section 1

a) $x^2 - 10x + y^2 + 2y = 23$

$(x-5)^2 - 25 + (y+1)^2 - 1 = 23$

$(x-5)^2 + (y+1)^2 = 49$

i) centre: $(5, -1)$

ii) radius $= \sqrt{49} = 7$

b) $y = x + 2$

$x^2 - 10x + (x+2)^2 + 2(x+2) = 23$

$x^2 - 10x + x^2 + 4x + 4 + 2x + 4 = 23$

$2x^2 - 4x - 15 = 0$

$x = \frac{-(-4) \pm \sqrt{(-4)^2 - 4(2)(-15)}}{2 \times 2}$

$= \frac{4 \pm \sqrt{168}}{4}$

$= \frac{4 \pm 2\sqrt{34}}{4}$

$= 1 \pm \frac{\sqrt{34}}{2}$

$y = 1 \pm \frac{\sqrt{34}}{2} + 2$

$y = 3 \pm \frac{\sqrt{34}}{2}$

$\left(1 + \frac{\sqrt{34}}{2}, 3 + \frac{\sqrt{34}}{2}\right), \left(1 - \frac{\sqrt{34}}{2}, 3 - \frac{\sqrt{34}}{2}\right)$

2 a) $3\begin{pmatrix} 6 \\ -1 \end{pmatrix} + 5\begin{pmatrix} -3 \\ 4 \end{pmatrix} = \begin{pmatrix} 18 \\ -3 \end{pmatrix} + \begin{pmatrix} -15 \\ 20 \end{pmatrix}$

$= \begin{pmatrix} 3 \\ 17 \end{pmatrix}$

b) $\hat{\underline{i}} = \frac{\underline{a}}{|\underline{a}|}$ $\quad |\underline{a}| = \sqrt{(-3)^2 + (4)^2}$

$= 5$

$\hat{\underline{a}} = \begin{pmatrix} -3 \\ 4 \end{pmatrix} \div 5$

$= \begin{pmatrix} -0.6 \\ 0.8 \end{pmatrix}$

3 $x = \frac{1}{2t+1}$ $\quad y = \frac{2}{3-t}$

$2tx + x = 1$
$2tx = 1 - x$
$t = \frac{1-x}{2x}$

$\therefore y = \frac{2}{3-\left(\frac{1-x}{2x}\right)} \times \frac{2x}{2x}$

$y = \frac{4x}{6x-(1-x)}$

$y = \frac{4x}{7x-1}$

4a) $\sec^4 x - \tan^4 x \equiv \sec^2 x + \tan^2 x$

$LHS \equiv \sec^4 x - \tan^4 x$
$\equiv (\sec^2 x + \tan^2 x)(\sec^2 x - \tan^2 x)$ $\quad [\tan^2 x + 1 \equiv \sec^2 x \;\therefore\; \sec^2 x - \tan^2 x \equiv 1]$
$\equiv \sec^2 x + \tan^2 x$
$\equiv RHS$
□

b) $\sec^4 x - \tan^4 x = 5 + \tan^2 x$
$\sec^2 x + \tan^2 x = 5 + \tan^2 x$
$(\tan^2 x + 1) + \tan^2 x = 5 + \tan^2 x$
$\tan^2 x = 4$
$\tan x = \pm 2$

$\tan^{-1}(2) = 1.107, -2.034$ $\quad \tan^{-1}(-2) = -1.107, 2.036$
($-\pi$ / $+\pi$)

$x = -2.034, -1.107, 1.107, 2.034$

5) $5xy - y^3 = 7$

$u = 5x \quad v = y$
$u' = 5 \quad v' = \frac{dy}{dx}$

$5x\frac{dy}{dx} + 5y - 3y^2\frac{dy}{dx} = 0$

$\frac{dy}{dx}(5x - 3y^2) = -5y \Rightarrow \frac{dy}{dx} = \frac{5y}{3y^2 - 5x}$

6) ~~$y = a^x \ln a$~~

$y = a^x$

$\ln y = \ln a^x$

$\ln y = x \ln a$ } differentiate

$\frac{1}{y}\frac{dy}{dx} = \ln a$

$\frac{dy}{dx} = y \ln a$ } $y = a^x$

$\frac{dy}{dx} = a^x \ln a$

Section 2

1 **a** $\dfrac{3x+5}{(x+1)(x+3)} \equiv \dfrac{A}{x+1} + \dfrac{B}{x+3}$

$3x + 5 \equiv A(x+3) + B(x+1)$

$x = -1 \Rightarrow 2 = 2A \Rightarrow A = 1$

$x = -3 \Rightarrow -4 = -2B \Rightarrow B = 2$

$\therefore \dfrac{3x+5}{(x+1)(x+3)} \equiv \dfrac{1}{x+1} + \dfrac{2}{x+3}$

b $= \int \left(\dfrac{1}{x+1} + \dfrac{2}{x+3}\right) dx$

$= \ln|x+1| + 2\ln|x+3| + c$ (8)

2 $\dfrac{3}{(t-2)(t+1)} \equiv \dfrac{A}{t-2} + \dfrac{B}{t+1}$

$3 \equiv A(t+1) + B(t-2)$

$t = 2 \Rightarrow 3 = 3A \Rightarrow A = 1$

$t = -1 \Rightarrow 3 = -3B \Rightarrow B = -1$

$\therefore \int \dfrac{3}{(t-2)(t+1)}\, dt$

$= \int \left(\dfrac{1}{t-2} - \dfrac{1}{t+1}\right) dt$

$= \ln|t-2| - \ln|t+1| + c$

$= \ln\left|\dfrac{t-2}{t+1}\right| + c$ (7)

3 **a)** $\dfrac{14-x}{x^2+2x-8} \equiv \dfrac{A}{x+4} + \dfrac{B}{x-2}$

$14 - x \equiv A(x-2) + B(x+4)$

$x = -4 \Rightarrow 18 = -6A \Rightarrow A = -3$

$x = 2 \Rightarrow 12 = 6B \Rightarrow B = 2$

$\therefore \int \dfrac{14-x}{x^2+2x-8}\, dx$

$= \int \left(\dfrac{2}{x-2} - \dfrac{3}{x+4}\right) dx$

$= 2\ln|x-2| - 3\ln|x+4| + c$ (7)

b) $\dfrac{3x^2-5}{x^2-1} \equiv A + \dfrac{B}{x+1} + \dfrac{C}{x-1}$

$3x^2 - 5 \equiv A(x+1)(x-1) + B(x-1) + C(x+1)$

$x = -1 \Rightarrow -2 = -2B \Rightarrow B = 1$

$x = 1 \Rightarrow -2 = 2C \Rightarrow C = -1$

coeffs of x^2 $\Rightarrow A = 3$

$\therefore \int \dfrac{3x^2-5}{x^2-1}\, dx = \int \left(3 + \dfrac{1}{x+1} - \dfrac{1}{x-1}\right) dx$

$= 3x + \ln|x+1| - \ln|x-1| + c = 3x + \ln\left|\dfrac{x+1}{x-1}\right| + c$ (7)

c) $\dfrac{x(4x+13)}{(2+x)^2(3-x)} \equiv \dfrac{A}{2+x} + \dfrac{B}{(2+x)^2} + \dfrac{C}{3-x}$

$x(4x+13) \equiv A(2+x)(3-x) + B(3-x) + C(2+x)^2$

$x = -2 \Rightarrow -10 = 5B \Rightarrow B = -2$

$x = 3 \Rightarrow 75 = 25C \Rightarrow C = 3$

coeffs of x^2 $\Rightarrow 4 = -A + C \Rightarrow A = -1$

$\therefore \int \dfrac{x(4x+13)}{(2+x)^2(3-x)}\, dx = \int \left(\dfrac{3}{3-x} - \dfrac{1}{2+x} - \dfrac{2}{(2+x)^2}\right) dx$

$= -3\ln|3-x| - \ln|2+x| + 2(2+x)^{-1} + c$ (9)

4 **a)** $\dfrac{x+3}{x(x+1)} \equiv \dfrac{A}{x} + \dfrac{B}{x+1}$

$x + 3 \equiv A(x+1) + Bx$

$x = 0 \Rightarrow A = 3$

$x = -1 \Rightarrow 2 = -B \Rightarrow B = -2$

$\therefore \int_1^3 \dfrac{x+3}{x(x+1)}\, dx = \int_1^3 \left(\dfrac{3}{x} - \dfrac{2}{x+1}\right) dx$

$= [3\ln|x| - 2\ln|x+1|]_1^3$

$= (3\ln 3 - 2\ln 4) - (0 - 2\ln 2) = 3\ln 3 - 2\ln 2$ (6)

b)

$$\frac{5x+7}{(x+1)^2(x+3)} \equiv \frac{A}{x+1} + \frac{B}{(x+1)^2} + \frac{C}{x+3}$$

$5x + 7 \equiv A(x + 1)(x + 3) + B(x + 3) + C(x + 1)^2$

$x = -1 \quad \Rightarrow \quad 2 = 2B \quad \Rightarrow \quad B = 1$

$x = -3 \quad \Rightarrow \quad -8 = 4C \quad \Rightarrow \quad C = -2$

coeffs of $x^2 \quad \Rightarrow \quad 0 = A + C \quad \Rightarrow \quad A = 2$

$$\therefore \int_0^1 \frac{5x+7}{(x+1)^2(x+3)}\,dx = \int_0^1 \left(\frac{2}{x+1} + \frac{1}{(x+1)^2} - \frac{2}{x+3}\right) dx$$

$$= \left[2\ln|x+1| - (x+1)^{-1} - 2\ln|x+3|\right]_0^1$$

$$= (2\ln 2 - \tfrac{1}{2} - 2\ln 4) - (0 - 1 - 2\ln 3)$$

$$= \tfrac{1}{2} - 2\ln 2 + 2\ln 3$$

9

Question Number	Scheme					Marks
5 (a)	x	1	1.25	1.5	1.75	2
	y	1.414	**1.601**	1.803	2.016	2.236
	{At $x = 1.25$,} $y = 1.601$ (only)			**1.601 (May not be in the table and can score if seen as part of their working in (b))**		B1 cao
						[1]
(b)	$\frac{1}{2}\times 0.25;\times\{1.414 + 2.236 + 2(\text{their } 1.601 + 1.803 + 2.016)\}$					B1; M1 A1ft
	B1; for using $\frac{1}{2}\times 0.25$ or $\frac{1}{8}$ or equivalent.		M1: Structure of $\{\ldots\ldots\ldots\ldots\}$		A1ft: for the correct expression as shown following through candidate's y value found in part (a).	
	M1 requires the correct structure for the y values. It needs to contain first y value **plus** last y value and the second bracket to be multiplied by 2 and to be the summation of the remaining y values in the table with no additional values. If the only mistake is a copying error or is to omit one value from 2(.....) bracket this may be regarded as a slip and the M mark can be allowed (nb: an extra repeated term, however, forfeits the M mark). M0 if any values used are x values instead of y values. A1ft: for the correct underlined expression as shown following through candidate's y value found in part (a). Bracketing mistakes: e.g. $\left(\frac{1}{2}\times\frac{1}{4}\right)(1.414 + 2.236) + 2(\text{their } 1.601 + 1.803 + 2.016)\ (=11.29625)$ $\left(\frac{1}{2}\times\frac{1}{4}\right)1.414 + 2.236 + 2(\text{their } 1.601 + 1.803 + 2.016)\ (=13.25275)$ Both score B1 M1 A0 unless the final answer implies that the calculation has been done correctly (then full marks could be given). **Alternative:** Separate trapezia may be used, and this can be marked equivalently. $\left[\frac{1}{8}(1.414+1.601) + \frac{1}{8}(1.601+1.803) + \frac{1}{8}(1.803+2.016) + \frac{1}{8}(2.016+2.236)\right]$ B1 for $\frac{1}{8}$ (aef), M1 for correct structure, 1st A1ft for correct expression, ft their 1.601					
	$\{=\frac{1}{8}(14.49)\} = 1.81125$		1.81 or awrt 1.81			A1
	Correct answer only in (b) scores no marks **If required accuracy is not seen in (a), full marks can still be scored in (b) (e.g. uses 1.6)**					
						[4]
						Total 5

Question Number	Scheme		Marks
6	$y = 8 - 2^{x-1}$, 0 ,, x ,, 4		
(a)	7	7	B1 cao
			[1]
(b)	$\left(\int_0^4 \left(8 - 2^{x-1}\right)dx \approx\right) \frac{1}{2}\times 1;\times\{7.5+2("\text{their } 7" + 6 + 4)+0\}$	Outside brackets $\frac{1}{2}\times 1$ or $\frac{1}{2}$ For structure of trapezium rule $\{.....\}$ for a candidate's y-ordinates.	B1; M1
	$\left\{=\frac{1}{2}\times 41.5\right\} = 20.75$ o.e.	20.75	A1 cao
			[3]
(c)	Area $(R) = "20.75" - \frac{1}{2}(7.5)(4)$		M1
	$= 5.75$	5.75	A1 cao
			[2]
			6

7 2 (a)	$\sqrt{7}$ and $\sqrt{15}$	B1 **(1)**
(b)	Area $(R) \approx \frac{1}{2}\times 2;\times\left\{\sqrt{3}+2\left(\sqrt{7}+\sqrt{11}+\sqrt{15}\right)+\sqrt{19}\right\}$ Note decimal values are $\frac{1}{2}\times 2;\times\left\{\sqrt{3}+\sqrt{19}+2\left(\sqrt{7}+\sqrt{11}+\sqrt{15}\right)\right\} = \frac{1}{2}\times 2;\times\{6.0909..+19.6707...\}$ $= 1\times 25.76166865... = 25.76166... = \underline{25.76}$ (2dp)	B1; M1 A1 cao **(3)**
(c)	underestimate	B1 **(1)**
		[5]

TOTAL: 69