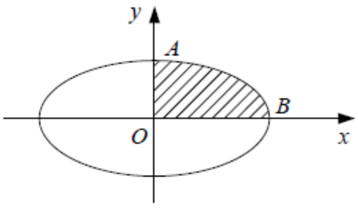
a) Find the value of the parameter t when $x = 0$ and when $x = 2$. (3 marks)

b) Show that the area of the shaded region is given by $\int_2^3 \frac{2}{t}\, dt$. (4 marks)

c) Hence, find the area of the shaded region. (3 marks)

d) Verify your answer to part c) by first finding the cartesian equation for the curve (4 marks)

4.



The diagram shows the ellipse with parametric equations $x = 4cos\theta, y = 2sin\theta$, $0 \leq \theta < 2\pi$,

which meets the positive coordinate axes at the points A and B.

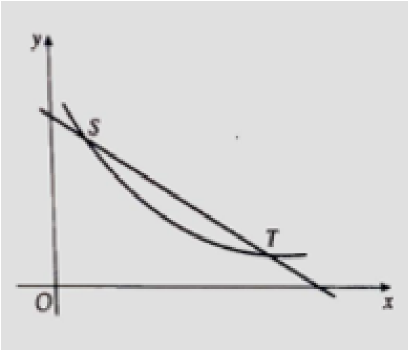
a) Find the value of the parameter θ at the points A and B (2 marks)

b) Show that the area of the shaded region bounded by the curve and the positive coordinate axes is given by $\int_0^{\frac{\pi}{2}} 8sin^2\theta \, d\theta.$ (4 marks)

c) Hence, show that the area of the region enclosed by the ellipse is 8π (4 marks)

5. The diagram shows a sketch of the curve with parametric equations $x = at, y = \frac{4a}{t}, t > 0$, and the line $y = 5a - x$, where a is a constant.

The line meets the curve at S and T.



a) Find, in terms of a, the co-ordinates of the points S and T (5 marks)

b) Show that $\int y \frac{dx}{dt} dt = 4a^2 lnt + c$, where c is a constant. (2 marks)

c) Hence find, in terms of a, the exact area of the fiite region between the curve and the line. (5 marks)

Total: 53 marks