SECTION 1

1. $y = x^2 - 8x^{-\frac{1}{2}} \quad \therefore \frac{dy}{dx} = 2x + 4x^{-3/2}$

at $x = 4 \quad \frac{dy}{dx} = 2\times 4 + 4\times 4^{-3/2} = \frac{17}{2}$

$\therefore$ gradient of normal $= -\frac{2}{17}$

at $x = 4,\ y = 12 \quad \therefore y - 12 = -\frac{2}{17}(x-4)$

$$\therefore \underline{\underline{y = \frac{-2x + 212}{17}}}$$

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2. $f(x) = 3x^2 + 8x + 2$

$\therefore f'(x) = 6x + 8$

$f(x)$ is an increasing function when $f'(x) > 0$

$\therefore \quad 6x + 8 > 0$

$\therefore \quad \underline{\underline{x > -\frac{4}{3}}}$

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3. $y = 3x^5 + 4x^{-2} \quad \therefore \frac{dy}{dx} = 15x^4 - 8x^{-3}$

$$\therefore \underline{\underline{\frac{d^2y}{dx^2} = 60x^3 + 24x^{-4}}}$$

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4. $f(x) = px^3 - 3px^2 + x^2 - 4$

$\therefore f'(x) = 3px^2 - 6px + 2x$

$f''(x) = 6px - 6p + 2$

$f''(2) = 12p - 6p + 2 = 6p + 2 = -1$

$\therefore \underline{\underline{p = -\frac{1}{2}}}$

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5. a) $f(x) = 2x^3 - 15x^2 + 24x + 6$

$f'(x) = 6x^2 - 30x + 24$

at stationary points $f'(x) = 0$

$\therefore 6x^2 - 30x + 24 = 0$

$\therefore x^2 - 5x + 4 = 0$

$\therefore (x-1)(x-4) = 0$

$\therefore x = 1, 4$

$f(1) = 2 - 15 + 24 + 6 = 17$

$f(4) = 2 \times 64 - 15 \times 16 + 24 \times 4 + 6 = -10$

Stationary points are $\underline{\underline{(1, 17) \text{ and } (4, -10)}}$

5b) $f''(x) = 12x - 30$

$f''(1) = 12 - 30 = -18 \quad \therefore$ a maximum

$f''(4) = 48 - 30 = 18 \quad \therefore$ a minimum

$(1, 17)$ is a maximum, $(4, -10)$ is a minimum

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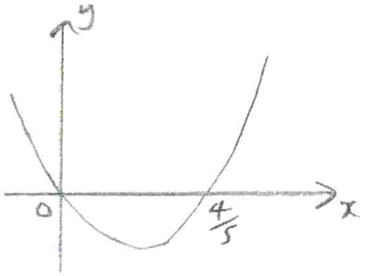
6. $5 > \frac{4}{x}$

$\therefore \quad 5x^2 > \frac{4}{x} \times x^2$

$\therefore \quad 5x^2 - 4x > 0$

$\therefore \quad x(5x - 4) > 0$

critical values are $x = 0$, $x = \frac{4}{5}$



$\therefore \quad x < 0$ or $x > \frac{4}{5}$

$\{x : x < 0\} \cup \{x : x > \frac{4}{5}\}$

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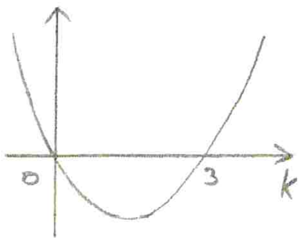
7. $kx^2 - 2kx + 3 = 0$ discriminant < 0

$\therefore \quad (-2k)^2 - 4k \times 3 < 0 \quad \therefore \quad 4k^2 - 12k < 0$

7. $\therefore \quad 4k(k-3) < 0 \quad \therefore$ critical values are $k = 0, 3$



negative when $0 < k < 3$

at $k = 0$ we get $3 = 0$ which is clearly not true
at $k = 3$ we get $3x^2 - 6x + 3 = 0 \quad \therefore (x-1)^2 = 0 \quad \therefore x = 1$
which is a real root

Hence $\underline{\underline{0 \leq k < 3}}$

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8. $x + by + c = 0 \quad \therefore \quad y = -\frac{x}{6} - \frac{c}{6}$

gradient $= \frac{4-3}{a-3a} = \frac{-1}{6} \quad \therefore \quad \underline{\underline{a = 3}}$

line passes through $(3, 4)$ and $(9, 3)$

$\therefore \quad y - 4 = -\frac{1}{6}(x - 3)$

$\therefore \quad y = -\frac{x}{6} + \frac{9}{2}$

$\therefore \quad -\frac{c}{6} = \frac{9}{2}$

$\therefore \quad \underline{\underline{c = -27}}$

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9. $3x + 8y - 11 = 0 \quad \therefore \quad y = -\frac{3x}{8} + \frac{11}{8}$

$\therefore$ gradient $= -3/8$

$\therefore$ gradient of perpendicular is $8/3$

$$\therefore \quad y + 8 = \frac{8x}{3}$$

$$\therefore \quad \underline{\underline{3y - 8x + 24 = 0}}$$

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10. for point P $\quad x^2 + y^2 = 34$

$$\therefore \quad x^2 + (4 - 3x)^2 = 34$$

$$\therefore \quad 10x^2 - 24x - 18 = 0$$

$$\therefore \quad 5x^2 - 12x - 9 = 0$$

$$\therefore \quad (5x + 3)(x - 3) = 0$$

$$\therefore \quad x = -3/5,\ 3$$

$x = -3/5 \quad y = 4 - -\frac{3}{5} \times 3 = 29/5$

$x = 3 \quad y = 4 - 3 \times 3 = -5$

$$\underline{\underline{\left(-3/5,\ \tfrac{29}{5}\right) \text{ and } (3, -5)}}$$

SECTION 2

1. a) $2x e^{x^2}$ b) $2\sin x \cos x$ c) $2\cos x \, e^{2\sin x}$

d) $f(x) = \sin x^{\frac{1}{2}} + (\sin x)^{\frac{1}{2}}$

$\therefore f'(x) = \left(\frac{1}{2}x^{-\frac{1}{2}}\right)\cos x^{\frac{1}{2}} + \frac{1}{2}(\sin x)^{-\frac{1}{2}} \cos x$

$\therefore f'(x) = \dfrac{1}{2\sqrt{x}} \cos\sqrt{x} + \dfrac{\cos x}{2\sqrt{\sin x}}$

e) $\dfrac{6}{3x+5}$ 6

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2. a) $\dfrac{dy}{dx} = 3(3x^2+1)^2 \times 6x \times 5x + 5(3x^2+1)^3$

$= 90x^2(3x^2+1)^2 + 5(3x^2+1)^3$

$= 5(3x^2+1)^2(21x^2+1)$

b) $f'(x) = 4\cos 4x \cos 5x - 5\sin 4x \sin 5x$

c) $\dfrac{dy}{dx} = \dfrac{\sin 3x}{x} + 3\cos 3x \ln 2x$ 7

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3 a) $\dfrac{dy}{dx} = \dfrac{(x^2-1) - x(2x)}{(x^2-1)^2} = \dfrac{-x^2-1}{(x^2-1)^2}$

3b) $f'(x) = \dfrac{e^x \cos x + e^x \sin x}{\cos^2 x}$

c) $\dfrac{dy}{dx} = \dfrac{(\sqrt{x}+1)(\frac{1}{2}x^{-\frac{1}{2}}) - \sqrt{x}(\frac{1}{2}x^{-\frac{1}{2}})}{(\sqrt{x}+1)^2}$

$= \dfrac{1}{2\sqrt{x}(\sqrt{x}+1)^2}$

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4.a) $\dfrac{dy}{dx} = 2xe^{-3x} - 3x^2e^{-3x} = xe^{-3x}(2-3x)$

b) $f'(x) = (\frac{1}{2}x^{-\frac{1}{2}})\ln 3x + \dfrac{x^{\frac{1}{2}}}{x} = \dfrac{1}{\sqrt{x}}(1 + \frac{1}{2}\ln 3x)$

c) $y = x\ln\left(\dfrac{x-1}{x+1}\right) = x\ln(x-1) - x\ln(x+1)$

$\therefore \dfrac{dy}{dx} = \dfrac{x}{x-1} + \ln(x-1) - \dfrac{x}{x+1} - \ln(x+1)$

$= \dfrac{2x}{x^2-1} + \ln\left(\dfrac{x-1}{x+1}\right)$

d) $f'(x) = \dfrac{-\sin x}{\cos x} = -\tan x$

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5. $y = xe^{x^2}$

a) $\dfrac{dy}{dx} = e^{x^2} + 2x^2 e^{x^2} = e^{x^2}(1 + 2x^2)$

$x = 1 \quad \dfrac{dy}{dx} = 3e$

$y = e$

$\therefore \quad y - e = 3e(x - 1)$

$\therefore \quad \underline{y = 3ex - 2e}$

b) cut x axis $\Rightarrow y = 0 \quad \therefore \quad x = 2/3 \quad \underline{(2/3, 0)}$

cut y axis $\Rightarrow x = 0 \quad \therefore \quad y = -2e \quad \underline{(0, -2e)}$

c) Area $= \frac{1}{2} \times 2/3 \times 2e = \underline{\dfrac{2e}{3}}$

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6. crosses the x axis at $x = -3, 1$

$\underline{P\ (-3, 0)} \qquad \underline{Q\ (1, 0)}$

$\dfrac{dy}{dx} = (x-1)^3 + 3(x+3)(x-1)^2$

$= 4(x-1)^2(x+2)$

6. at R $\frac{dy}{dx} = 0$ $\therefore$ $x = 1, -2$

$$\frac{d^2y}{dx^2} = 8(x-1)(x+2) + 4(x-1)^2$$

$$= 12(x-1)(x+1)$$

$x = 1 \Rightarrow \frac{d^2y}{dx^2} = 0$

$x = -2 \Rightarrow \frac{d^2y}{dx^2} = 36$ ie a minimum

$\therefore$ R is a minimum with coordinates $(-2, -27)$

(11)

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7. a) $f(x) = x(2x+12)^{\frac{1}{2}}$

$$f'(x) = (2x+12)^{\frac{1}{2}} + \tfrac{1}{2}x(2x+12)^{-\frac{1}{2}} \times 2$$

$$= (2x+12)^{\frac{1}{2}} + x(2x+12)^{-\frac{1}{2}}$$

b) $f''(x) = \tfrac{1}{2}(2x+12)^{-\frac{1}{2}}(2) + (2x+12)^{-\frac{1}{2}} - \tfrac{1}{2}x(2x+12)^{-\frac{3}{2}}(2)$

$$= 2(2x+12)^{-\frac{1}{2}} - x(2x+12)^{-\frac{3}{2}}$$

$$= (2x+12)^{-\frac{1}{2}}\left(2 - x(2x+12)^{-1}\right)$$

$$= (2x+12)^{-\frac{1}{2}}\left(\frac{2(2x+12) - x}{2x+12}\right)$$

7. $$f''(x) = \frac{4x + 24 - x}{(2x+12)^{3/2}}$$

$$= \frac{3x + 24}{(2x+12)^{3/2}}$$

$$\therefore \quad f''(x) = \frac{3(x+8)}{(2x+12)^{3/2}}$$

c) $f'(x) = 0 \quad \therefore \quad (2x+12)^{1/2} = -x\,(2x+12)^{-1/2}$

$$\therefore \quad 2x + 12 = -x$$

$$\therefore \quad x = -4$$

$f(-4) = -4\sqrt{-2\times 4 + 12} = -8 \Rightarrow (-4, -8)$

$f''(-4) = \dfrac{3(-4+8)}{(2\times -4 + 12)^{3/2}} = \dfrac{3}{2} > 0 \quad \therefore$ a minimum

turning point is at $(-4, -8)$ and is a minimum

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TOTAL 61