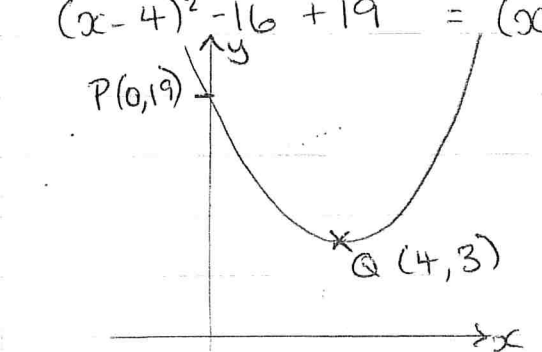


SOLUTIONS.

-COMPOUND ANGLES

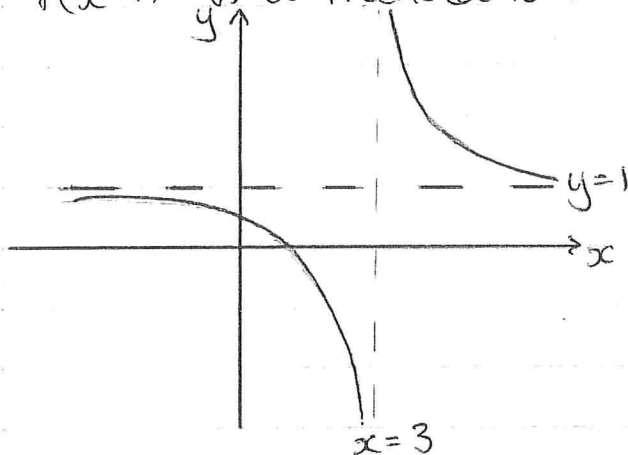
Section 1

1a) $(x-4)^2 - 16 + 19 = (x-4)^2 + 3$

b)  c) $\sqrt{(0-4)^2 + (19-3)^2}$
 $= 4\sqrt{17}$

2) Need radius: $r = \sqrt{(-1-0)^2 + (7-0)^2} = \sqrt{50}$
 $(x+1)^2 + (y-7)^2 = 50$

3) $f(x-1)$ is a translation 1 right.



c) $f(x-1) = \frac{x-1}{x-1-2} = \frac{x-1}{x-3}$

when $x=0$ $y = \frac{1}{3}$

when $y=0$ $x=1$
 $(0, \frac{1}{3})$ $(1, 0)$

4) $(\cot^2 x + 1) + \cot^2 x = 3 \Rightarrow 2\cot^2 x = 2$
 $\Rightarrow \cot^2 x = 1 \Rightarrow \tan^2 x = 1 \Rightarrow \tan x = \pm 1$

$x = \frac{\pi}{4}, \frac{3\pi}{4}, \frac{5\pi}{4}, \frac{7\pi}{4}$

5) a) $\overset{\text{LHS}}{\sec^2 x - \sin^2 x} \equiv (\tan^2 x + 1) - (1 - \cos^2 x)$
 $\equiv \tan^2 x + \cos^2 x$ RHS

b) LHS: $(\sin x - \sec x)^2 \equiv \sin^2 x - 2\sin x \sec x + \sec^2 x$
 $\equiv \sin^2 x - 2\tan x + \tan^2 x + 1$
 $\equiv \sin^2 x + \tan^2 x - 2\tan x + 1$
 $\equiv \sin^2 x + (\tan x - 1)^2$

Section 2

(1) 1a) $\sin 10 \cos 30 + \cos 10 \sin 30 = \sin (10+30) = \sin 40$ ✓

(2) b) $\cos 14 \cos 39 - \sin 14 \sin 39 = \cos (14+39) = \cos 53 = \sin 37$ ✓

(1) 2) $\tan 7A$ ✓

(3) 3a) $\cos(x-30)$ ✓ $\max = 1$ ✓ when $x = 30$ (shifted 30° right) ✓

(3) b) $3(\sin x \cos 45 + \cos x \sin 45) = 3 \sin(x+45)$ (shifted 45° left) ✓
 $\max = 3$ ✓ when $x = 45$ ✓

(3) 4a) $\sin(x - \frac{\pi}{3})$ ✓ $\min = -1$ ✓ when $x = \frac{11\pi}{6}$ (shifted $\frac{\pi}{3}$ right) ✓

b) $2(\cos x \cos \frac{\pi}{6} - \sin x \sin \frac{\pi}{6}) = 2 \cos(x + \frac{\pi}{6})$ ✓

(3) $\min = -2$ ✓ when $x = \frac{5\pi}{6}$ (shifted $\frac{\pi}{6}$ left) ✓

5a) $\sin \theta \cos 15 + \cos \theta \sin 15 = 0.4$

$\Rightarrow \sin(\theta + 15) = 0.4$ ✓

(3) $\theta + 15 = 23.6, 156.42 \Rightarrow \theta = 8.6^\circ, 141.4^\circ$ ✓✓

b) $\tan(2\theta - 60) = 1$ ✓ $(-60 \leq 2\theta - 60 \leq 660)$

(5) $2\theta - 60 = 45, 225, 405, 585$ ✓✓

$\theta = 52.5^\circ, 142.5^\circ, 232.5^\circ, 322.5^\circ$ ✓✓

c) $\cos \theta \cos 60 + \sin \theta \sin 60 = \sin \theta$ ✓

$\frac{1}{2} \cos \theta + \frac{\sqrt{3}}{2} \sin \theta = \sin \theta$ ✓

$\Rightarrow \frac{1}{2} \cos \theta = \sin \theta \left(1 - \frac{\sqrt{3}}{2}\right)$ ✓

(6) $\frac{\frac{1}{2}}{1 - \sqrt{3}/2} = \frac{\sin \theta}{\cos \theta} = \tan \theta$ ✓

$\tan \theta = 2 + \sqrt{3}$ ✓

$\theta = 75^\circ, 255^\circ$ ✓

6)a) $2(\cos x \cos 50 - \sin x \sin 50) = \sin x \cos 40 + \cos x \sin 40$ ✓

$2 \cos x \sin 40 - 2 \sin x \cos 40 = \sin x \cos 40 + \cos x \sin 40$ ✓

$\cos x \sin 40 = 3 \sin x \cos 40$ } $\div \cos x \cos 40$

$\frac{\sin 40}{\cos 40} = \frac{3 \sin x}{\cos x}$ ✓

$\tan x = \frac{1}{3} \tan 40$ ✓

(4)

$$b) \quad 2\cos(2\theta + 50) = \sin(2\theta + 40)$$

$$\Rightarrow \tan 2\theta = \frac{1}{3} \tan 40 \quad \checkmark$$

(4)

$$\Rightarrow 2\theta = 15.63, 195.63, 375.63, 555.63 \quad \checkmark$$

$$\theta = 7.8^\circ, 97.8^\circ, 187.8^\circ, 277.8^\circ \quad \checkmark$$

$$7a) \quad \tan(A+B) = \frac{\sin(A+B)}{\cos(A+B)} = \frac{\sin A \cos B + \cos A \sin B}{\cos A \cos B - \sin A \sin B} \quad \checkmark$$

(4)

$$\left(\begin{array}{l} \div \text{ all terms by} \\ \cos A \cos B \end{array} \right) = \frac{\frac{\sin A \cos B}{\cos A \cos B} + \frac{\cos A \sin B}{\cos A \cos B}}{\frac{\cos A \cos B}{\cos A \cos B} - \frac{\sin A \sin B}{\cos A \cos B}} \quad \checkmark$$

$$= \frac{\tan A + \tan B}{1 - \tan A \tan B} \quad \checkmark$$

$$b) \quad \tan\left(\theta + \frac{\pi}{6}\right) = \frac{\tan \theta + \tan \frac{\pi}{6}}{1 - \tan \theta \tan \frac{\pi}{6}} = \frac{\tan \theta + \frac{\sqrt{3}}{3}}{1 - \frac{\sqrt{3}}{3} \tan \theta} \quad \checkmark$$

(3)

$$\left(\times \text{ all terms by } \sqrt{3} \right) = \frac{\sqrt{3} \tan \theta + 1}{\sqrt{3} - \tan \theta} \quad \checkmark$$

$$c) \quad \frac{1 + \sqrt{3} \tan \theta}{\sqrt{3} - \tan \theta} = \tan(\pi - \theta)$$

(6)

$$\tan\left(\theta + \frac{\pi}{6}\right) = \tan(\pi - \theta) \quad \checkmark$$

$$\Rightarrow \theta + \frac{\pi}{6} = \pi - \theta \quad \checkmark$$

$$2\theta = \frac{5\pi}{6}, \frac{11\pi}{6} \quad \checkmark \checkmark$$

$$\theta = \frac{5\pi}{12}, \frac{11\pi}{12} \quad \checkmark \checkmark$$

Total 51 marks.

