

(1)

PURE 13 SECTION 1

$$1. \int 2x^{-3} - 3x^{\frac{1}{2}} dx = \underline{\underline{-x^{-2} - 2x^{\frac{3}{2}}}} (+c)$$

$$2. \int 4x^2 + x^{-\frac{3}{2}} + 5x^{-2} dx = \underline{\underline{\frac{4x^3}{3} - 2x^{-\frac{1}{2}} - 5x^{-1}}} (+c)$$

$$3. f(x) = \int f'(x) dx = \int 9x^2 + 4x - 3 dx$$

$$= 3x^3 + 2x^2 - 3x + c$$

$$f(-1) = 0 \quad \therefore -3 + 2 + 3 + c = 0$$

$$\therefore c = -2$$

$$\underline{\underline{f(x) = 3x^3 + 2x^2 - 3x - 2}}$$

$$4. \int_1^3 x^2 + 2x dx = \left[\frac{x^3}{3} + x^2 \right]_1^3 = \underline{\underline{\frac{50}{3}}}$$

$$5. \int_1^4 2x^{-2} + x^{-\frac{3}{2}} dx = \left[-2x^{-1} - 2x^{-\frac{1}{2}} \right]_1^4 = \underline{\underline{\frac{5}{2}}}$$

$$6. \int_1^k x^{-\frac{1}{2}} dx = \left[2x^{\frac{1}{2}} \right]_1^k = 2\sqrt{k} - 2 = 3$$

$$\therefore \underline{\underline{k = \frac{25}{4}}}$$

$$7. \text{ Area} = \int_{-4}^1 (4 - 3x - x^2) dx = \left[4x - \frac{3x^2}{2} - \frac{x^3}{3} \right]_{-4}^1 = \underline{\underline{\frac{125}{6}}}$$

$$8. y = x(x^2 - 5x + 6) = x(x-2)(x-3)$$

\therefore roots at $x = 0, 2, 3$

Total area = area from $x = 0$ to $x = 2$ plus
area from $x = 2$ to $x = 3$

$$\text{Area} = \int_{x=0}^2 (x^3 - 5x^2 + 6x) dx = \left[\frac{x^4}{4} - \frac{5x^3}{3} + 3x^2 \right]_0^2 = \frac{8}{3}$$

$$\text{Area} = \int_{x=2}^3 (x^3 - 5x^2 + 6x) dx = \left[\frac{x^4}{4} - \frac{5x^3}{3} + 3x^2 \right]_2^3 = -\frac{5}{12}$$

$$\therefore \text{ Total area} = \frac{8}{3} + \frac{5}{12} = \underline{\underline{\frac{37}{12}}}$$

9. at points of intersection

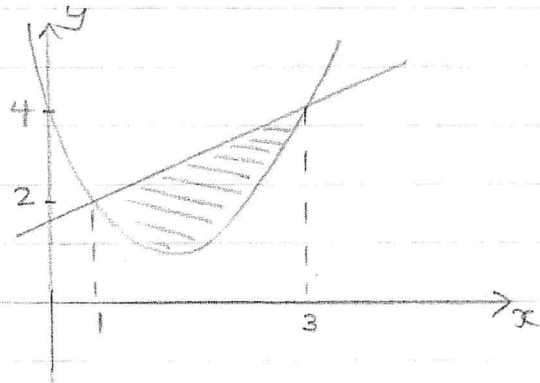
$$x^2 - 3x + 4 = x + 1$$

$$\therefore x^2 - 4x + 3 = 0$$

$$\therefore (x-3)(x-1) = 0$$

$$\therefore x = 3, 1$$

∴ points of intersection are $(1, 2)$, $(3, 4)$

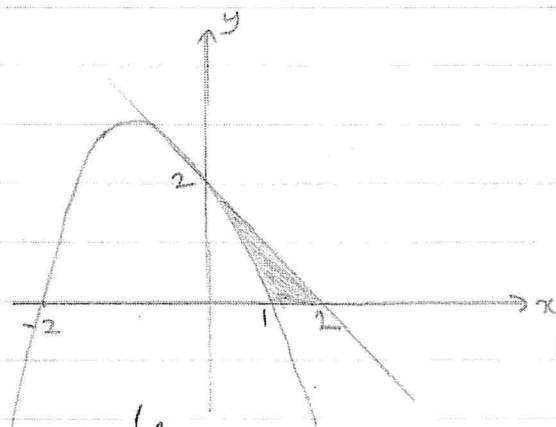


Area of trapezium
 $= \frac{1}{2} (2+4) = 6$

Area under curve = $\int_1^3 x^2 - 3x + 4 \, dx = \left[\frac{x^3}{3} - \frac{3x^2}{2} + 4x \right]_1^3$
 $= \frac{14}{3}$

∴ Required area = $6 - \frac{14}{3} = \underline{\underline{\frac{4}{3}}}$

10. $y = 2 - x - x^2$ ∴ at $y = 0$ $x^2 + x - 2 = 0$
 $\therefore (x+2)(x-1) = 0$
 $\therefore x = -2, 1$ (roots)



$\frac{dy}{dx} = -1 - 2x$

on y-axis $x = 0$ ∴ $\frac{dy}{dx} = -1$

ie tangent has gradient = -1 and passes through (0, 2)

Area under curve = $\int_0^1 2 - x - x^2 \, dx$

$= \left[2x - \frac{x^2}{2} - \frac{x^3}{3} \right]_0^1 = \frac{7}{6}$

∴ equation of tangent is $y - 2 = -x$

∴ $y = 2 - x$

∴ x intercept is (2, 0)

∴ Shaded area = $2 - \frac{7}{6} = \underline{\underline{\frac{5}{6}}}$ ∴ Area of triangle = $\frac{1}{2} \times 2 \times 2 = 2$

Pure 20 – Section 2

1. (a) e^x ✓ (b) $\ln|x|$ ✓ (c) $\sin x$ ✓ (d) $\tan x$ ✓ (e) $-\operatorname{cosec} x$ ✓

2. (a) $5x - 3\ln x$ ✓ ✓ (b) $\frac{2e^x}{5} + \frac{x}{5}$ ✓ ✓ (c) $3x + \ln x$ ✓ ✓

(d) $\int \frac{\sin x}{\cos^2 x} \cdot dx = \int \sec x \tan x \cdot dx = \sec x$ ✓ ✓

(e) $\int \frac{\cos x}{\sin^2 x} \cdot dx = \int \operatorname{cosec} x \cot x \cdot dx = -\operatorname{cosec} x$ ✓ ✓

3. (a) $\int (x-2)^7 \cdot dx = \frac{1}{8}(x-2)^8$ ✓ ✓

(b) $\int (2x+5)^3 \cdot dx = \frac{1}{4 \times 2}(2x+5)^4 = \frac{1}{8}(2x+5)^4$ ✓ ✓

(c) $\int (4x-1)^{\frac{1}{2}} \cdot dx = \frac{1}{\frac{3}{2} \times 4}(4x-1)^{\frac{3}{2}} = \frac{1}{6}(4x-1)^{\frac{3}{2}}$ ✓ ✓

(d) $\int (\frac{x}{4}-2)^5 \cdot dx = \frac{1}{6 \times \frac{1}{4}}(\frac{x}{4}-2)^6 = \frac{2}{3}(\frac{x}{4}-2)^6$ ✓ ✓

(e) $\int 5(3-2x)^{-2} \cdot dx = \frac{1}{-1 \times -2} \times 5(3-2x)^{-1} = \frac{5}{2}(3-2x)^{-1}$ ✓ ✓

4. (a) $\frac{1}{2} \int \frac{2}{2x-1} \cdot dx = \frac{1}{2} \ln|2x-1|$ ✓ ✓

(b) $\frac{2}{3} \int \frac{3}{3x+5} \cdot dx = \frac{2}{3} \ln|3x+5|$ ✓ ✓

(c) $-\frac{3}{7} \int \frac{-7}{2-7x} \cdot dx = -\frac{3}{7} \ln|2-7x|$ ✓ ✓

5. (a) $\frac{1}{5} \sin(5x-2) + 2e^{x+3}$ ✓ ✓

(b) $-\frac{1}{2} e^{5-2x} + \frac{1}{3} \tan 3x$ ✓ ✓

(c) $(e^{2x} + 1)^2 = e^{4x} + 1 + 2e^{2x}$
Hence $-\frac{5}{2} \cos(2x+3) + \frac{3}{5} \ln|5x-1| + \frac{1}{4} e^{4x} + x + e^{2x}$ ✓ ✓

6. (a) 0.75 ✓

(b) $0.75 \times (4.25^2 - 4 \times 4.25 + 5) = 4.55$ (2 d.p.) ✓ ✓

(c) the limit of the sum of the areas of rectangles as the width tends to zero..so the exact area under the curve

(d) $\int_2^5 x^2 - 4x + 5 \cdot dx = [\frac{x^3}{3} - 2x^2 + 5x]_2^5 = 12$ ✓ ✓

TOTAL 52 MARKS