Pure 16 SOLUTIONS

Section 2

1) a $u = x,\ \frac{du}{dx} = 1;\ \frac{dv}{dx} = e^x,\ v = e^x$

$\int xe^x\ dx = xe^x - \int e^x\ dx$ ✓

$= xe^x - e^x + c$

$= e^x(x - 1) + c$ ✓

b $u = 4x,\ \frac{du}{dx} = 4;\ \frac{dv}{dx} = \sin x,\ v = -\cos x$

$\int 4x \sin x\ dx = -4x\cos x - \int -4\cos x\ dx$ ✓

$= -4x\cos x + \int 4\cos x\ dx$

$= -4x\cos x + 4\sin x + c$ ✓

c $u = x,\ \frac{du}{dx} = 1;\ \frac{dv}{dx} = e^{-3x},\ v = -\frac{1}{3}e^{-3x}$

$\int \frac{x}{e^{3x}}\ dx = -\frac{1}{3}xe^{-3x} - \int -\frac{1}{3}e^{-3x}\ dx$ ✓

$= -\frac{1}{3}xe^{-3x} + \int \frac{1}{3}e^{-3x}\ dx$

$= -\frac{1}{3}xe^{-3x} - \frac{1}{9}e^{-3x} + c$

$= -\frac{1}{9}e^{-3x}(3x + 1) + c$ ✓

(6)

2)
$$u = e^x, \ \frac{du}{dx} = e^x; \ \frac{dv}{dx} = \sin x, \ v = -\cos x$$

$$\int e^x \sin x \ dx = -e^x \cos x - \int -e^x \cos x \ dx$$
$$= -e^x \cos x + \int e^x \cos x \ dx$$ ✓

for $\int e^x \cos x \ dx$, $\quad u = e^x, \ \frac{du}{dx} = e^x; \ \frac{dv}{dx} = \cos x, \ v = \sin x$

$$\int e^x \cos x \ dx = e^x \sin x - \int e^x \sin x \ dx$$ ✓

$$\therefore \int e^x \sin x \ dx = -e^x \cos x + e^x \sin x - \int e^x \sin x \ dx$$ ✓

$$2\int e^x \sin x \ dx = -e^x \cos x + e^x \sin x + c$$ ✓

$$\int e^x \sin x \ dx = \tfrac{1}{2} e^x(\sin x - \cos x) + c$$ ✓

(5)

3) **a** $u = \ln 2x, \ \frac{du}{dx} = \frac{1}{x}; \ \frac{dv}{dx} = 1, \ v = x$

$$\int \ln 2x \ dx = x \ln 2x - \int \frac{1}{x} \times x \ dx$$ ✓
$$= x \ln 2x - \int dx$$
$$= x \ln 2x - x + c$$
$$= x(\ln 2x - 1) + c$$ ✓

b $u = \ln x, \ \frac{du}{dx} = \frac{1}{x}; \ \frac{dv}{dx} = 3x, \ v = \frac{3}{2}x^2$

$$\int 3x \ln x \ dx = \tfrac{3}{2}x^2 \ln x - \int \frac{1}{x} \times \tfrac{3}{2}x^2 \ dx$$ ✓
$$= \tfrac{3}{2}x^2 \ln x - \int \tfrac{3}{2}x \ dx$$
$$= \tfrac{3}{2}x^2 \ln x - \tfrac{3}{4}x^2 + c$$
$$= \tfrac{3}{4}x^2(2 \ln x - 1) + c$$ ✓

c $u = (\ln x)^2, \ \frac{du}{dx} = 2(\ln x) \times \frac{1}{x}; \ \frac{dv}{dx} = 1, \ v = x$

$$\int (\ln x)^2 \ dx = x(\ln x)^2 - \int 2 \ln x \ dx$$

for $\int 2 \ln x \ dx$, $\quad u = \ln x, \ \frac{du}{dx} = \frac{1}{x}; \ \frac{dv}{dx} = 2, \ v = 2x$

$$\int 2 \ln x \ dx = 2x \ln x - \int 2 \ dx$$
$$= 2x \ln x - 2x + c$$ ✓

$$\therefore \int (\ln x)^2 \ dx = x(\ln x)^2 - (2x \ln x - 2x) + c$$
$$= x[(\ln x)^2 - 2 \ln x + 2] + c$$ ✓

(6)

4) a $u = x+2,\ \frac{du}{dx} = 1;\quad \frac{dv}{dx} = e^x,\ v = e^x$

$$\int_{-1}^{0} (x+2)e^x\ dx$$

$$= [(x+2)e^x]_{-1}^{0} - \int_{-1}^{0} e^x\ dx$$

$$= [(x+2)e^x - e^x]_{-1}^{0}$$

$$= (2-1) - (e^{-1} - e^{-1})$$

$$= 1$$

b $u = \ln(2x+3),\ \frac{du}{dx} = \frac{2}{2x+3};\quad \frac{dv}{dx} = 1,\ v = x$

$$\int_0^3 \ln(2x+3)\ dx$$

$$= [x\ln(2x+3)]_0^3 - \int_0^3 \frac{2x}{2x+3}\ dx$$

$$= [x\ln(2x+3)]_0^3 - \int_0^3 \frac{(2x+3)-3}{2x+3}\ dx$$

$$= [x\ln(2x+3)]_0^3 - \int_0^3 \left(1 - \frac{3}{2x+3}\right) dx$$

$$= [x\ln(2x+3) - x + \tfrac{3}{2}\ln|2x+3|]_0^3$$

$$= (3\ln 9 - 3 + \tfrac{3}{2}\ln 9) - (0 - 0 + \tfrac{3}{2}\ln 3)$$

$$= \tfrac{15}{2}\ln 3 - 3$$

c $u = e^{3x},\ \frac{du}{dx} = 3e^{3x};\quad \frac{dv}{dx} = \sin 2x,\ v = -\frac{1}{2}\cos 2x$

$$\int e^{3x}\sin 2x\ dx = -\tfrac{1}{2}e^{3x}\cos 2x - \int -\tfrac{3}{2}e^{3x}\cos 2x\ dx$$

$$= -\tfrac{1}{2}e^{3x}\cos 2x + \int \tfrac{3}{2}e^{3x}\cos 2x\ dx$$

for $\int \frac{3}{2}e^{3x}\cos 2x\ dx$, $\quad u = \frac{3}{2}e^{3x},\ \frac{du}{dx} = \frac{9}{2}e^{3x};\quad \frac{dv}{dx} = \cos 2x,\ v = \frac{1}{2}\sin 2x$

$$\int \tfrac{3}{2}e^{3x}\cos 2x\ dx = \tfrac{3}{4}e^{3x}\sin 2x - \int \tfrac{9}{4}e^{3x}\sin 2x\ dx$$

$$\therefore \int e^{3x}\sin 2x\ dx = -\tfrac{1}{2}e^{3x}\cos 2x + \tfrac{3}{4}e^{3x}\sin 2x - \int \tfrac{9}{4}e^{3x}\sin 2x\ dx$$

$$\tfrac{13}{4}\int e^{3x}\sin 2x\ dx = -\tfrac{1}{2}e^{3x}\cos 2x + \tfrac{3}{4}e^{3x}\sin 2x + c$$

$$\therefore \int_0^{\frac{\pi}{4}} e^{3x}\sin 2x\ dx = \tfrac{4}{13}[-\tfrac{1}{2}e^{3x}\cos 2x + \tfrac{3}{4}e^{3x}\sin 2x]_0^{\frac{\pi}{4}}$$

$$= \tfrac{4}{13}[(0 + \tfrac{3}{4}e^{\frac{3\pi}{4}}) - (-\tfrac{1}{2} + 0)]$$

$$= \tfrac{1}{13}(3e^{\frac{3\pi}{4}} + 2)$$

(9)

5) $u = \ln x \qquad \frac{du}{dx} = \frac{1}{x}$

$\frac{dv}{dx} = x^2 \qquad v = \frac{1}{3}x^3 \qquad \Rightarrow$

$\frac{1}{3}x^3 \ln x - \int \frac{1}{3}x^2 \, dx = \left[\frac{1}{3}x^3 \ln x - \frac{1}{9}x^3\right]_1^3$

$= (9\ln 3 - 3) - (0 - \frac{1}{9}) = 9\ln 3 - \frac{26}{9}$ (6)

6) (a) $u = x \qquad \frac{du}{dx} = 1$

$\frac{dv}{dx} = \cos 2x \qquad v = -\frac{1}{2}\sin 2x \qquad \Rightarrow$

$\frac{1}{2}x\sin 2x - \frac{1}{2}\int \sin 2x \, dx =$ (4)

$\frac{1}{2}x\sin 2x + \frac{1}{4}\cos 2x + c$

(b) $\frac{1}{2}x(2\sin x\cos x) + \frac{1}{4}(\cos^2 x - \sin^2 x) + c$

$= \frac{1}{2}x(2\sin x\cos x) + \frac{1}{4}(1 - 2\sin^2 x) + c$

$= \frac{1}{2}\sin x\left[2x\cos x - \sin x\right] + k$ (3)

$k = c + \frac{1}{4}$

Total 39