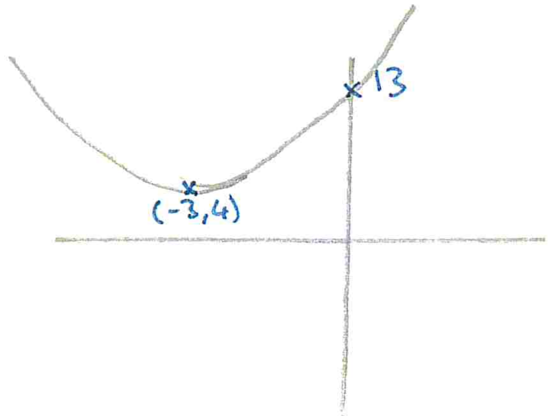
section 1

1) a) $x^2+6x+13 = (x+3)^2-9+13$

$= (x+3)^2+4$

b)



2) a) when $t=0$, $M=50$

b) Mass will be 25 after one half-life

$25 = 50e^{-0.3t}$

$\frac{1}{2} = e^{-0.3t}$

$\ln\frac{1}{2} = -0.3t$

$\ln\frac{1}{2} \div -0.3 = t$

$2.3 = t$

the half life is 2.3 days.

3 a) $f(x) = \frac{x}{x-3}$

$y = \frac{x}{x-3}$

$y(x-3) = x$

$yx - 3y = x$

$yx - x = 3y$

$x(y-1) = 3y$

$x = \frac{3y}{y-1}$

$\therefore f^{-1}(x) = \frac{3x}{x-1}$

b) $gf(x) = g\left(\frac{x}{x-3}\right)$

$$= \frac{5\left(\frac{x}{x-3}\right) - 2}{\frac{x}{x-3}} \qquad \times \frac{x-3}{x-3}$$

$$= \frac{5x - 2(x-3)}{x}$$

$$= \frac{5x - 2x + 6}{x}$$

$$= \frac{3x+6}{x}$$

c) $f^{-1}(x) = gf(x)$

$$\frac{3x}{x-1} = \frac{3x+6}{x}$$

$$3x^2 = (3x+6)(x-1)$$

$$3x^2 = 3x^2 + 3x - 6$$

$$0 = 3x - 6$$

$$x = 2$$

4 a) $u_1 = 3$

$u_2 = 4 - 3k$

b) $u_3 = 4 - k(4 - 3k)$

$= 4 - 4k + 3k^2$

c) $3 + 4 - 3k + 4 - 4k + 3k^2 = 9$

$3k^2 - 7k + 2 = 0$ $\qquad$ [$3 \times 2 = 6$ and $-1 \times -6 = 6$

$3k^2 - k - 6k + 2 = 0$ $\qquad$ $-1 + -6 = -7$

$k(3k-1) - 2(3k-1) = 0$

$(k-2)(3k-1) = 0$

$k = 2$ or $k = \frac{1}{3}$

ii) $f(x) = \frac{3x^2}{\cos x}$

$u = 3x^2 \qquad v = \cos x$

$u' = 6x \qquad v' = -\sin x$

$$f'(x) = \frac{-3x^2(-\sin x) + 6x\cos x}{(\cos^2 x)}$$

$$= \frac{3x(x\sin x + 2\cos x)}{\cos^2 x}$$

OR

$f(x) = \frac{3x^2}{\cos x} = 3x^2 \mathrm{se}$

$u = 3x^2 \qquad v = \sec x$

$u' = 6x \qquad v' = \tan x$

(product rule)

$f'(x) = 3x^2(\tan x \sec x) +$

$= 3x\sec x(x\tan x +$

iii) $f(x) = (3x^3+5)e^x$

$u = 3x^3+5 \qquad v = e^x$

$u' = 9x^2 \qquad v' = e^x$

$$f'(x) = (3x^3+5)e^x + 9x^2e^x$$

$$= e^x(3x^3+9x^2+5)$$

b) $f(x) = \frac{x^2+3x}{x-5}$

$u = x^2+3x \qquad v = x-5$

$u' = 2x+3 \qquad v' = 1$

$$f'(x) = \frac{(x-5)(2x+3) - (x^2+3x)}{(x-5)^2}$$

$$= \frac{2x^2-7x-5-x^2-3x}{(x-5)^2}$$

$$= \frac{x^2-10x-15}{(x-5)^2}$$

$a = 1, \; b = -10, \; c = -15$

5 a) i) $\frac{\cos x}{\sin x} - \frac{\sin x}{1-\cos x} \equiv -\operatorname{cosec} x$

$$\text{LHS} \equiv \frac{\cos x(1-\cos x)}{\sin x(1-\cos x)} - \frac{\sin x(\sin x)}{(1-\cos x)(\sin x)}$$

$$\equiv \frac{\cos x - \cos^2 x - \sin^2 x}{\sin x(1-\cos x)}$$

$$\equiv \frac{\cos x - 1}{-\sin x(\cos x - 1)}$$

$$\equiv \frac{1}{-\sin x}$$

$$\equiv -\operatorname{cosec} x$$

$$\equiv \text{RHS}$$

$\square$

ii) $x \in \mathbb{R}$, $x \neq \pi k$, k is an integer.
(x can't be any multiple of π)

b) $\frac{\cos x}{\sin x} - \frac{\sin x}{1-\cos x} = 3 \Rightarrow -\operatorname{cosec} x = 3$

$$-\frac{1}{\sin x} = 3$$

$$-\frac{1}{3} = \sin x$$

$x = -0.340, 3.48, 5.94$

6 a) i) $f(x) = \frac{x}{x+2}$ let $u = x$, $v = x+2$
$u' = 1$ $v' = 1$

$$f'(x) = \frac{1(x+2) - 1(x)}{(x+2)^2}$$

$$= \frac{2}{(x+2)^2}$$

Section 2

1 a $= 2\sin x + c$ b $= -\frac{1}{4}\cos 4x + c$ c $= 3\cos(\frac{\pi}{3} - x) + c$ d $= \sec x + c$

e $= -\cot x + c$ f $= -4\,\text{cosec}\,\frac{1}{4}x + c$

2 a $= [\sin x]_0^{\frac{\pi}{2}}$
$= 1 - 0 = 1$

b $= [\frac{1}{3}\tan 3x]_{\frac{\pi}{4}}^{\frac{\pi}{3}}$
$= 0 - (-\frac{1}{3}) = \frac{1}{3}$

c $= [-\text{cosec}\,x]_{\frac{\pi}{2}}^{\frac{2\pi}{3}}$
$= -\frac{2}{\sqrt{3}} - (-1) = 1 - \frac{2}{3}\sqrt{3}$

3 a $\tan^2\theta = \sec^2\theta - 1$

b $\int \tan^2 x\ dx = \int (\sec^2 x - 1)\ dx = \tan x - x + c$

4 a $= \int \frac{1}{2}\sin 2x\ dx$
$= -\frac{1}{4}\cos 2x + c$

b $= \int (2 + 2\cos 6x)\ dx$
$= 2x + \frac{1}{3}\sin 6x + c$

c $= \int \frac{1}{\sin 2x} \times \frac{\cos x}{\sin x}\ dx$
$= \int \frac{1}{2\sin x\cos x} \times \frac{\cos x}{\sin x}\ dx$
$= \int \frac{1}{2}\text{cosec}^2 x\ dx$
$= -\frac{1}{2}\cot x + c$

5 a $= \frac{1}{4}(x^3 - 2)^4 + c$

b $= e^{\sin x} + c$

c $= \frac{1}{2}\int \frac{2x}{x^2+1}\ dx$
$= \frac{1}{2}\ln|x^2 + 1| + c$
$[= \frac{1}{2}\ln(x^2 + 1) + c]$

d $= -\int \cot^3 x\,(-\text{cosec}^2 x)\ dx$
$= -\frac{1}{4}\cot^4 x + c$

e $= \ln|1 + e^x| + c$
$[= \ln(1 + e^x) + c]$

f $= \frac{1}{4}\int \frac{4x^3}{(x^4 - 2)^2} + c$
$= \frac{1}{4} \times [-(x^4 - 2)^{-1}] + c$
$= -\frac{1}{4(x^4 - 2)} + c$

g $= \frac{1}{4}(\ln x)^4 + c$

h $= \frac{2}{3}\int \frac{3}{2}x^{\frac{1}{2}}(1 + x^{\frac{3}{2}})^2\ dx$
$= \frac{2}{3} \times \frac{1}{3}(1 + x^{\frac{3}{2}})^3 + c$
$= \frac{2}{9}(1 + x^{\frac{3}{2}})^3 + c$

6 a $= -\int_0^{\frac{\pi}{2}} (-\sin x)(1 + \cos x)^2\ dx$
$= -[\frac{1}{3}(1 + \cos x)^3]_0^{\frac{\pi}{2}}$
$= -\frac{1}{3}(1 - 8)$
$= \frac{7}{3}$

b $= -\frac{1}{2}\int_{-1}^{0} \frac{-2e^{2x}}{2 - e^{2x}}\ dx$
$= -\frac{1}{2}[\ln|2 - e^{2x}|]_{-1}^{0}$
$= -\frac{1}{2}[0 - \ln(2 - e^{-2})]$
$= \frac{1}{2}\ln(2 - e^{-2})$

c $= -\int_{\frac{\pi}{6}}^{\frac{\pi}{4}} (-\cot x\,\text{cosec}\,x)\,\text{cosec}^3 x\ dx$
$= -[\frac{1}{4}\text{cosec}^4 x]_{\frac{\pi}{6}}^{\frac{\pi}{4}}$
$= -\frac{1}{4}(4 - 16)$
$= 3$

d $= \frac{1}{2}\int_2^4 \frac{2x + 2}{x^2 + 2x + 8}\ dx$
$= \frac{1}{2}[\ln|x^2 + 2x + 8|]_2^4$
$= \frac{1}{2}(\ln 32 - \ln 16)$
$= \frac{1}{2}\ln 2$