# SECTION 1

1. $\frac{dy}{dx} = 3x^2 - 6x + 3$

stationary where $\frac{dy}{dx} = 0$ $\therefore$ $x^2 - 2x + 1 = 0$

$\therefore$ $(x-1)(x-1) = 0$

$\frac{d^2y}{dx^2} = 6x - 6$

$\therefore$ $x = 1$

at $x = 1$ $\frac{d^2y}{dx^2} = 6\times1 - 6 = 0$ $\therefore$ $x = 1$ is a possible point of inflection.

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2. $\frac{dy}{dx} = -\frac{1}{x^2} + 81x^2$

stationary points at $-\frac{1}{x^2} + 81x^2 = 0$

$\therefore$ $x = \pm \frac{1}{3}$ ie $a = \frac{1}{3}$

$\frac{d^2y}{dx^2} = \frac{2}{x^3} + 162x$

$x = \frac{1}{3}$ $\frac{d^2y}{dx^2} = 2\times27 + 162\times\frac{1}{3} = 108 > 0$ $\therefore$ minimum

$x = -\frac{1}{3}$ $\frac{d^2y}{dx^2} = -2\times27 - 162\times\frac{1}{3} = -108 < 0$ $\therefore$ maximum

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3. midpoint of $AB = \left(\frac{-4+2}{2}, \frac{6+8}{2}\right) = (-1, 7)$

gradient of $AB = \frac{8-6}{2--4} = \frac{1}{3}$

$\therefore$ gradient of perpendicular $= -3$

$\Rightarrow \quad y - 7 = -3(x+1)$

$\therefore \quad \underline{\underline{y = -3x + 4}}$

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4. midpoint $(6, 2)$ gradient $= \frac{12}{-2} = -6$

$\therefore$ gradient of perpendicular $= \frac{1}{6}$

$y - 2 = \frac{1}{6}(x - 6)$

$\therefore \quad y = \frac{x}{6} + 1$

at $Q$ $y = 0$ $\therefore$ $x = -6$ $\qquad \underline{\underline{(-6, 0)}}$

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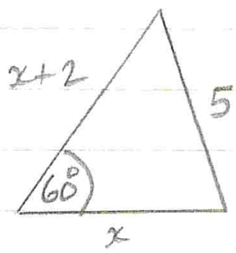
5.




Cosine Rule: $5^2 = x^2 + (x+2)^2 - 2x(x+2)\cos 60$

$\therefore \; 25 = 2x^2 + 4x + 4 - 2x(x+2)\cdot\frac{1}{2}$

$\therefore \quad x^2 + 2x - 21 = 0$

$\therefore \quad x = \frac{-2 \pm \sqrt{4 + 4 \times 21}}{2}$

$\therefore \quad x = -1 \pm \sqrt{22} = \underline{\underline{3.69}}$

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6. $\sin^2\theta + \cos^2\theta = 1$

$\therefore \cos^2\theta = 1 - \left(\frac{3}{5}\right)^2 = \frac{16}{25}$

$\therefore \underline{\cos\theta = \frac{4}{5}}$

$\tan\theta = \frac{\sin\theta}{\cos\theta} = \frac{3/5}{4/5} = \underline{3/4}$

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7. a) $\sin^2 3\theta + \cos^2 3\theta = \underline{1}$

b) $\frac{\sin 2\theta}{\sqrt{1-\sin^2 2\theta}} = \frac{\sin 2\theta}{\cos 2\theta} = \underline{\tan 2\theta}$

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8. $\sin\theta = 5/7 \qquad 0 < \theta \leq 360$

$\underline{\theta = 45.6^\circ,\ 134.4^\circ}$

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9. $\sin(3\theta - 45) = \frac{1}{2} \qquad 0 \leq \theta \leq 180$

$\therefore 3\theta - 45 = \sin^{-1}\left(\frac{1}{2}\right) \qquad -45 \leq 3\theta - 45 \leq 495$

$\therefore 3\theta - 45 = 30,\ 150,\ 390,$

$\therefore \underline{\theta = 25^\circ,\ 65^\circ,\ 145^\circ}$

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10. $2\sin^2\theta = 3 - 3\cos\theta \qquad 0 \leq \theta \leq 180$

$2(1-\cos^2\theta) = 3 - 3\cos\theta$

$\therefore\ 2\cos^2\theta - 3\cos\theta + 1 = 0$

let $c = \cos\theta \quad \therefore\ 2c^2 - 3c + 1 = 0$

$\therefore\ (2c-1)(c-1) = 0$

$\therefore\ c = \frac{1}{2},\ 1$

$\therefore\ \cos\theta = \frac{1}{2},\ 1$

$\cos\theta = \frac{1}{2} \qquad \theta = 60^\circ,$

$\cos\theta = 1 \qquad \theta = 0^\circ$

$\underline{\underline{\theta = 0^\circ, 60^\circ}}$

## SECTION 2

1 a) $y = \frac{2}{\cos x}$ $\therefore \frac{dy}{dx} = \frac{2\sin x}{\cos^2 x} = \underline{\underline{2\tan x \sec x}}$

b) $f(x) = \frac{1}{\sin 4x}$ $\therefore f'(x) = -\frac{4\cos 4x}{\sin^2 4x} = \underline{\underline{-4\cot 4x \operatorname{cosec} 4x}}$

c) $y = \frac{5}{\tan\frac{\pi x}{3}}$ $\therefore \frac{dy}{dx} = -\frac{\frac{5\pi}{3}\sec^2\frac{\pi x}{3}}{\tan^2\frac{\pi x}{3}} = \underline{\underline{-\frac{5\pi}{3}\operatorname{cosec}^2\frac{\pi x}{3}}}$

d) $f(x) = \frac{3}{\cos(x-3)}$ $\therefore f'(x) = \frac{3\sin(x-3)}{\cos^2(x-3)} = \underline{\underline{3\tan(x-3)\sec(x-3)}}$

e) $y = \frac{1}{\tan(2x-3)}$ $\therefore \frac{dy}{dx} = \frac{-2\sec^2(2x-3)}{\tan^2(2x-3)} = \underline{\underline{-2\operatorname{cosec}^2(2x-3)}}$

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2. $y = e^x \cot x$ $\therefore \frac{dy}{dx} = -e^x \operatorname{cosec}^2 x + e^x \cot x$

$= e^x\left(\frac{\cos x}{\sin x} - \frac{1}{\sin^2 x}\right)$

$= \frac{e^x}{\sin^2 x}(\sin x \cos x - 1)$

$\therefore \frac{dy}{dx} = 0$ only if $\sin x \cos x = 1$

$\therefore \sin 2x = 2$

Since we cannot have $\sin 2x > 1$ we cannot have $\frac{dy}{dx} = 0$ and $\therefore$ there are no turning points.

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3 a) $x = \tan^2 y \quad \therefore \quad \dfrac{dx}{dy} = 2\tan y \sec^2 y$

$\sin^2 y + \cos^2 y = 1 \quad \therefore \quad \tan^2 y + 1 = \sec^2 y$

$$\therefore \quad \frac{dx}{dy} = 2\tan y\,(1 + \tan^2 y)$$

$$= 2\sqrt{x}\,(1 + x)$$

$$\therefore \quad \frac{dy}{dx} = \underline{\underline{\frac{1}{2\sqrt{x}\,(1 + x)}}}$$

b) at $y = \frac{\pi}{4} \quad x = \tan^2 \frac{\pi}{4} = 1 \quad \therefore \quad \dfrac{dy}{dx} = \frac{1}{4}$

$\therefore$ gradient of normal $= -4$

$$y - \frac{\pi}{4} = -4(x - 1)$$

$$\therefore \quad \underline{\underline{y = -4x + 4 + \frac{\pi}{4}}}$$

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4 a) $y = \dfrac{1}{\cos^2 2x} \quad \therefore \quad \dfrac{dy}{dx} = \dfrac{-2}{\cos^3 2x}(-2\sin 2x) = \underline{\underline{4\tan 2x \sec^2 2x}}$

b) $f(x) = \dfrac{1}{\tan^3 x} \quad \therefore \quad f'(x) = \dfrac{-3}{\tan^4 x}(\sec^2 x) = \underline{\underline{-3\cot^2 x \operatorname{cosec}^2 x}}$

c) $y = \dfrac{1}{\sin^2(2x+1)} \quad \therefore \quad \dfrac{dy}{dx} = \dfrac{-2\,(2\cos(2x+1))}{\sin^3(2x+1)} = \underline{\underline{-4\cot(2x+1)\operatorname{cosec}^2(2x+1)}}$

d) $f'(x) = \dfrac{4\sec^2 4x}{\tan 4x} = 4\sec^2 4x \cot 4x = \underline{\underline{8 \operatorname{cosec} 8x}}$

e) $\frac{dy}{dx} = 3\cos 3x \, e^{\sin 3x}$

(10)

5. $y = \operatorname{cosec}\left(x - \frac{\pi}{6}\right) \quad \therefore \frac{dy}{dx} = -\frac{\cos\left(x-\frac{\pi}{6}\right)}{\sin^2\left(x-\frac{\pi}{6}\right)}$

a) at $P$ $\quad x = 0 \quad \therefore \frac{dy}{dx} = -\frac{\cos\left(-\frac{\pi}{6}\right)}{\sin^2\left(-\frac{\pi}{6}\right)} = \frac{-\sqrt{3}/2}{1/4} = -2\sqrt{3}$

$y = \operatorname{cosec}\left(-\frac{\pi}{6}\right) = -2$

$\therefore$ gradient of normal at $P = \frac{+1}{2\sqrt{3}}$

$\therefore y + 2 = \frac{+1}{2\sqrt{3}} x$

$\therefore y = \frac{x}{2\sqrt{3}} - 2$

b) at $Q$ $\quad x = \frac{\pi}{3} \quad \therefore y = \operatorname{cosec}\left(\frac{\pi}{3} - \frac{\pi}{6}\right) = 2$

$\frac{dy}{dx} = -\frac{\cos\left(\frac{\pi}{6}\right)}{\sin^2\left(\frac{\pi}{6}\right)} = \frac{-\sqrt{3}/2}{1/4} = -2\sqrt{3}$

$y - 2 = -2\sqrt{3}\left(x - \frac{\pi}{3}\right)$

$y = -2\sqrt{3}x + 2 + \frac{2\pi\sqrt{3}}{3}$

5c) at R : $\frac{x}{2\sqrt{3}} - 2 = -2\sqrt{3}\,x + 2 + 2\pi\frac{\sqrt{3}}{3}$ ✓

$\therefore\ x\left(2\sqrt{3} + \frac{1}{2\sqrt{3}}\right) = 4 + 2\pi\frac{\sqrt{3}}{3}$

$\therefore\ x\left(\frac{(2\sqrt{3})^2 + 1}{2\sqrt{3}}\right) = \frac{12 + 2\pi\sqrt{3}}{3}$

$\therefore\ \frac{13x}{2\sqrt{3}} = \frac{12 + 2\pi\sqrt{3}}{3}$ ✓

$\therefore\ x = \frac{2\sqrt{3}}{13}\left(\frac{12 + 2\pi\sqrt{3}}{3}\right)$

$\therefore\ x = \frac{8\sqrt{3} + 4\pi}{13}$ ✓ 11