

# PURE 4 - SOLUTIONS

## Section 1

1 a)  $\cos x = 0.9$

$$x = \underline{25.8^\circ, 334^\circ}$$

b)  $\sin x = 0.41$

$$x = \underline{24.2, 156^\circ}$$

c)  $\tan x = 0.74$

$$x = \underline{36.5^\circ, 217^\circ}$$

2 a)  $-10 \leq x - 10 \leq 350$

$$x - 10 = \sin^{-1}(0.5) = 30, 150$$

$$x = \underline{40^\circ, 160^\circ}$$

b)  $0 \leq 2x \leq 720$

$$2x = \cos^{-1}(0.5)$$

$$= 60, 300, 420, 660$$

$$x = \underline{30^\circ, 150^\circ, 210^\circ, 330^\circ}$$

3) a)  $\tan(x-15) = 1.85$

$$x-15 = 61.6, -118.4$$

$$x = \underline{76.6^\circ, -103.4^\circ}$$

b)  $\cos 3x = \frac{1}{2}$

$$3x = 60, 300, 420, -60, -300, -4$$

$$x = \underline{-140^\circ, -100^\circ, -20^\circ, 20^\circ, 100^\circ, 140^\circ}$$

c)  $2x + 30 = \tan^{-1}(-1) = -45, 135, 315, -255$

$$x = \underline{-37.5^\circ, 52.5^\circ, 142.5^\circ, -127.5^\circ}$$

4) a)  $\cos x (2\cos x + 1) = 0$

$$\cos x = 0 \quad \text{or} \quad \cos x = -\frac{1}{2}$$

$$x = \underline{90^\circ, 270^\circ, 120^\circ, 240^\circ}$$

b)  $\tan x = \frac{-3 \pm \sqrt{13}}{2}$

$$x = \underline{106.8^\circ, 286.8^\circ, 16.8^\circ, 196.8^\circ}$$

5 a)  $\sin^2 x + \cos^2 x = 1$

b)  $\tan x = \frac{\sin x}{\cos x}$

6 a)  $\cos x = 3 \frac{\sin x}{\cos x}$

$$\cos^2 x = 3 \sin x$$

$$1 - \sin^2 x = 3 \sin x$$

$$\sin^2 x + 3 \sin x - 1 = 0$$

$$\sin x = \frac{-3 \pm \sqrt{13}}{2} \quad \text{no solution for } \sin x = \frac{-3 - \sqrt{13}}{2}$$

$$x = \sin^{-1}\left(\frac{-3 + \sqrt{13}}{2}\right) = \underline{17.6^\circ, 162.4^\circ}$$

$$6b) 2(1 - \cos^2 x) + 3\cos x - 3 = 0$$

$$2 - 2\cos^2 x + 3\cos x - 3 = 0$$

$$2\cos^2 x - 3\cos x + 1 = 0$$

$$(2\cos x - 1)(\cos x - 1) = 0$$

$$\cos x = \frac{1}{2}$$

$$\cos x = 1$$

$$x = \underline{60^\circ, 300^\circ, 0^\circ, 360^\circ}$$

$$c) 3\sin x \times \frac{\sin x}{\cos x} = 8 \Rightarrow 3\sin^2 x = 8\cos x$$

$$\Rightarrow 3(1 - \cos^2 x) = 8\cos x$$

$$\Rightarrow 3 - 3\cos^2 x = 8\cos x$$

$$3\cos^2 x + 8\cos x - 3 = 0$$

$$(3\cos x - 1)(\cos x + 3) = 0$$

$$\cos x = \frac{1}{3}$$

no solution

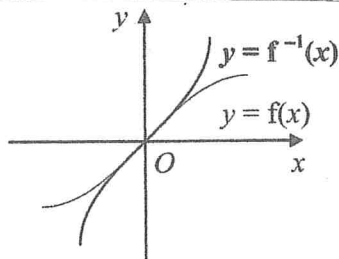
$$x = \underline{70.5^\circ, 289.5^\circ}$$

## Section 2

$$1)a) -1 \leq f(x) \leq 1$$

$$b) f^{-1}(x) \equiv \arcsin x, x \in \mathbb{R}, -1 \leq x \leq 1$$

c)



✓✓ (2) For  $\sin x$

✓✓ (2) For  $\arcsin x$

$$2a) x = \cos 2 = -0.416$$

$$b) 3x = \tan 0.96 = 1.42836 \Rightarrow x = 0.476$$

$$c) \arccos 2x = 3 \Rightarrow 2x = \cos 3 = -0.98999$$

$$x = -0.495$$

3 a)  $f(-\frac{1}{2}) = \frac{2\pi}{3} - \frac{\pi}{3} = \frac{\pi}{3}$  ✓

b)  $\arccos x = \frac{\pi}{3} \Rightarrow x = \cos \frac{\pi}{3} = \frac{1}{2}$  ✓

c)  $y = \arccos x - \frac{\pi}{3}$

$x = \arccos y - \frac{\pi}{3}$  ✓

$x + \frac{\pi}{3} = \arccos y$  ✓

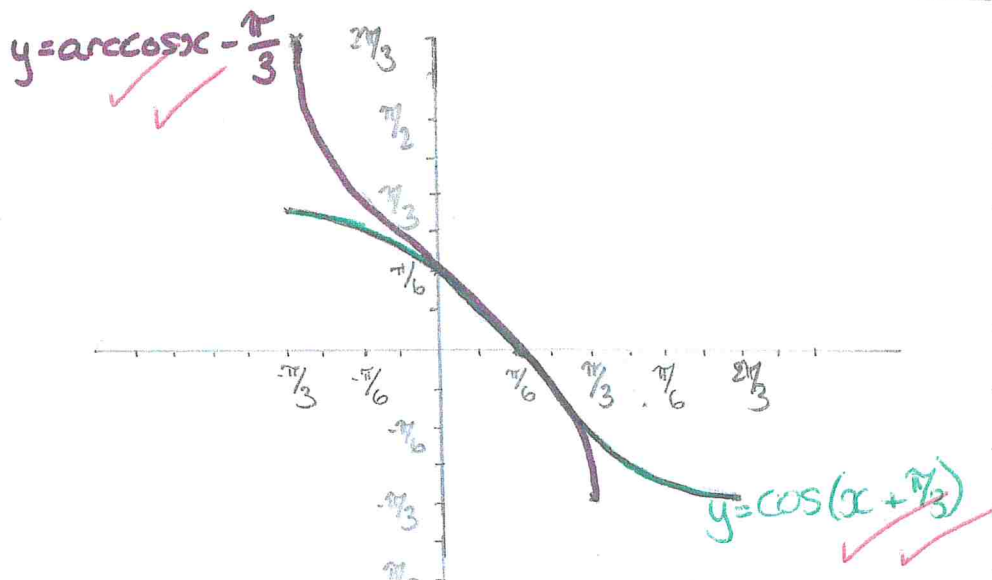
$y = \cos(x + \frac{\pi}{3})$  ✓

$f^{-1}(x) = \cos(x + \frac{\pi}{3})$  ✓

$x \in \mathbb{R}$

$-\frac{\pi}{3} \leq x \leq \frac{2\pi}{3}$  ✓

d)



4a) 1.09 ✓

b) -11.47 ✓

c) 0.19 ✓

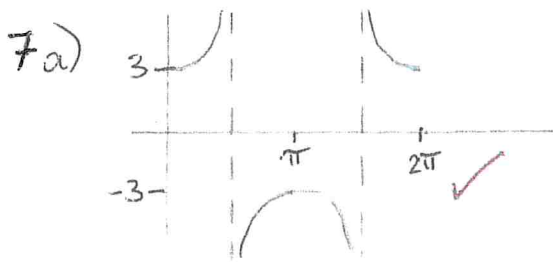
5a)  $\sqrt{2}$  ✓

b) -1 ✓

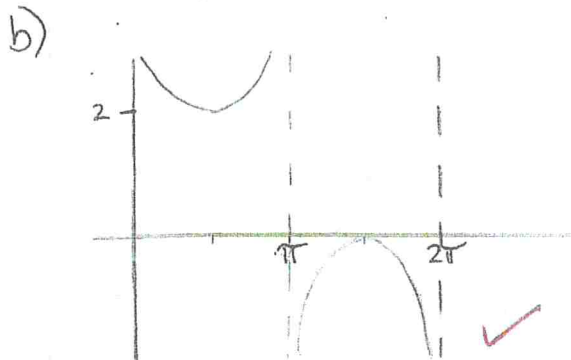
c) -2 ✓

6a)  $(0, 1), (180, -1), (360, 1), (540, -1), (720, 1)$  ✓

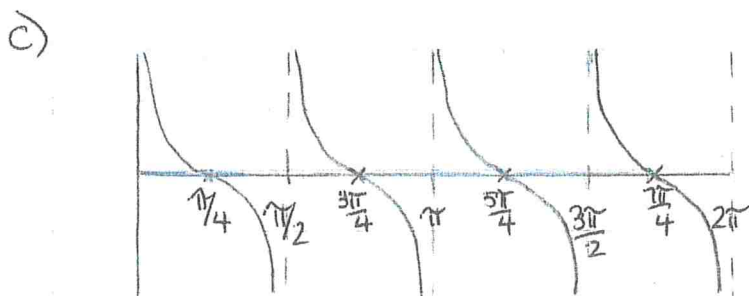
b)  $x=90, x=270, x=450, x=630$  ✓



turning points:  
 $(0, 3)$ ,  $(\pi, -3)$ ,  $(2\pi, 3)$  ✓  
 Asymptotes:  
 $x = \pi/2$  and  $x = 3\pi/2$  ✓



turning points:  
 $(\pi/2, 2)$ ,  $(3\pi/2, 0)$  ✓  
 Asymptotes  
 $x = 0$ ,  $x = \pi$ ,  $x = 2\pi$  ✓



✓ for shape  
 ✓ for axes labelled.

Asymptotes at  $x = \pi/2$ ,  $x = \pi$ ,  $x = 3\pi/2$ ,  $x = 2\pi$  ✓

8 a)  $\frac{1}{\cos \theta} = 1.8 \Rightarrow \cos \theta = 5/9$  ✓  
 $\theta = 0.982^\circ, 5.30^\circ$  ✓

b)  $\frac{1}{\sin(\theta + \pi/6)} = 2 \Rightarrow \sin(\theta + \pi/6) = 1/2$  ✓  
 $\theta + \pi/6 = \pi/6, 5\pi/6, 13\pi/6$  ✓  
 $\theta = 0, 2\pi/3, 2\pi$  ✓

c)  $\frac{1}{\sin^2 \theta} = 4 \Rightarrow \sin^2 \theta = 1/4$  ✓  
 $\sin \theta = \pm 1/2$  ✓  
 $\theta = \pi/6, 5\pi/6, 7\pi/6, 11\pi/6$  ✓

d)  $(\sec \theta - 3)(\sec \theta + 1) = 0$  ✓  
 $\sec \theta = 3$        $\sec \theta = -1$  ✓  
 $\cos \theta = 1/3$        $\cos \theta = -1$  ✓  
 $\theta = 1.23, 5.05, \pi$  ✓

$$\begin{aligned}
 9) a) \text{ LHS: } \sec x - \cos x &\equiv \frac{1}{\cos x} - \cos x \quad \checkmark \\
 &\equiv \frac{-\cos^2 x}{\cos x} \quad \checkmark \\
 &\equiv \frac{\sin^2 x}{\cos x} \quad \checkmark \\
 &\equiv \sin x \times \frac{\sin x}{\cos x} \quad \checkmark \\
 &\equiv \sin x \tan x \quad \checkmark \text{ (RHS)}
 \end{aligned}$$

$$\begin{aligned}
 b) \text{ LHS } (1 + \cos x)(\operatorname{cosec} x - \cot x) & \\
 &\equiv \operatorname{cosec} x - \cot x + \cos x \operatorname{cosec} x - \cos x \cot x \\
 &\equiv \operatorname{cosec} x - \cot x + \cos x \times \frac{1}{\sin x} - \cos x \times \frac{\cos x}{\sin x} \quad \checkmark \\
 &\equiv \frac{1}{\sin x} - \cot x + \cot x - \frac{\cos^2 x}{\sin x} \quad \checkmark \\
 &\equiv \frac{1 - \cos^2 x}{\sin x} \quad \checkmark \\
 &\equiv \frac{\sin^2 x}{\sin x} \quad \checkmark \\
 &\equiv \sin x \quad \checkmark \text{ (RHS)}
 \end{aligned}$$